Internal-Evolution Driven Growth in Creation-Annihilation Cyclic Games

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Abstract. In this paper, the domain growth process in a novel kind of cyclic game is investigated by similation method. Different with the classical cyclic games, this process is called "creation-annihilation process", in which is just like the autocatalysis system. The results of numerical simulations show that the domain growth in such cyclic games with four or five states has a special feature: the growing domain usually has a stable boundary, and the growth is driven by the internal-evolution of the domain. Considering with the widespread of the cyclic autocatalysis in organism activities, such internal-evolution driven growth could be universal in many organism systems.

Keywords: domain growth, cyclic autocatalysis, internal-evolution, creation-annihilation process, cyclic games.

1 Introduction

Cyclic games, which is well mimic the cyclic struggle interactions of different populations, are attracting much research interests recently [1,2,3,4,5,6,7,8]. Generally speaking, the cyclic game has many different but coordinate states (or say strategy), and each state/strategy denotes a kind of populations, and the common algorithm of cyclic games is that the struggle interactions of these states/strategies construct a closed cycle. For example, a three states cyclic game can be expressed as A win/displace/annihilate B, B win/displace/annihilate C, and C win/replace/annihilate A, where A, B, and C denotes the three states/strategies. This is the well-known rock-paper-scissors game [1,2,3,6,8] that is the typical example of cyclic games. Based on this basic frame, except for the modeling for the struggle interactions in ecology systems such as the cyclic predator-prey system, we can introduce many other elements into the game to fit our research issue, or view it from many other points, such as the chemical reaction [6,7]. In additional, many cyclic game type processes can be easily realized by the microbial laboratory experiments [8]. Thus these factors lead to a much wide significance for the research. The recent empirical, experimental and theoretical studies about cyclic games reveal rich interesting phenomena in nature,

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laboratory experiments, and computer simulations. For example, some resent cyclic games model generated rich orderly time-spatial structures [6], such as the spiral waves, which is important for the statistic physics about the dynamical equilibrium of the self-organized systems; in another way, such cyclic struggle relationships is found out in real-world ecology systems outside the food chain system, such as the mating strategy of different male lizard [1,2]. Many recent studies extend algorithm of cyclic games with more than one kind of relations such as the heterogeneous invasion relationships in the cyclic games [9], or more than one cycle such as a more complex relation web in the game [9,10]. Here, we present another extended version of cyclic games. Let us consider the cyclic autocatalysis process that plays a much important role in the biochemical dynamical processes. Considering the simplest case that the catalysis reaction is just like A + B A + C, where A is the catalyst, this reaction can be treated as two step in theory: A annihilates B, and then create C. Therefore, we can introduce the creation relation into the cyclic game to mimic such cyclic autocatalysis reactions. In this paper, we propose a new model of such kind of cyclic game with the creation interaction, which can be called the creation-annihilation game in the following discussions. The creation-annihilation process has not only the general cyclic annihilation interactions just like that in the paper-rock-scissors game, but also the creation relations (neither replication nor production), and also the creation relations is cyclic too. Such cyclic games are much similar with the cyclic autocatalysis process, or say hypercycle system [11,12]. Different with most of the previous researches, the global spatial structure is not the focal point of our discussion. On the contrary, the domain growth process in our model is mainly investigated based on computer simulations. Our results indicate that the growth process in the cyclic autocatalysis system surprisingly exhibits a new feature that is different with our general understanding of growth processes.

2 The Creation-Annihilation Cyclic Games

The simplest case of our model is the three-state creation-annihilation cyclic game. The basic rules includes two kind of relationships: one is annihilation (win/displace), which is similar with the relationship in the rock-paper-scissors game, expressed as A win/displace/annihilate B, B win/displace/annihilate C, and C win/displace/annihilate A, which is shown by the black arrows in Fig. 1 (a), where A, B, and C denotes the three states/strategies in the game; another is creation, shown by the blue arrows in Fig. 1 (a), expressed as A create C, B create A, and C create B. Assuming the annihilation and creation is simultaneous for a pair reactable particles, the creation-annihilation process can be regarded as a catalysis reaction. For the above example in Fig. 1 (a), the corresponding catalysis reactions of the cyclic creation-annihilation effect is shown in Fig. 1 (b). In the following discussions, also for the above example, state A is called the catalyst of B, and state C is called the resultant of B. The creation-annihilation cyclic games can be also extended to four, five or more states. All of the basic types of the game with three, four and five states and their corresponding cyclic

