Immunization of Geographical Networks

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Abstract. We numerically investigate the epidemic spread phenomena and efficient immunization strategies on complex networks embedded in geometry. It is assumed that there exists an unavoidable time delay (we call it the detection time) between the actual infection and the beginning of immunization, and we implement two different immunization strategies: one is based on topological connection neighbors (CN) of the infected vertex and the other on geographical spatial neighbors (SN). It is found that the decrease of the detection time is very important for a successful immunization. Our results suggest that within the limitation of the network models considered here, in which the infection probability is assumed to decrease with the geographic distance, the simple SN strategy works almost equally or better than the CN strategy, especially when the detection time is longer.

1 Introduction

Since the seminal papers on the scale-free and the small-world networks [1], strong research interest has been put on dynamical processes on complex networks [2]. In particular, the epidemic spread on networks has been drawn much attention and the susceptible-infected (SI) model [3,4] has often been used to describe the epidemic spread. In the SI model, the disease propagates through links between infected and susceptible vertices. One practically possible application example of the SI model is when the disease is very hard to be cured.

Designing the efficient immunization strategy to prevent the epidemic outbreak is very important. It has been revealed that the simple immunization strategy in which vertices to be immunized are picked completely at random is inefficient especially for networks with broad degree distributions, since the hub vertices with very high values of degree have very small chance to be chosen [5]. In comparison, the target immunization strategy focusing on vertices with high degrees can stop the disease spreading effectively [5]. However, the direct application of this strategy requires the full knowledge of the network structure, which is very difficult to obtain in many cases. Several efficient but still local immunization methods have been been suggested [6].

Most of real networks, e.g., the power grid, are built in real space, usually on the two-dimensional surface of the globe. In such networks, it is often desirable if

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the sum of the lengths of links (the sum of Euclidean distances between vertices connected by the links) is minimized from various reasons, like the reduction of the construction cost and the better performance of the network. The geographically embedded networks have been studied in terms of the robustness [7], the synchronization [8], and epidemic spread [9,10]. A number of spatial models have been used to characterize these large-scale spatial transmission disease including patch model for measles, distance transmission model for foot-and-mouth disease, multigroup models for influenza and network models for smallpox (see [10] for details). However, we believe that the detailed study of the immunization strategy on geographical spread still requires further attention.

In the present paper, we present and compare two different immunization strategies in geographically embedded scale-free (SF) networks. If the network structure is fully known, the disease spreading path can easily be predicted. In such case, an efficient way to stop disease spreading is to immunize a certain number of vertices connected to the initial infected source through the network structure. We call this method as connection neighbors immunization (CN), in which vertices separated from the initial source by shorter network distances are immunized first. In real situations, although finding the actual network structure is very important it is often hardly accessible. From this reasoning, the direct application of the CN strategy based on the actual connection structure of a network is almost impossible to apply for networks such as human sexual network. In this case, we can take an alternative approach based on geographic distance, instead of the path length in network, to control the disease spread. More specifically, it is possible to immunize vertices within the local range of the infected vertex, no matter whether these vertices are connected to the infected source or not via edges in the network. We call this method based on the real geographical distance as spatial neighbors (SN) strategy.

Hinted by the real situations in which the immunization often takes place after the detection of the initial outbreak of epidemic, we also allow the time delay between the actual infection and the start of immunization and call it as the detection time. We believe that the introduction of the detection time in the study of epidemic spread and immunization is quite important since if the detection time is too large, any immunization strategy becomes worthless. It should be noted that the detection time and the number of vertices to be immunized are strongly related to each other. Accordingly, effects of the detection time needs to be investigated in combination with the identification of efficient immunization strategy, which constitutes our main motivation of the present work.

In the present work, we use the simple SI model and immunize local neighbors of the first infected vertex after the detection time. The results show that the SN strategy performs better than the CN strategy, especially at the late detection time. The dependence on detection time is found to be significant, implying that technical and scientific advancements for the reduction of the detection time must be very important.

This paper is organized as follows: In Sec. 2, we briefly review the method of generating geographical networks, and in Sec. 3 discuss the SI model with

the distance-dependent infection rate. The used immunization strategies are explained in Sec. 4, and we describe the simulation procedure in Sec. 5. In Sec. 6, we present our results for immunization strategies applied for SF networks. Finally, in Sec. 7, we summarize our results and present some discussions.

