A Generating Method for Internet Topology with Multi-ASes and Multi-tiers*

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Abstract. We have understood more precise about the Internet characteristics by the rapid development of the measure, which help us to design new network. In this paper, we analysis the characteristics, including the power-law, hiberarchy and community structure mainly. We propose a new model based on hierarchical framework in accordance with the actual process of internet construct. The model can generate a network topology with multi-ASes and multi-Tiers(MAMT). The result can be applied on new protocol design or network performance evaluate.

Keywords: Internet, topology modeling, complex network, router level.

1 Introduction

Nowadays, Internet has become more and more complex. The research on the generating method of Internet topology is very important for (1) some new applications or experiments is not allow on Internet, for some of them will hurt Internet itself, such as the simulation of worm spread in the large scale network. The simulation require a virtue network topology for experiment. (2) some protocols require different network topology to evaluate their performance, Such as multicast protocol.(3)The research on network topology is important to the network security. We require to control the action on network online from the nation security point, such as the project of NMS(Network Modeling and Simulation) [1]in charged by DARPA.

The earliest Internet topology model is Waxman [2], proposed by Waxman in 1988. It is a random model and applied for many years till 1996. In 1996, Doar proposed Tiers model [3], which reflect the hiberarchy character of Internet. Zegura et. proposed Transit-Stub model [4] for the hiberarchy character. In 1999, Faloutsos et al. discover that there exist power-law in the topology of Internet [5], which arouse a climax of research on Internet topology. The arithmetic of power-law can be classed into two types: one arithmetic assign the node degree with power-low, such as PLOD [6] and PLGR(Model A) [7], and another make the node degree satisfied power-low by simulating the process of internet evolving, such as BA [8], ESP [9] and GPL [10].

In fact, a good arithmetic of network topology should not only reflect the characters of the network, but also reflect the fact of the network construct process and satisfy the

* This work was supported in part by the grants from National Basic Research Program of China (973 Program) with No.2007CB307102 and National Hi-tech Research and Development Program of China (863 program) with No. 2008AA01A323.

J. Zhou (Ed.): Complex 2009, Part II, LNICST 5, pp. 1363 – 1373, 2009.

requires of network design, such as design some protocols. BA shows the mechanism of power-low of topology: growth and preferential attachment. [11] discovery that preferential attachment is not effect in whole network but in local world of the node [12] describe a model of Internet construct with multi-local-world. It reflects the characters more similar to the real internet data.

The topology of Internet has not only power-law character, but also the hiberarchy and community structure. We often divided the network into some autonomous system. When we design new routing protocol(include BGP and IGP), the topology of multi-ASes and multi-Tiers is demand.

In this paper, we aimed to propose a new model to reflect the process of internet construct and satisfy the demand of some new protocols.

The remainder of the paper is organized as follows. We first introduce the main character and explain the reason to the point of network construct, then we present our MAMH model of network topology generating. Finally we simulate it and the result show that the model reflect the characters.

2 Topology Characters Analysis

As a complex network, Internet shows some complex characters, such as hiberarchy, power-law and modularity.

2.1 Hiberarchy

Whereas Internet is a self-organize system, Internet Service Provider(ISP) provide the service. The inter-connection between ISPs is hiberarchy. A strict hiberarchy is that a small global ISP at the top, the lower is national ISP, regional ISP and local ISP.

But in fact, with the new hosts, routers and links appear or disappear, or to acquire the shorter path, ISP will make some shortcut links between the routers(or hosts),which will destroy the Internet hiberarchy to some extent.

To analysis the hiberarchy of internet topology, the model of Tiers [3]and Transit-stub[4] is proposed.