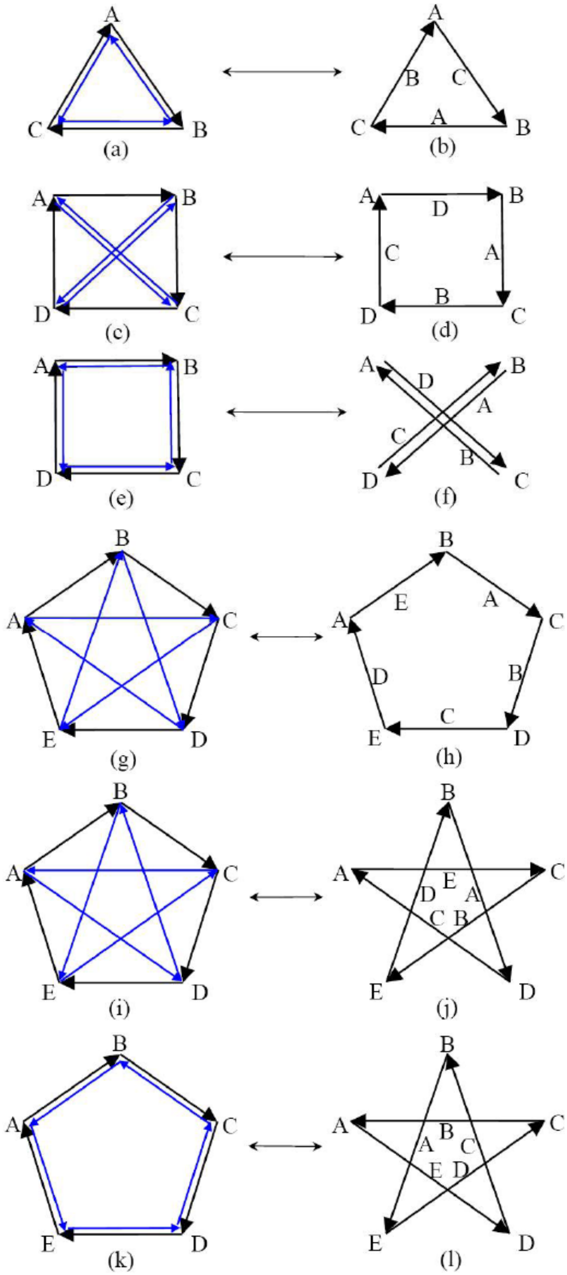

Fig. 1. Sketchs of the rule of creation-annihilation cyclic games with three, four and five states. The left four panels (a), (c), (e), (g), (i) and (k) show the relationship of creation (the blue arrows) and annihilation (the black arrows) of Type III, IV-A, IV-B, V-A, V-B and V-C respectively, and the right four panels (b), (d), (f), (h), (j) and (l) show the corresponding cyclic autocatalysis, where the alphabets beside the arrows denote the state of catalyst. The middle double-arrows denote the corresponding relations.

catalysis reactions are exhibited in Fig. 1, and named by Type III, Type IV-A, Type IV-B, Type V-A, Type V-B and Type V-C, respectively. There are many other categories of the game with the states no more than five, but they are all equivalent to one of the above six categories. Our numerical simulations are embedded in a 128128 square lattices. The time of our simulations is discrete, and the asynchronous updating scheme is used, namely each lattice point is sequentially picked and focused to treat with random order in each time step. In the treated process, the focal lattice point randomly chooses one of its four nearest surrounding neighbors. If the chosen neighbor is catalyst of the focus lattice point, the focal lattice point will be updated along the arrows of its catalysis reaction to its resultant (for example, in Type III, the focal lattice point is A, and the chosen neighbor is C, the focal lattice point will change to B). The initial states of the central $N \times N$ lattices are randomly chosen (this field is called the core in the following discussions) from all the possible states, and others are all set as state A. In other words we set the core growing in an environment with state A. The growth and the surface structure of the core is the focus of our discussions.