2 Network Model in Geographical Space

We first build the SF network in a geographical space as in Ref. [11], in which connections are made on the basis of geographical distances aiming to reduce the total length of links. In a 2D square lattice of the linear size L (with the network size $N = L \times L$) under the periodic boundary condition, each site is assigned a degree k from the given degree distribution function, $p(k) \sim k^{-\gamma}$ with $\gamma \geq 2$. The construction of the geographically embedded SF network proceeds as follows (see Ref. [11] for details): for a randomly selected site, links connecting the site and its closest neighbors are created until either its preassigned degree is realized or all sites within the geographic distance $r(k) = Ak^{1/2}$ have been explored with a suitably given control parameter A. Repetitions of the above step for all sites produce a geographically embedded SF network of uniformly and regularly positioned vertices. The larger γ is, the more homogeneous the degree distribution is.

3 SI Model in Geographically Embedded Network

Suppose that chicken farms are spread and those farms are connected by visiting vehicles, humans, and so on. Taking farms as vertices in the network, it is plausible that the disease spread between farms is somehow related with travel patterns of humans. It has been recently found that the probability for a person to travel a distance d decays following the power-law form with the exponent about 1.6 [12]. We note that the spread of some disease should be closely related with the pattern of human travel, and write the infection rate λ_{ij} in the SI epidemic spread model as

$$\lambda_{ij} = \frac{\lambda a_{ij}}{d_{ij}^{\alpha}}, \tag{1}$$

where d_{ij} is the Euclidean geographic distance between vertices i and j, and $\alpha \geq 0$, $\lambda \in (0, 1]$. The element of the adjacency matrix a_{ij} takes the value unity if i and j are connected and zero otherwise. In terms of α, there are two limiting cases: if $\alpha = 0$, the infection rate is independent of the geographic distance ($\lambda_{ij} = \lambda$), corresponding to the original SI model. If $\alpha \to \infty$, on the other hand, only nearest neighbors at a unit lattice distance can be infected at the infection rate λ. In reality, $\alpha \approx 1.6$ corresponds to the empirical findings based on the travel of bank notes in Ref. [12]. It is obvious that larger λ or smaller α will increase the infection rate λ_{ij} if there is a link between vertex i and j. In the simulation of the SI model, we choose $\lambda = 0.2$ and $\alpha = 2$, which, we believe, is a reasonable choice in comparison to $\alpha = 1.6$ in Ref. [12].

We start from the initial configuration in which all vertices are in the susceptible states. A randomly chosen vertex is infected, and then the disease spreads at the infection rate in Eq. (1) across the geographically embedded network structure.

4 Detection Time and Immunization Strategies

In a real situation of epidemic spread, such as SARS, HIV, we often observe that it takes some time to identify an infected individual among population and tend to overlook the possibility of being epidemic. This unavoidable time delay, we call the detection time τ throughout the present study, between the actual infection and the start of the immunization should be taken into account to design an efficient immunization strategy. For instance, suppose that the state-run public health organization is armed with well developed immunization techniques. Even in this case, in order to stop the spread of the disease, the immunization needs to be done as soon as possible. Otherwise, the number of people to be immunized can be huge, making the spending tremendous. In an extreme case, any immunization is worthless if it starts after all people are already infected. We believe that the use of nonzero τ is a realistic extension of existing studies on immunization strategies in a geographically embedded network.

After the detection time τ since one randomly chosen vertex v was infected, we immunize the fraction f of the whole vertices by using the following two

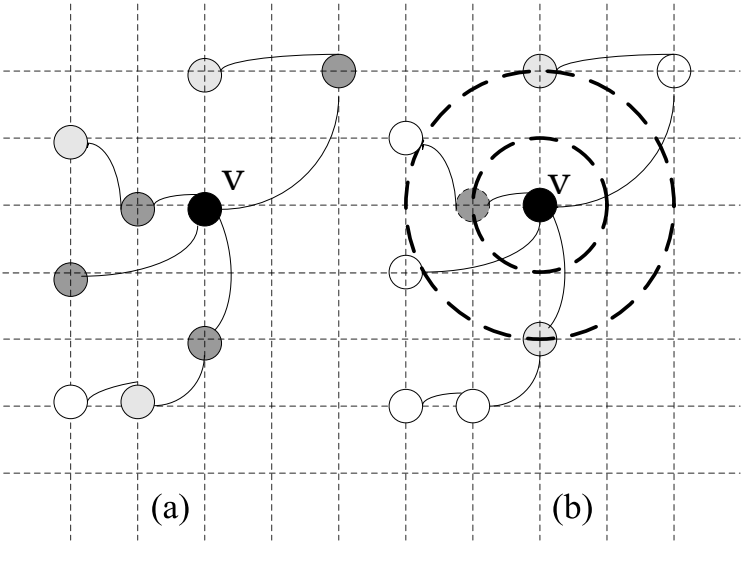

Fig. 1. (a) Connection neighbor (CN) and (b) spatial neighbor (SN) strategies. The infected vertex v is immunized first, and then further immunizations are made in the ascending order of the network distance for CN, and the geographical Euclidean distance in SN.

