

# Two-Dimensional Coupling Model on Social Deprivation and Its Application

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**Abstract.** This paper qualitatively describes the deprivation under different coupling situations of two-dimensional indicators and then establishes the two-dimensional coupling model on social deprivation, using the social welfare function approach and Foster-Greer-Thorbecke  $P_\alpha$  method. Finally, this paper applies the model to evaluate the social deprivation of 31 provinces in China under the coupling state of capita disposable income and housing price.

**Keywords:** social deprivation, positive indicator, negative indicator, coupling mode.

## 1 Introduction

At present, the people's material life has been more greatly improved than the past, but people's life satisfaction haven't increased with the improvement of the material conditions. This is because that most people have a jealous type of preference function, and other people's income is negatively correlated with his well-being. The higher is the income of other people, the more he is not happy and the more he feels painful and resent [1]. Therefore, a person's satisfaction with his income does not only depend on the absolute income, but also depends on the relative income. Nowadays, China is in the historical period of economic and social transformation, and the urban-rural gap and the income gap between regions make some people have relative deprivation. The rapidly expansive relative deprivation is one of the sources of social conflict and the enemy of social stability and harmony [2]. As a result, it is a meaningful and important thing for us to study relative deprivation.

Broadly speaking, relative deprivation is a kind of emotion that arises from inequalities within social groups. The concept was first put forward by the Stoufer and others [3], but they did not give the normative definition and measurement methods of relative deprivation. Crosby [4] reviewed different deprivation theories and stressed the relativity of deprivation: people's deprivation is relevant with the

reference groups. Runciman [5] gave the definition of relative deprivation from an economic perspective which laid a solid foundation for the subsequent quantitative researches. However, those scholars look only at income attribute, which is not enough. We need also to look at other attributes, such as access to education and life expectancy.

This paper establishes the two-dimensional coupling model on social deprivation, using the social welfare function approach. And then, this paper discusses the deprivation under the four two-dimensional coupling states. Finally, this paper uses the model evaluate the social deprivation of 31 provinces in China.

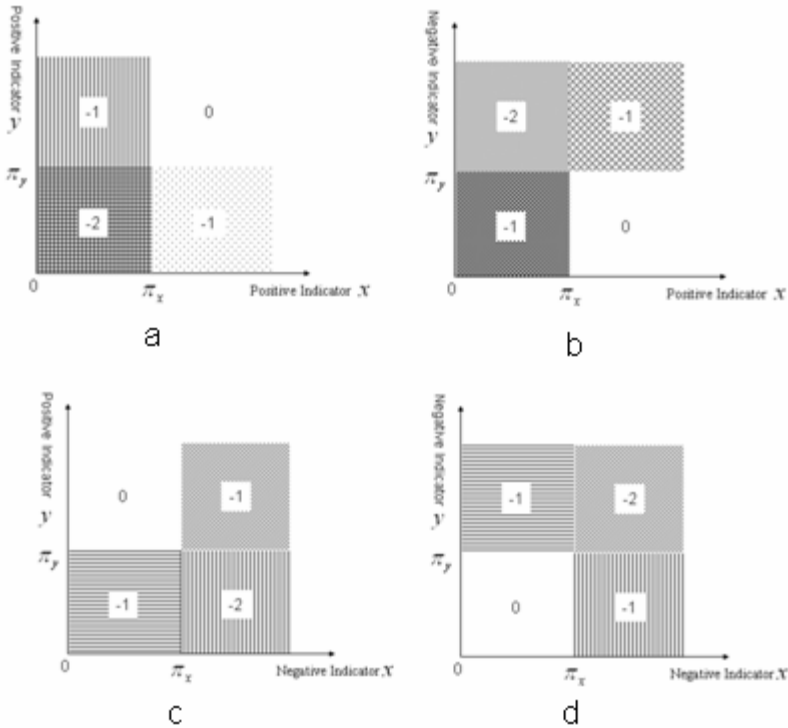
## 2 Model

Some scholars have tried to evaluate social deprivation using the social welfare function approach. Bourguignon and Chakravarty use the approach to describe the measurement of multivariate poverty [6]; Duclos, Sahn and Younger establish a multi-dimensional poverty comparison method [7]; Atkinson establishes a method of evaluating social deprivation using this approach [8]. However, they don't solve the deprivation problem under the interaction of the positive and negative indicators. The two-dimensional coupling model on social deprivation which is established in this paper is to solve this problem. We will discuss the different measurement method of social deprivation according to four two-dimensional coupling states including positive indicator- positive indicator, positive indicator-negative indicator, negative indicator-positive indicator and negative indicator-negative indicator.