2.1 General Feature of the Growth of Core

The features of the growth process of the six basic types of the model shown in Fig. 1 are gotten via computer simulations. Here we set the initial size of the core is 30×30 ($N = 30$). The phenomenon of each type is discussed in the follows:

Type III: In this type, the core grows very fast, and there is not clearly surface structure in the boundary of the core, as shown in Fig. 2.

Type IV-A: The growth process of this type shows many special properties. As shown in Fig. 3, in the growth process, there are two clearly kind of fields in the core, one is composed by the state A and C, the other is state B and D. The front field (state A and C) cannot react with the environment (state A). The latter field (state B and D) can keep moving and distorting in the core. The growth is only occurred when the latter field contacts the boundary of the core, as shown in Fig. 3 (b). Comparing with the speed of growth of Class III, the core

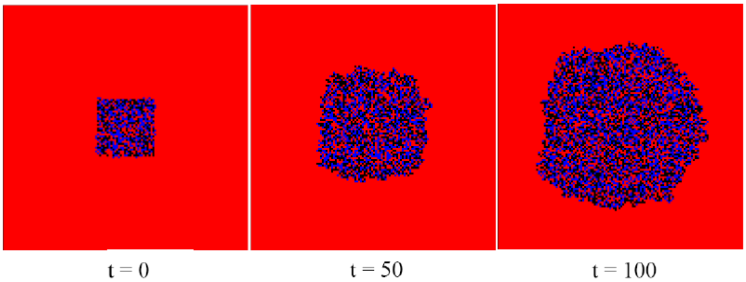

Fig. 2. The growth of Type III in 128×128 square space, where red, blue and black grids denote state A, B and C, respectively

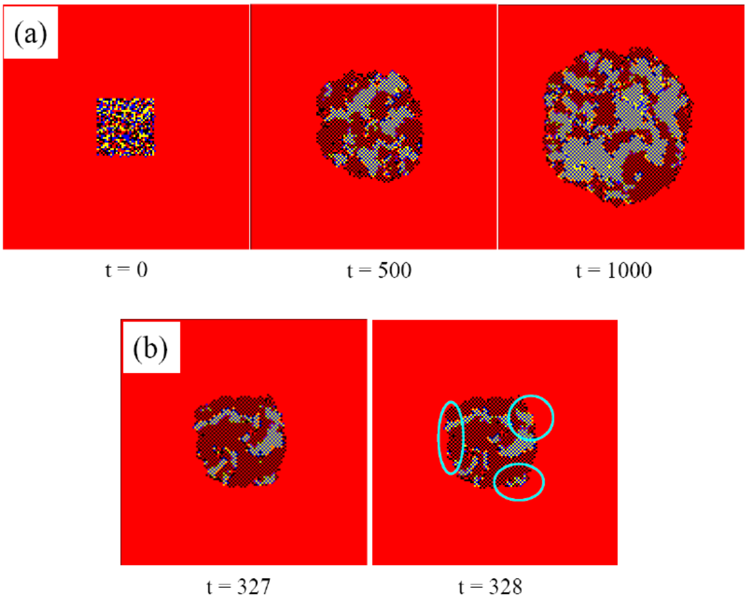

Fig. 3. The growth of Type IV-A in 128128 square space, where red, blue, black and yellow grids denote state A, B, C and D, respectively. (a), the pattern of the system at $t = 0$, 500 and 1000. Obviously, the core domain has two kind of field: one is constructed by state A and C, another is B and D. The fields of B and D keep moving in the core domain which is just like wind. The growth is only occurred when the fields of B and D contact the boundary. (b), the pattern at $t = 327$ and 328, where the growth points are encircled in the light blue circles.

growth is much slow. If at any time step the latter field disappears (means the evolution inside the core stops), the growth of the core stops too. In this regard, the growth process of the core can be regard as internal-evolution driven.