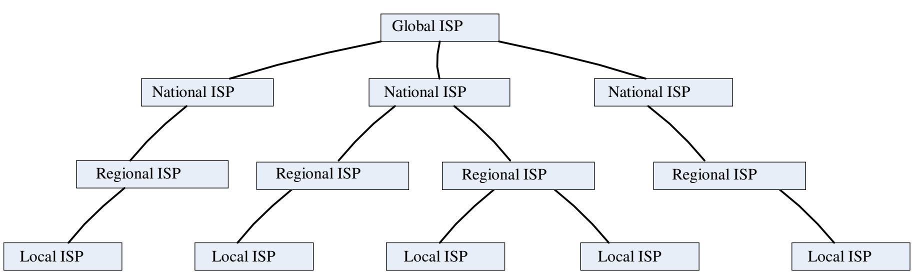

Fig. 1. Strict hiberarchy of Internet topology

2.2 Power Law

In 1999, Faloutsos discover that the Internet topology present power law from the data of NLANR(National Lab for Applied Network Research), three pieces of BGP data collected in 1997 to 1998 and a piece of traceroute data in 1995.

Barabasi and Albert analysis the mechanism and declare that growth and preferential attachment is the two basic reason. And the model which can generate the topology with power law is called model BA. For some special condition, more and more model is made based on model BA[6-10].

2.3 Community Structure

Community structure in complex network refer that there exist some flocks, nodes in the same flock connect very compactly, but between different flock connect thin. There is clear borderline between communities or community and non-community.

[13] shows that there is community structure in Internet topology.

2.4 The Reason Anslysis

In fact, the three character is not isolated, but a entia. The topology of internet has not only power-law character, but also the hiberarchy and community structure. We can explain it with the style of internet construction.

One of the internet traits is self-organize. But the process of constructing internet is by some corporations with special principle, which will bring the next three characters in network topology:

- Hiberarchy of network programming: to the ISP, it is usually obey on the hiberarchy principle of group company to divided company to branch company, which will arouse the hiberarchy of network topology.
- Modularization of network construction: for the protection of economy and guarantee of QoS and satisfaction of network security, the company will add the nodes(routers and other network equipments) and links in the network of itself.
- preferential attachment of network construction: the connection, whether the network in one company or between different company, will attachment with the node of larger degree. When a node connect with another node in other company, 'the other company' usually has more users.

A typical topology can be showed with the next sketch map.

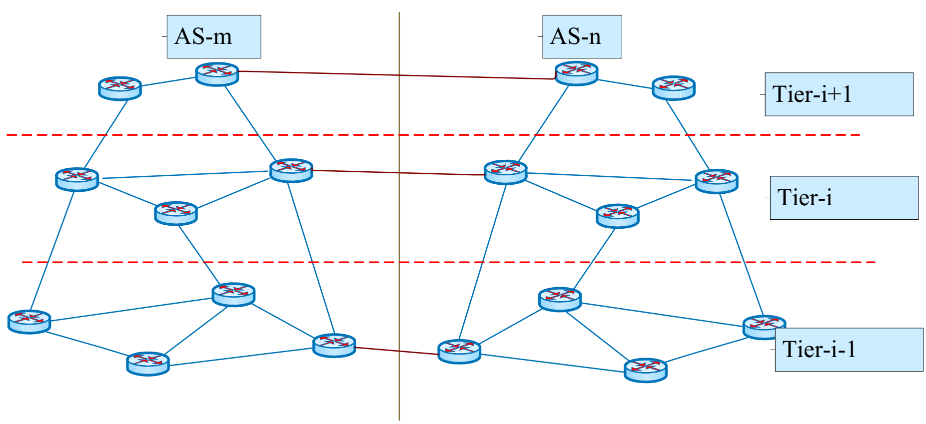

Fig. 2. Real network with hiberarchy

3 A Multi-ASes-Multi-Tiers Internet Topology Model(MAMT)

We propose the next model of network topology evolving, which called MAMT in this paper.

3.1 MAMT

Initial condition: a network topology has m_0 nodes and e_0 edges, which located in m AS and n tiers.