Before discussing, we firstly illustrate some parameters.  $\pi_x$  is indicator threshold of  $x$ . If  $x$  is a positive indicator, evaluation objects are not subject to deprivation when  $x \geq \pi_x$ ; or subject to deprivation when  $x < \pi_x$ . If  $x$  is a negative indicator, evaluation objects are not subject to deprivation when  $x \leq \pi_x$ ; or subject to deprivation when  $x > \pi_x$ . Likewise,  $\pi_y$  is Indicator threshold of  $y$ , and its deprivation situation of positive and negative indicators is the same as indicator  $x$ . In order to illustrate the deprivation intensity under different situations, we use -2 to show the situation that evaluation objects are subject to deprivation in two dimensions at the same time, -1 to indicate deprivation in only one dimension, and 0 to indicate deprivation in no dimension.

For the coupling situation of positive indicator and positive indicator (figure 1(a)), there is deprivation in no dimension when  $x \geq \pi_x$  and  $y \geq \pi_y$ ; deprivation in only  $y$  dimension when  $x > \pi_x$  and  $y < \pi_y$ ; deprivation in only  $x$  dimension when  $x < \pi_x$  and  $y > \pi_y$ ; deprivation in two dimensions when  $x < \pi_x$  and  $y < \pi_y$ . Atkinson has described this situation in the literature [8].

For the coupling situation of positive indicator and negative indicator (figure 1(b)), there is deprivation in no dimension when  $x \geq \pi_x$  and  $y \leq \pi_y$ ; deprivation in only



**Fig. 1.** The deprivation on different coupling conditions of two-dimensional indicators

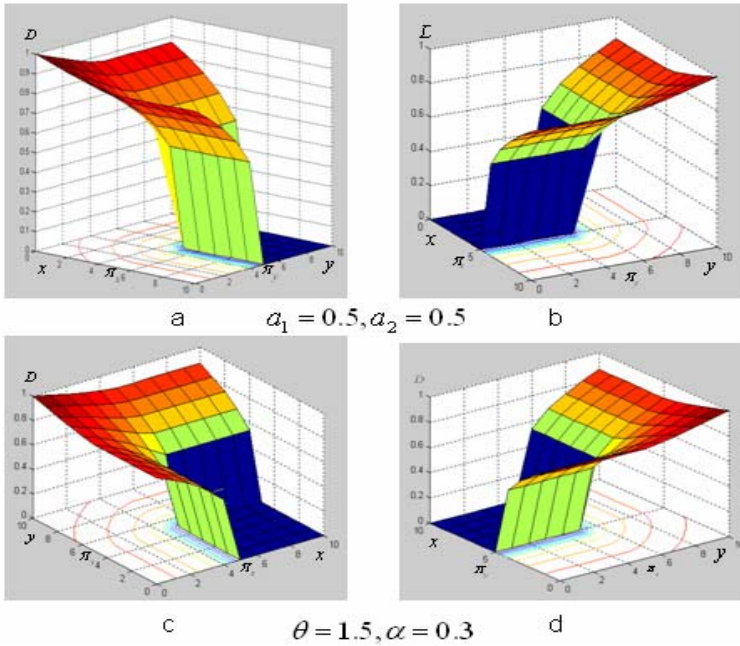
y dimension when  $x > \pi_x$  and  $y > \pi_y$  ; deprivation in only x dimension when  $x < \pi_x$  and  $y < \pi_y$  ; deprivation in two dimensions when  $x < \pi_x$  and  $y > \pi_y$  .

For the coupling situation of negative indicator and positive indicator (figure 1(c)), there is deprivation in no dimension when  $x \leq \pi_x$  and  $y \geq \pi_y$  ; deprivation in only y dimension when  $x < \pi_x$  and  $y < \pi_y$  ; deprivation in only x dimension when  $x > \pi_x$  and  $y > \pi_y$  ; deprivation in two dimensions when  $x > \pi_x$  and  $y < \pi_y$  .

For the coupling situation of negative indicator and negative indicator (figure 1(d)), there is deprivation in no dimension when  $x \leq \pi_x$  and  $y \leq \pi_y$  ; deprivation in only y dimension when  $x < \pi_x$  and  $y > \pi_y$  ; deprivation in only x dimension when  $x > \pi_x$  and  $y < \pi_y$  ; deprivation in two dimensions when  $x > \pi_x$  and  $y > \pi_y$  .