Type IV-B: In this type, because of the rule of catalysis doesnt construct a single loop, the core cannot grow.

Type V-A: As show in Fig. 4 (a), The evolution of the core leads to a maze-like structure in the inside, and the core has a surface constructed by state C (majority) and D in most positions. The surface cannot grow until the evolution inside the surface breaks the surface. The break process of the surface is: firstly state C change to D, and then change to E (the surface is broken), then a neighbor position outside the core (state A) change to B (the core grows under the effect of state E), and then change to C after few steps, namely a new surface outside the old one is founded. Obviously, in this class, the growth of the core is also driven by the internal evolution.

Type V-B: Similar with the Type V-A, the core has also a surface in the growth process, but the surface is surprisingly much clearly and thick, as shown in Fig. 4. (b). The surface has obvious multi-layer structure: from the most outside layer

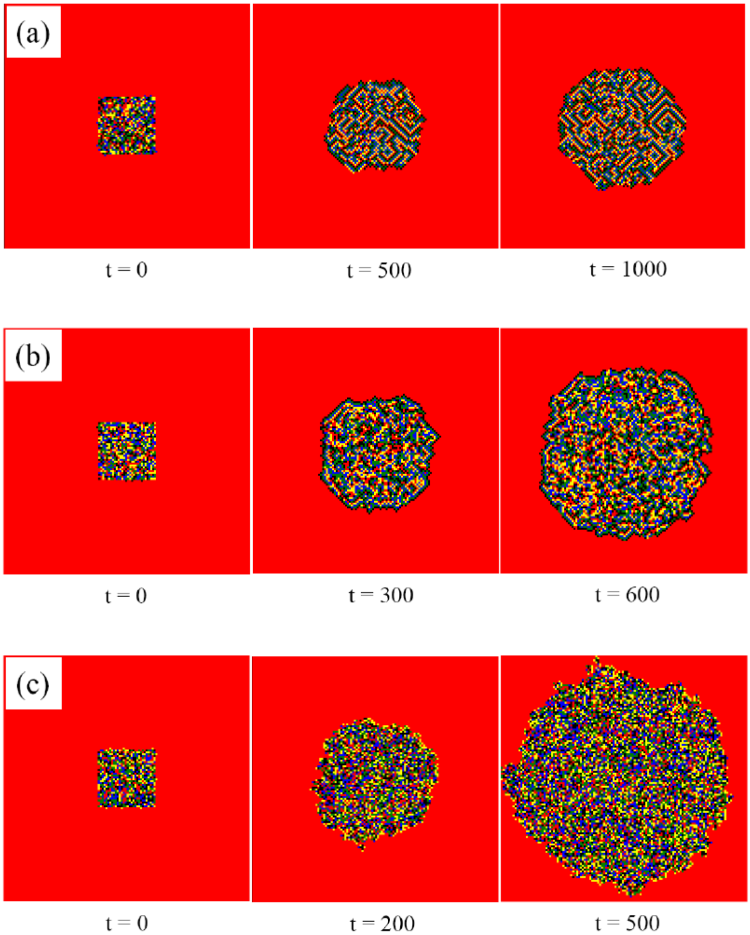

Fig. 4. The growth of Type V-A (a), V-B (b) and V-C (c) in 128128 square space, where red, blue, black, yellow and green grids denote state A, B, C, D and E, respectively.

of the core to the inner, the main state of each layer of the surface successively is C, E, B, and D, as shown in Fig. 4 (b) and Fig. 5. The inside of the core are filled with filamentous structures. The growth of the core occurs also only when the surface is broken by the internal evolution at any position. A typical break-growth-rebuild process of the surface is shown in Fig. 5. Different with the Type V-A, the break process generally is: A position in the second layer with state E changes to B under the effect of state D on its neighbor, then it react with its another neighbor with state C on the first layer and change it to state E, then the growth is occurred.