Step 1: A new node added the $s-th$ AS with probability p ;

Step 2: Add this node to the $t-th$ tier of $s-th$ AS with probability q ;

We call the $t-th$ tier of $s-th$ AS Ω_{st}

Step 3: Operating in same AS and same tier, which reflect the change of ISP programming the MAN :

1) add m_1 edges form the new node to the other node in Ω_{st} , with the probability α_{at} , the other node is chosen with the probability given by

$$P_{\Omega_{st}}(k_i) = \frac{k_i}{\sum_{j \in \Omega_{st}} k_j} \tag{1}$$

This process repeat m_1 .

2) delete m_2 edges in the same AS and same tier, with the probability β_{at} . To do this, first,a node is selected randomly, and then a node is chosen with the probability given by

$$P_{\Omega_{st}}(k_i) = \frac{1}{N_{\Omega_{st}} - 1}\left(1 - \frac{k_i}{\sum_{j \in \Omega_{st}} k_j}\right) \tag{2}$$

Delete the edge connected the two node. This process repeat m_2 .

Step 4: Operating between tiers but in same AS, which reflect the change of ISP programming the WAN.

1) add m_3 edges to the next up tier in the same AS with the probability α_a . To do this, first,a node is selected randomly, and then a node is chosen with the probability given by (1), add anew edge between the two node. This process repeat m_3 .

2) delete m_4 edges in the same AS and same tier, with the probability β_a. To do this, first, a node is selected randomly in Ω_{st}, and then a node in $\Omega_{s,t-1}$ is chosen with the probability given by(2). This process repeat m_4.

Step 5: Operating between ASes, which reflect the connecting the change that ISPs inter-connect

1) add m_5 edges to the next up tier in the same AS with the probability α_t. To do this, first,a node in Ω_{st} is selected randomly, and then a node in the different AS is chosen with the probability given by,

$$p(a_j) = \frac{N^2(a_j)}{\sum_i N^2(a_i)} \tag{3}$$

add a new edge between the two node. This process repeat m_5.

For Eqs.(3), the benefit of ISP is squared to the number of users. We assume the number of users is direct ratio to the number of routers.

2) add m_6 edges to the next up tier in the same AS with the probability β_t. To do this, first,a node in Ω_{st} is selected randomly, and then a node in the different AS is chosen with the probability given by,

$$P(a_j) = 1 - \frac{N^2(a_j)}{\sum_i N^2(a_i)} \tag{4}$$

Delete the exist edge between the two node. This process repeat m_6.

In the up process $\alpha_{at} + \beta_{at} + \alpha_a + \beta_a + \alpha_t + \beta_t = 1$.

3.2 Arithmetic Analysis

We analysis the arithmetic by mean-field theory. One can obtain the degree distribution of a node i in the Ω_{st} can be derived analytically as follows.

$k_i(t)$ is the degree of node i at time t and we assume it is continuous.

Step 1: A new node added the $s-th$ AS with probability p;

$$\frac{\partial k_i}{\partial t} = 0 \tag{5}$$

Step 2: Add this node to Ω_{st} with probability q;

$$\frac{\partial k_i}{\partial t} = 0 \tag{6}$$

Step 3: Operating in Ω_{st} :

1) add m_1 edges form the new node to the other node in Ω_{st} , with the probability α_{at}

$$\left(\frac{\partial k_i}{\partial t}\right) = \frac{\alpha_{at} m_1}{mn} \frac{k_i}{\sum_{j \in \Omega_{st}} k_j} \tag{7}$$

2) delete m_2 edges in the same AS and same tier, with the probability β_{at} .

$$\frac{\partial k_i}{\partial t} = -\frac{\beta_{at} m_2}{mn} \left[\frac{1}{N_{\Omega_{st}}} + \left(1 - \frac{1}{N_{\Omega_{st}}}\right) \frac{1}{N_{\Omega_{st}} - 1} \left(1 - \frac{k_i}{\sum_{j \in \Omega_{st}} k_j}\right) \right] \tag{8}$$

Step 4: Operating between tiers but in same AS

1)add m_3 edges to the next up tier in the same AS with the probability α_a

$$\frac{\partial k_i}{\partial t} = \frac{\alpha_a m_3}{mn} \frac{1}{N_{\Omega_{st}}} + \frac{\alpha_a m_3}{mn - 1} \cdot \frac{k_i}{\sum_{j \in \Omega_{st}} k_j} \tag{9}$$

2) delete m_4 edges in the same AS and same tier, with the probability β_a

$$\frac{\partial k_i}{\partial t} = -\frac{\beta_a m_4}{mn} \frac{1}{N_{\Omega_{st}}} - \frac{\beta_a m_4}{mn - 1} \left(1 - \frac{k_i}{\sum_{j \in \Omega_{st}} k_j}\right) \tag{10}$$

Step 5: Operating between ASes.

