Stabilities of Stock States in Chinese Stock Markets

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Abstract. We study the evolution of the correlation-based clusters of stocks, which usually accord with business groups. By segmenting the whole time series into several overlapping segments, we trace the dynamical evolution of each business sectors in terms of the multi-factor model and especially treat the stock prices of Shanghai composites that are not incorporated into developed markets of the financial time stock exchange index.

Keywords: random matrix theory, correlation matrix, multi-factor model.

1 Introduction

A financial market is most exciting among other complex systems [1-5] because of the fact that its constituents are unceasingly increased and at times behaved in an unpredictable way. Especially, if they may be judged and adjusted by the environment and the condition that the agents have just made, the complexity emerges along with these. In fact, the market has a hierarchical structure of investors, such as the banks, pensions, mutual funds, hedge funds, and individual investors (with a small amount of capital). They are interwoven in a complicated manner via various debts and competing to win the financial game by using their own portfolio strategies. Owing to the globalization of financial markets, invested money can freely go beyond the border. A financial market allows for various investment horizons, which denote the time of holding shares of a specific stock. The long-positions are mostly aimed for gaining from dividends. However, there are many short-term investment strategies, such as tick-by-tick, minutely, daily, etc. One fact is that all these things are incorporated in price fluctuations.

Apart from investors, there is a sea of portfolios, each of which incorporates many stocks in order to control latent risks. The components of a portfolio are usually consisted of anti-correlated stocks because, in that case, the future risk can be reduced to a limit. However, the market has other means to control risks

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by using derivative securities, such as futures, options, swaps, etc [6-9]. The existence of derivatives increases the complexity of a financial market and yields the variety of investment strategies. For these reasons, the cross-correlation matrix approach to a financial market dynamics is meaningful and sufficiently justified. Previous works [1-5] have shown that a stock market has a hierarchical structure based on inter-dependence among stocks. This inter-dependence is probably due to business groups and the recognition by traders. According to behavioral economics [10], humans tend to take a similar thing when he/she is exposed to an unfamiliar environment. This behavior is termed by heuristics [11], and the implicit hierarchical structure of a financial market can be due to all these things mentioned so far.

2 Multi-factor Model and Numerical Calculations

We examine the evolution of business groups over time by using the stock prices of Shanghai composites, which is composed by 696 stocks with 1400 returns. According to the financial time stock exchange index, this stock market can be considered as the secondary emerging market, in which money flows occur more frequent than in developed market because of the recognition of foreign investors. The foreign investors ordinarily think of emerging markets as checking accounts, and they can withdraw their invested money whenever it is necessary. We can check the effects by tracing out the change of each business group over time. To this end, we take the multi-factors model, which is a basic method of modeling the stock price dynamics. For any stock i this model is represented in terms of

$$r_i(t) = \alpha_i + \sum_{j=1}^{N_g} \beta_{ij} M_j(t) + \epsilon_i(t), \tag{1}$$

where N_g denotes the number of factors, and α_i and β_{ij} are real parameters. The noise terms $\epsilon_i(t)$ of different stocks are assumed to be uncorrelated. Herein, N_g is determined by the number of eigenvalues deviating from the predictions of the random matrix theory about the empirical cross-correlation matrices. However, we consider 4 large eigenvalues including the largest one, which is known to indicate the market-wide effect.

In order to determine N_g, we have to solve the characteristic equation of the cross-correlation matrix C expressed by

$$C = \frac{1}{L} G G^T \tag{2}$$

where T denotes the transposition and G denotes the $N \times L$ normalized return matrix. And the characteristic equation is given as

$$det(C - \lambda \mathbf{1}) = 0 \tag{3}$$

where $\mathbf{1}$ denotes the identity matrix. Also, for comparison, we consider a random correlation matrix