Type V-C: The growth of this type does not generate clearly surface of the core. Though state D is majority in the most outside of the core, it is not connect with each other to construct a stable surface. In parallel, there is not any orderly structure in the core, as shown in Fig. 4 (c).

2.2 Discussions

There are the following problems about the present model should be discussed.

(i) Boundary conditions. The reflection boundary condition is used in all above simulations, namely the outer neighbor of the lattice point on the boundary is its own. Actually, because we mainly discuss the growth process of the core, the boundary condition cannot effect the growth before the core contact the boundary.

(ii) The updating scheme. The updating scheme used in the above simulations is asynchronous, and each lattice point is only affected by one of its neighbors. We also considered many other updating schemes as follows:

The second asynchronous updating scheme. (We call the above updating scheme as the first asynchronous updating scheme in our discussions). Comparing with the above scheme, it is considering the effect of all the four neighbors, namely, the focal point must be updated if there is the catalyst of the focal point in its four neighbors. For the Type V-A and V-B, comparing with the above scheme, this scheme does not introduce any different phenomena on the structure of the core, only the growth speed is quicker. Type V-C has some differences that the surface of the core that is mainly constructed by state D is clearer using this update rule. The most noticeable things is the two-arm spiral wave in the Type IV-A. It can be clearly observed in the growing core using this updating scheme, as shown in Fig. 6. The wave is not much stable: it can divide

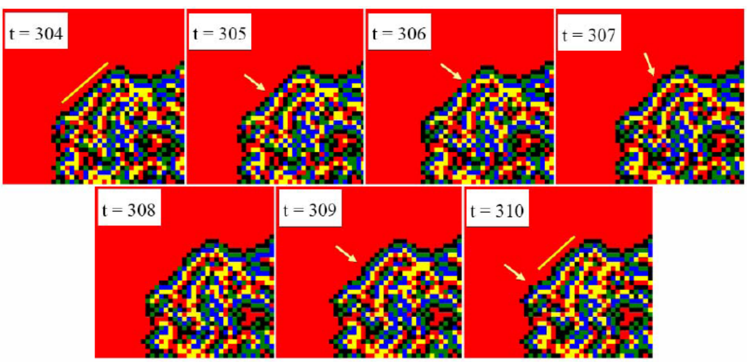

Fig. 5. A typical growth case of Type V-B at the left-upper boundary from t = 304 to 310, obviously, the boundary is broken from t = 305 to t = 306, and then is rebuilt outside the old boundary. The yellow line at t = 304 shows the initial status of the boundary, notice the layered structure of the boundary: the outside is state C (black), next mainly is E (green), and the inner of the boundary mainly is B (blue) and D (yellow). The arrows show the changing point at the boundary. The yellow line at t = 310 shows the rebuild boundary.

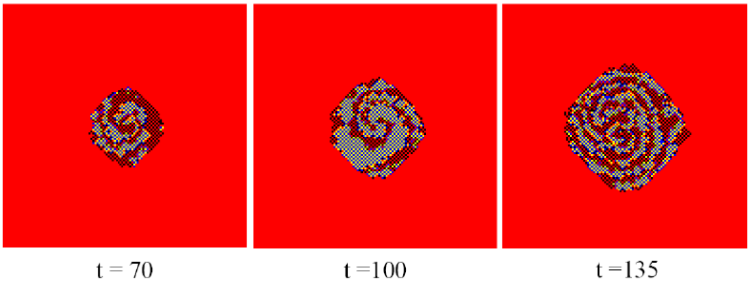

Fig. 6. The spiral wave in the growing core based on Type IV-A using the second asynchronous updating scheme. The initial size of the core is 1010. There are clearly two-arm spiral wave in the left two panels, and the spiral wave breaks in the right panel.