1) add m_5 edges to the next up tier in the same AS with the probability α_t

$$\frac{\partial k_i}{\partial t} = \frac{\alpha_t m_5}{mn} \frac{1}{N_{\Omega_{st}}} + \frac{\alpha_t m_5}{mn - 1} \frac{N_{a_i}^2}{\sum_j N_{a_j}^2} \frac{k_i}{\sum_j k_j} \tag{11}$$

2) add m_6 edges to the next up tier in the same AS with the probability β_t

$$\frac{\partial k_i}{\partial t} = -\frac{\beta_t m_6}{mn}\frac{1}{N_{\Omega_{st}}} - \frac{\beta_t m_6}{mn-1}\left(1-\frac{N_{a_i}^2}{\sum_j N_{a_j}^2}\right)\left(1-\frac{k_i}{\sum_j k_j}\right) \tag{12}$$

By combining Eqs(5~12). Together, one has

$$\frac{\partial k_i}{\partial t} = \frac{\alpha_{at} m_1}{mn}\frac{k_i}{\sum_{j\in\Omega_{st}} k_j} - \frac{\beta_{at} m_2}{mn}\left[\frac{1}{N_{\Omega_{st}}} + \left(1-\frac{1}{N_{\Omega_{st}}}\right)\frac{1}{N_{\Omega_{st}}-1}\left(1-\frac{k_i}{\sum_{j\in\Omega_{st}} k_j}\right)\right]$$

$$+\frac{\alpha_a m_3}{mn}\frac{1}{N_{\Omega_{st}}} + \frac{\alpha_a m_3}{mn-1}\cdot\frac{k_i}{\sum_{j\in\Omega_{st}} k_j}$$

$$-\frac{\beta_a m_4}{mn}\frac{1}{N_{\Omega_{st}}} - \frac{\beta_a m_4}{mn-1}\left(1-\frac{k_i}{\sum_{j\in\Omega_{st}} k_j}\right) + \frac{\alpha_t m_5}{mn}\frac{1}{N_{\Omega_{st}}}$$

$$+\frac{\alpha_t m_5}{mn-1}\frac{N_{a_i}^2}{\sum_j N_{a_j}^2}\frac{k_i}{\sum_j k_j} - \frac{\beta_t m_6}{mn}\frac{1}{N_{\Omega_{st}}} - \frac{\beta_t m_6}{mn-1}\left(1-\frac{N_{a_i}^2}{\sum_j N_{a_j}^2}\right)\left(1-\frac{k_i}{\sum_j k_j}\right)$$

$$= \frac{k_i}{\sum_j k_j}\left(\frac{\alpha_{at} m_1}{mn} + \frac{\alpha_a m_3}{mn-1} + \frac{\beta_a m_4}{mn-1} + \frac{\beta_{at} m_2}{mnN_{\Omega_{st}}} + \left(\frac{\alpha_t m_5}{mn-1} - \frac{\beta_t m_6}{mn-1}\right)\cdot\frac{N_{a_i}^2}{\sum N_{a_j}^2}\right)$$

$$+\left((-2\beta_{at} m_2 + \alpha_a m_3 - \beta_a m_4 + \alpha_t m_5 - \beta_t m_6)\cdot\frac{1}{mn}\cdot\frac{1}{N_{\Omega_{st}}} + \frac{\beta_t m_6}{mn-1}\cdot\frac{N_{a_i}^2}{\sum N_{a_j}^2}\right)$$

$$+\left(-\frac{\beta_a m_4}{mn-1} - \frac{\beta_t m_6}{mn-1}\right)$$

$$\triangleq \frac{k_i}{\sum_j k_j}\left(P + \frac{Q}{N_{\Omega_{st}}} + R\cdot\frac{N_{a_i}^2}{\sum N_{a_j}^2}\right) + \left(\frac{S}{N_{\Omega_{st}}} + T\cdot\frac{N_{a_i}^2}{\sum N_{a_j}^2} + K\right) \tag{13}$$