$$C_{rm} = \frac{1}{L} A A^T \tag{4}$$

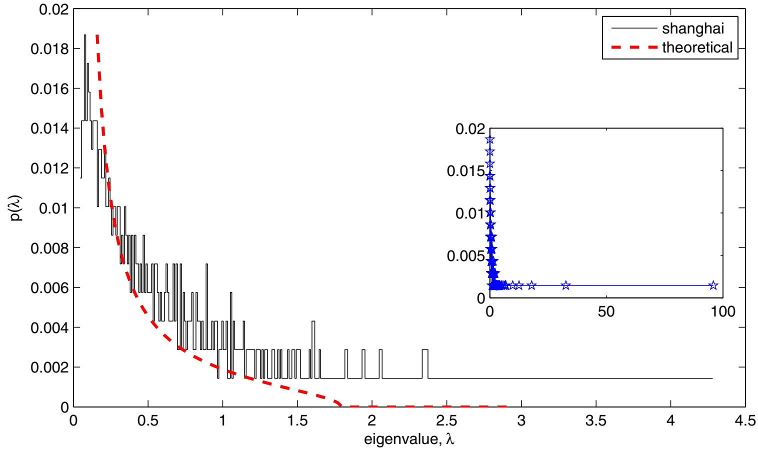

Fig. 1. The density function of the eigenvalues of a correlation matrix C is shown in comparison with the theoretical density P_{rm} of a Wishart matrix. The inset shows that the largest eigenvalue is about 33 times larger than the upper bound $\lambda_+ \simeq 2.9$.

where A is an $N \times L$ matrix containing N time series of L gaussian-distributed random elements a_{ij} with zero mean and unit variance, that are mutually uncorrelated. In the limit $N \to \infty$, $L \to \infty$ with $Q \equiv L/N (> 1)$ fixed, the probability density function $P_{rm}(\lambda)$ is analytically given [12,13] by

$$P_{rm}(\lambda) = \frac{Q}{2\pi\sigma^2} \frac{\sqrt{(\lambda_+ - \lambda)(\lambda - \lambda_-)}}{\lambda} \tag{5}$$

for $\lambda \in [\lambda_-, \lambda_+]$, where λ_- and λ_+ are the minimum and maximum eigenvalues of C_r, respectively, given by

$$\lambda_\pm = \sigma^2 (1 + \frac{1}{Q} \pm 2\sqrt{\frac{1}{Q}}). \tag{6}$$

Figure 1 shows the distribution of eigenvalues for Shanghai composites with the theoretical probability density function given by Eq. (4). The inset shows that the largest eigenvalue, λ_{max} is greater by about 33 times than $\lambda_+ \simeq 2.9$. From these statistics, we extract 4 largest eigenvalues to do our analysis. And the factors $\{M_j\}$ are computed as:

$$M_j(t) \equiv \sum_{k=1}^{696} < k|\lambda_j > r_k(t) \tag{7}$$

where k covers all the components of the eigenvector $|\lambda_j >$ corresponding to the eigenvalue λ_j. These factors represent each business group including the market-wide effect, respectively.

The goal of this study is to trace the evolution of each business group, represented by the eigenvector corresponding to an eigenvalue deviating from the

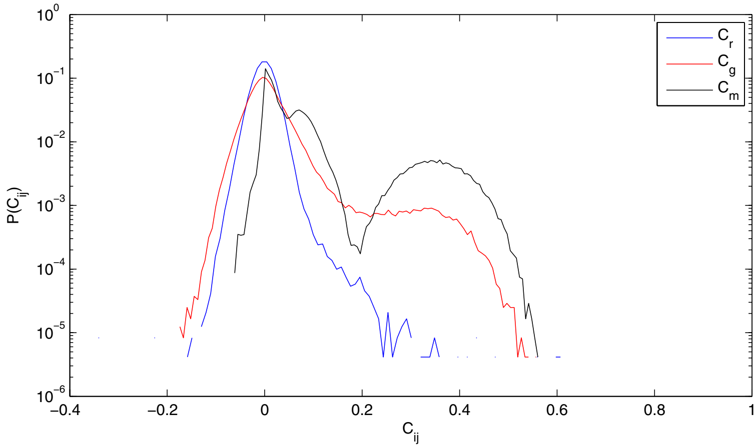

Fig. 2. The density function of cross-correlation coefficients are presented for C_r, C_g, and C_m. The critical C^* is estimated to be about 0.25. At the value, C_r and C_g is well separated.

predictions of the random matrix theory. Therefore, first, we divide the original return series into several segments, and then, by computing the cross-correlation β_{ij} between each return series $r_i(t)$ and the factor series $\{M_j(t)\}$, we examine the evolution of those correlation coefficients. In order to distinguish the significant correlation from random correlations, we use distributions of cross-correlation coefficients via the decomposition of C as follows,

$$C = \sum_{i=1}^{N_r} \lambda_i |\lambda_i >< \lambda_i| + \sum_{j=1}^{N_g} \lambda_j |\lambda_j >< \lambda_j| + \lambda_{max} |\lambda_{max} >< \lambda_{max}|, \qquad (8)$$

where N_r denotes the number of eigenvalues belonging to the bulk, which is inside the predictions of the random matrix theory as shown in Figure 1, and N_g denotes the deviating part except the largest one. From above three terms, we can define three correlation matrices, such as C_r, C_g, and C_m. Figure 2 shows distributions of cross-correlation coefficients for three of them. The critical value $C*$ is estimated at which C_r and C_g can be distinguished. As shown in Figure 2, the distribution of correlation coefficients of C_r almost decays at $C* \approx 0.25$ and C_r is well separated from C_g. The noticeable thing is the shape of the distribution of correlation coefficients C_m. Over the value of $C_{ij} = 2$, the deep groove is placed as if two regimes are divided.