Fig. 7. Accompanying with the growth, the pattern generated by the deterministic form of our model using synchronous updating scheme. From left to right, the panels are obtained by Type V-A, V-B and V-C, respectively. The initial size of the core is set as 3×3, these figures are obtained after run 200 time steps. The left panel is a spiral wave, obtained by Type V-A. The middle panel is a target wave converged by two closed spiral waves, generated by Type V-B. The right panel is a target wave propagating from outer to inner, generated by Type V-C.

into two or more new spiral waves, or breaks and disappears. The growth also occurs when the arm with state B and D of the spiral wave propagates to the surface of the core, is also driven by the internal evolution. According to this result, we conjecture that the wave-propagating structure could also be the evolution mechanism in the growth process using the first asynchronous updating scheme, but its wave is quite unstable.

The deterministic synchronous updating scheme. It is just like the updating in standard cellular automata, each lattice point synchronously changes its state to its resultant if there is its catalyst in the current states of its four neighbors. This updating scheme is non-probabilistic. Using this scheme, our model surprisingly generates many the regular and stable spiral waves or target waves. As shown in Fig. 7, from a small initial core with randomly states, Type V-A generally generates a spiral wave; Type V-B mainly can generate the target wave which is converged by two closed spiral waves; Type V-C shows a special target wave which is propagating from outer to inner of the core, but the outmost wave front is also propagating to out. In additional, the growth using this updating scheme is changed into the simple propagation of such spiral wave or target wave.

The probabilistic synchronous updating scheme. In this case, the lattice point synchronously changes its state, but for each point we only consider the effect of its one randomly chosen neighbor. The simulation results using this updating scheme is not any obvious differences with the the first asynchronous updating scheme.

3 Conclusions

In the present model, we set the core in a large environment with single ingredient, which supplies the resource for the growth of the core. The growth processes of the core in Type IV-A, V-A and V-B show clearly internal-evolution driven property. Although there are many differences in detail in the internal-evolution driven growth of different class, the common character of such growth process is emerges as follows:

(i) The growing core usually has a surface that is unreactable with the environment (for Type IV-A, the surface is the field composed by state A and C). The surface usually has some orderly structure or is composed mainly by some special compositions.

(ii) The surface exhibits break-growth-rebuild process in the growth. Because the surface is unreactable with the environment, it must be broken by the internal evolution to grow. Here, the evolution inside the core is not only the necessary condition but also the driver of the growth of the core. If the evolution inside the core is stop, the growth will stop too. In additional, the evolution inside the core implies the core should keep far from the equilibrium state in the growth process. In many cases the spatial structure shows wave like property.

(iii) When the growing core takes a new grid, the state on the grid must be changed into a new state which belong to the core. In other words, the new grid should be reacted with the core in the growth. It easily let us to think about the absorption of alimentation in living organ system.

These characters are evidently different with the general feature of much number of growth processes which had been studied. In many growth processes in the real-world systems or simulations, such as the growths in the well-known Diffusion Limited Aggregation (DLA) system [13,14], the crystal growths [15] and some domain growths in film [16,17], it is usually only driven by the interaction between the environment and the surface of the domain, and the field inside the domain trends to equilibrium state. Such characters are contrary to the properties of our model.

On the other side, there are also many natural growth processes which have many similar properties with our model, especially in large number of organism systems. For example, our general knowledge about the growth of cell tell us: firstly, the cell has a stable film, which is inactive with the environment; secondly, the cell must is living in growth (namely the cell is in nonequilibrium state and evolving), in other word, the growth of cell is also driven by the evolution of the cell. Such similar character can be found out in the growth of organ and living body. These characters of the growth of cell are in agreement with the new feature of growth indicated by our model. Therefore, the growth driven by the internal evolution proposed by our model could be a new type of mode of the growth.

Another noticeable point is that our model mimics the cyclic autocatalysis reactions, which is wide-spread in much organism systems [11,18]. Considering the similarity between the phenomenon in the toy model and the growth process of cell and many other organism systems in nature, we have to conjecture that such internal-evolution driven growth property could be universal in the evolution of organism systems. And also, it implies the growth of organism systems (maybe include many other self-organized systems) maybe has a much different mechanism comparing with the mainstream understanding. We hope our results can draw forth further studies about this issue.

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