At step t,

$$\sum_{j\in\Omega_{st}} k_j = [e_{st} + (\alpha_{at}m_1 + \alpha_a m_3 + \alpha_t m_5 - \beta_{at}m_2 - \beta_a m_4 - \beta_t m_6)t]/mn$$

$$\doteq (\alpha_{at}m_1 + \alpha_a m_3 + \alpha_t m_5 - \beta_{at}m_2 - \beta_a m_4 - \beta_t m_6)t/mn \quad \text{(for large } t\text{)}$$

$$\triangleq \frac{A_1 + ht}{B} \tag{14}$$

$$N_{\Omega_{st}} = m_{st} + tpq/mn \doteq tpq/mn \triangleq dt \quad \text{(for large } t\text{)} \tag{15}$$

$$N_{a_i} = \sum_s m_{st} + tpq/m \doteq tpq/m \triangleq ndt \quad \text{(for large } t\text{)} \tag{16}$$

$$\frac{N_{a_i}^2}{\sum_j N_{a_j}^2} \triangleq \frac{At^2 + Bt + C}{Dt^2 + Et + F} = \frac{A}{D} = \frac{1}{m} \tag{17}$$

Eqs.(13) at large time t is

$$\frac{\partial k_i}{\partial t} = \frac{k_i}{\sum_j k_j}\left(P + \frac{Q}{N_{\Omega_{st}}} + R\cdot\frac{N_{a_i}^2}{\sum N_{a_j}^2}\right) + \left(\frac{S}{N_{\Omega_{st}}} + T\cdot\frac{N_{a_i}^2}{\sum N_{a_j}^2} + K\right)$$

$$= \frac{B}{ht}\cdot k_i\left(P + \frac{Q}{dt} + \frac{R}{m}\right) + \left(\frac{S}{dt} + \frac{T}{m} + K\right)$$

$$= \frac{B}{ht}\cdot k_i\left(P' + \frac{Q}{dt}\right) + \left(K' + \frac{S}{dt}\right)$$

$$= \frac{B}{ht}\cdot k_i P' + \left(K' + \frac{S}{dt}\right)$$

$$\triangleq \frac{P'' k_i}{t} + \left(K' + \frac{S'}{t}\right) \tag{18}$$

One obtains from Eqs.(18):

$$k_i(t) = Ct^{P''} - \frac{K'}{P''-1}t - \frac{S'}{P''} \approx Ct^{P''} \text{ (for large } t\text{)} \tag{19}$$

Since $k_i(t_i) = m_1$

So that

$$k_i(t) = m_1 \cdot \left(\frac{t}{t_i}\right)^{P''} \tag{20}$$

The probability density of t_i is

$$p_i(t_i) = \frac{1}{m_0 + tp} \tag{21}$$

So that

$$\begin{aligned} p(k_i(t) < k) &= p(m_i \cdot \left(\frac{t}{t_i}\right)^{P''} < k) = p(t_i > (\frac{m_i}{k})^{1/P''} t) \\ &= 1 - p(t_i < (\frac{m_i}{k})^{1/P''} t) \\ &= 1 - (\frac{m_1}{k})^{1/P''} t \cdot \frac{1}{m_0 + tp} \end{aligned} \tag{22}$$

Using $p(k) = \frac{\partial(p(k_i(t) < k))}{\partial k}$ one has

$$p(k) = \frac{m_1^{1/P''} t}{(m_0 + tp)P''} \cdot k^{-\frac{1}{P''}-1} \tag{23}$$