To see the evolution of each business group, we set the moving window with the size of 800, corresponding to about 3 years. And this window moves by 30 steps, which correspond to one month. Further, to survey the structural transitions of business groups, if each β_{ij} is greater than C^*, then we give 1 to the parameter, or 0 otherwise. By doing so, we can trace out entry or exit of a specific stock to or out of the business groups represented by the eigenvector $|\lambda_j >$. Figures 3 and 4 show the evolution of business groups for 4 largest eigenvalues.

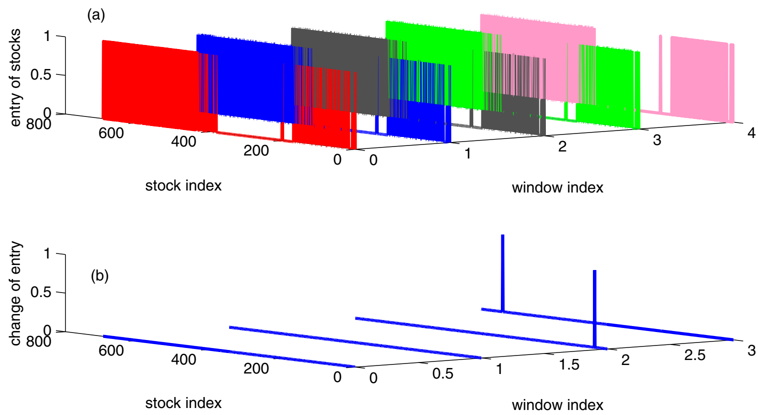

Fig. 3. The whole stock, represented by the eigenvector corresponding to the largest eigenvalue, are monitored. The presence of a bulk of stocks not effected by the market-wide effect is noticeable. (a) shows the composition of stocks belonging to the eigenvector over time, and (b) shows the change of its composition over time. There is no clear change.

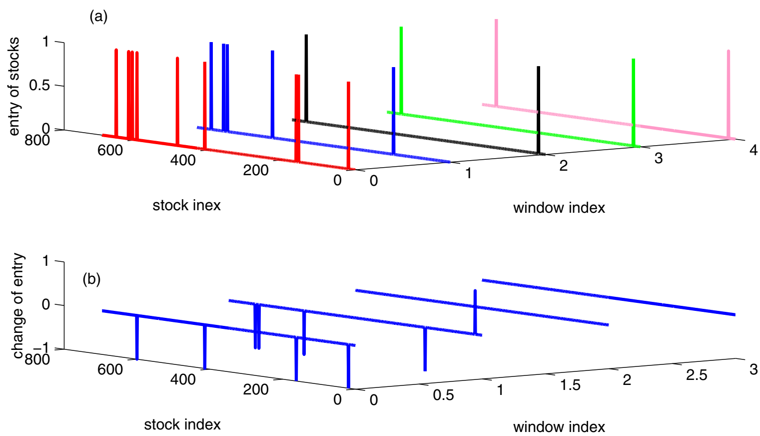

Fig. 4. The business groups, represented by the eigenvector corresponding to the second largest eigenvalue, are monitored. (a) shows the composition of stocks belonging to the eigenvector over time, and (b) shows the change of its composition over time. The change of composition is most explicit.

The eigenvector of the largest eigenvalues has no contributions from the center, which is well matched with previous observations. For the following three large eigenvalues, the eigenvector of the second largest eigenvalue shows more or less varying aspects compared to other two following eigenvectors while the rest two relatively show a stable aspect over time.

3 Conclusions

We have used the multi-factors model to examine the evolution of business groups over time. The first surprising observation is that Shanghai composites market shows a limitation of the market-wide effect different from other stock markets. It is also confirmed that the business groups belonging to the second largest eigenvalue are more or less vulnerable to the change of environment over time while the other groups seem to be more robust. This monitoring method is considered to be useful for comparing various stock markets, which are distinguished by the financial time stock exchange index. Through those studies, we shall get a clue about money-flow over the border by hedge funds or mutual funds.

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