different local strategies as shown in Fig. 1: the first strategy uses the connection structure of a given network by immunizing connection neighbors (CN) of v, so we call it the strategy CN. In CN, we first immunize the infected seed vertex, its directly connected neighbors with the shortest path length (called as the chemical distance or the network distance) $\ell = 1$, and vertices with $\ell = 2$, and so on, until fN vertices are immunized (we immunize vertices randomly for the outmost layer). When an already infected vertex is immunized, we assume that the vertex is first cured to become healthy again and then immunized. The second strategy, we call it SN since it is based on spatial neighbor information, uses the geographic distance d from the infected one, instead of the network distance ℓ: the shorter the geographic distance is, it is more likely that the vertex is immunized, until the given fraction f of vertices are immunized. These two immunization strategies are applied only once at the time τ, and no more immunizations are made afterwards.

5 Simulation Process

The simulation processes are performed as follows:

(i) A randomly chosen vertex is infected and the disease spreads with the infection rate described in Eq. (1) as time goes on.
(ii) After the detection time is reached ($t = \tau$), two different immunization strategies are carried out following either the SN or the CN strategy.
(iii) The SI model dynamics is run without further immunization.

The stationary value of the density of infected vertices is computed and the average is performed over 50 different network structures, 1000 random choices of initial vertex [step (i)], and 10 realizations of immunization strategies [step (ii)]. Without any immunization the SI model eventually makes all vertices infected with the density of infected vertices $\rho(t \to \infty) = 1$. Immunization of vertices may cut off some spreading paths, and one gets $\rho < 1$ in stationarity. The efficiency of an immunization strategy can be easily measured by ρ: the smaller ρ is, the more efficient the strategy is.

6 Results of Simulations

In Fig. 2(a) we present the results for the geographically embedded SF networks with $\gamma = 2.5$ of the network size $N = 6400 (L = 80)$: the stationary value ρ of the density of the infected vertices versus the fraction f of immunized vertices is displayed for various values of the detection time $\tau = 2, 4$, and 6. Although not shown here, it is observed that the disease spreads in a much less scale for $\gamma = 7$ than for $\gamma = 2.5$: this can be easily understood since the existences of hub nodes facilitate the epidemic spread. It is evidently seen that ρ is a decreasing function of f, while an increasing function of τ, indicating that immunization of more vertices at earlier stage of spread can keep the disease from being epidemic.