Where $P'' = \dfrac{\alpha_{at} m_1 + mn(\alpha_a m_3 + \beta_a m_4 + \frac{\alpha_t m_5 - \beta_t m_6}{m}) / (mn-1)}{\alpha_{at} m_1 + \alpha_a m_3 + \alpha_t m_5 - \beta_{at} m_2 - \beta_a m_4 - \beta_t m_6}$

The power-law of internet is 2~3 from the real data. So P'' should satisfied $\frac{1}{2} < P'' < 1$, one obtains

$$\begin{cases} (mn-1)\alpha_{at} m_1 + (2m^2 n - mn + 1)\alpha_a m_3 + (2m^2 n + mn - 1)\beta_a m_4 + (mn+1)\alpha_t m_5 \\ +(mn-1)(\beta_{at} m_2) > (mn+1)\beta_t m_6 \\ \alpha_a m_3 + (2mn-1)\beta_a m_4 + (mn-n-1)\beta_t m_6 + (mn-1)\beta_{at} m_2 < (mn-n-1)\alpha_t m_5 \end{cases} \tag{24}$$

3.3 Simulation

In order to validate the topology characters, let $m_0 = 10$, $e_0 = 19$, $m = 9$ $n = 3$, $p = 0.2$, $q = 0.3$, $\alpha_{at} = 0.3$, $\beta_{at} = 0.1$, $\alpha_a = 0.2$, $\beta_a = 0.1$, $\alpha_t = 0.25$,

$\beta_t = 0.05$. $m_1 = m_2 = m_3 = m_4 = m_6 = 1$, $m_5 = 2$. We obtain three topology with nodes number 1000.

The topology has hiberarchy natural for each new node has special tier.

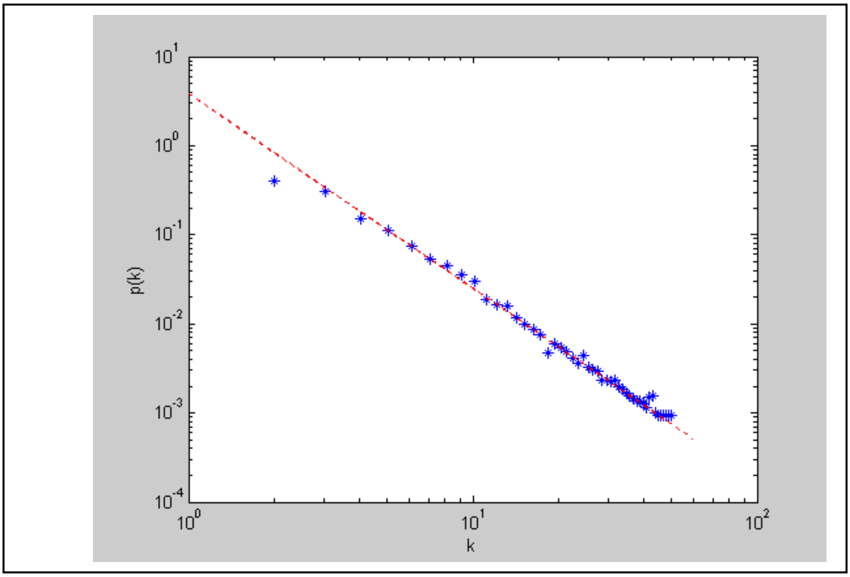

Fig. 3. The degree of a network generating by MAMT (red line is a beeline with slope -2.16)

The degree of nodes is figure as fig.3. It shows that the topology satisfy power law.

We generating another three topology with node number 2000, 5000 and 10000. Using the define of modularity in [13], the result about the modularity of the network can be describe to the next table.

Table 1. Average modularity of the simulation topology

Node scale	Average modularity
2000	0.408
5000	0.426
10000	0.455

The result shows that the topology generating by MAMT has power low character and modularity.

4 Conclusion

We analysis the characters of internet and derived from the process of internet construct. Our algorithm can obtain a MAMT topology, which satisfy power low and has modularity. It is helpful to design new type network and new routing protocol.

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