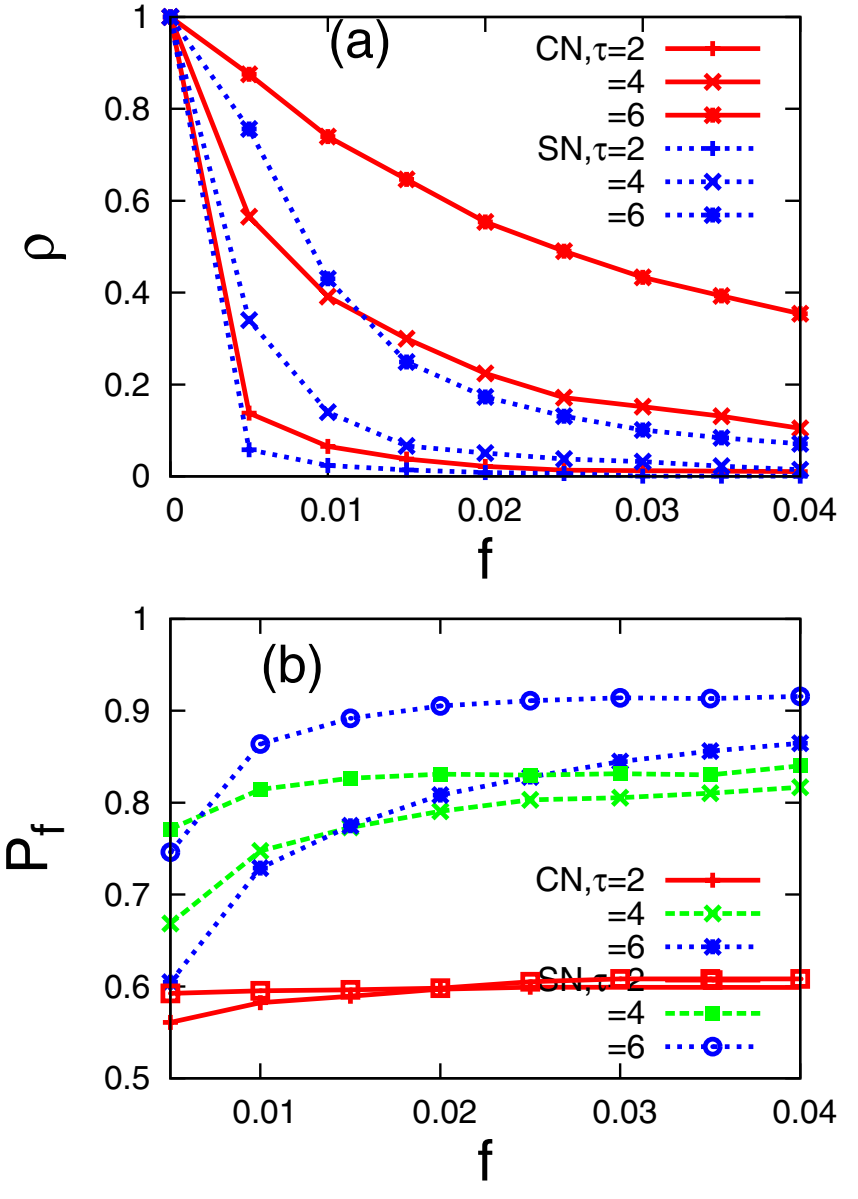

Fig. 2. (a) The density ρ of infected vertices at stationarity is shown as a function of the fraction f of immunized vertices. for scale-free networks of the size $N = 80 \times 80$ with the degree exponent $\gamma = 2.5$. The SN strategy shows better performance than the CN strategy, especially when the detection time becomes larger. (b) The ratio P_f of the number of immunized infected vertices to the total number of infected vertices is shown as a function of the immunization fraction f at different detection time τ for the two immunization strategies (CN and SN) for scale-free networks of the size $N = 80 \times 80$ with the degree exponent $\gamma = 2.5$.

It should be pointed out that the reduction of the detection time is crucial: for example, if we detect the disease at $\tau = 2$ and immunize 2.5% of nodes, we can wipe out the disease almost completely, while if the detection time is long, i.e., $\tau = 6$, the disease will eventually infect around 50% of the population [see the curves for CN in Fig. 2(a)]. In other words, the same immunization efficiency is achieved in Fig. 2(a) by the CN strategy, both at $\tau = 2$ for f less than 0.5% and at $\tau = 4$ for $f \approx 3\%$, indicating that by halving the detection time one can achieve the same level of immunization by immunizing much less number of people.

Figure 2(a) also exhibits that the differences between the CN and SN strategies are not discernible at early detection time such as $\tau = 2$. In contrast, with the increase of detection time τ, the SN immunization strategy performs more efficiently than the CN. The relative success of the SN over the CN can be understood as follows: a complex network in general often has a very small network diameter, which implies that the number of connection neighbors increases exponentially as the network distance from the original seed vertex is increased. Consequently, if we immunize the same number of vertices, the CN strategy is more likely to immunize the susceptible vertices and it is possible that many infected vertices escape from being immunized, while the SN strategy works relatively well by immunizing infected ones effectively. In order to verify this, we directly calculate the ratio P_f of the number of immunized infected vertices to the total number of infected vertices at $t = \tau$. In Fig. 2(b), we show P_f versus f for the CN and SN strategies, which corresponds to the data in (a). It can be seen that at $\tau = 2$, the difference between the CN and SN strategies is insignificant. However, as τ is increased, the difference becomes larger and the SN always yields a bigger value of P_f, which means that the SN is more efficient than the CN in the sense that it immunizes more infected vertices than the CN does. From the comparison of Fig. 2(a) and (b), we also note that the number of immunized infected vertices at the immunization stage (at $t = \tau$) appears to be closely related to the number of infected vertices at stationarity (at $t \to \infty$).

Considering that most real networks have the small-world property so that the number of connection neighbors increases very fast with the network distance, we believe that the efficiency of the SN strategy found above can also be true in reality. The application of the SN strategy requires only geographic information of the distances between vertices, which makes the SN useful especially when topological connection information is not available or hard to obtain.

7 Discussion and Conclusions

In the present work, we have investigated numerically the spread of the disease by using the SI model on the geographically embedded scale-free network with focus on the efficiency of immunization strategy. Different from the usual SI model, we have assumed that the infection spreads at a rate which has the inverse square form of the distance. Two immunization strategies, the CN (connection neighbor) strategy based on network distances and the SN (spatial neighbor) strategy based on geographic distances, have been compared. The results show

that the SN strategy outperforms the CN strategy, especially when the detection time is relatively large. We believe that our results may be helpful to stop the epidemic spread in real systems.

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