In order to get the value of deprivation in a single dimension, Atkinson uses the formula (1) to measure the value of deprivation in a positive indicator dimension:

$$d_x = \max(0, (1 - x/\pi_x)) \text{ and } d_y = \max(0, (1 - y/\pi_y)) . \tag{1}$$



**Fig. 2.** The three-dimensional graphics of the four situations with specific given parameters

Similarly, we use the formula (2) to measure the value of deprivation in a negative indicator dimension:

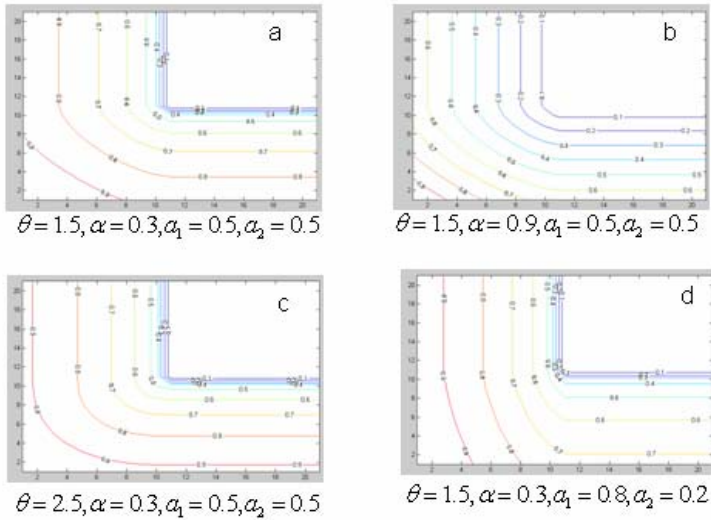
$$d_x = \max(0, (x/\pi_x - 1)) \text{ and } d_y = \max(0, (y/\pi_y - 1)) . \tag{2}$$

In order to get the overall deprivation in two dimensions, Bourguignon and Chakravarty take the weighted mean, and similarly Anand and Sen aggregate sub-indices on a individual scale [8]. This paper uses the method of Foster-Greer-Thorbecke  $P_\alpha$  index [9] to establish a deprivation model which is illustrated in formula (3):

$$D(x, y) = [a_1 d_x^\theta + a_2 d_y^\theta]^{\alpha/\theta} . \tag{3}$$

Where,  $a_1$  and  $a_2$  are the weights of  $x$  and  $y$  respectively, and  $a_1 + a_2 = 1$ . The parameter  $\theta$  alone governs the shape of the contours in  $(x, y)$  space, as illustrated in Figure 2. The parameter  $\alpha$  is used to measure the concavity of the function  $D(x, y)$ .

Figure 2 shows the three-dimensional graphics of the four situations with given parameters. The contours are equal deprivation curves in the region  $(x, y)$ . The contours are centered on  $(\pi_x, \pi_y)$  in the region of double deprivation; the farther the distance is from the point  $(\pi_x, \pi_y)$ , the more the deprivation is. The contours are



**Fig. 3.** The equal deprivation curves on the coupling condition of the positive and positive indicators with different parameters

straight lines in the region of single deprivation and the value of deprivation is zero in the region of no deprivation. Figure 3 shows the equal deprivation curves whose parameters are different on the coupling condition of positive indicator and positive indicator. We can see that the bigger the value of  $\theta$  is, the more curving the contours are and the smaller the value of  $\alpha$  is, the more concentrated the contours are. Comparing Figure 3 (b) with 3 (a), we see that the increase in the value of  $\alpha$  makes the distance between the equal deprivation curves uniform; comparing Figure 3 (c) with 3 (a), we know that the increase in value of  $\theta$  makes the curvature increase; comparing Figure 3 (d) with 3 (a), we see that the increase in the value of  $a_1$  and the decrease in the value of  $a_2$  make the curve peak migrate to the abscissa.

### 3 Application

This paper chooses capita disposable income (positive indicator) and housing price (negative indicator) as evaluation indicators, uses two-dimensional coupling model to evaluate the social deprivation of the 31 provinces in China, not including Taiwan, Hong Kong and Macao of China because of no data. The housing price indicator is obtained by sale revenue of commercial housing dividing sale area of commercial housing. The data of sale revenue of commercial housing, sale area of commercial housing and capita disposable income are from *China Statistical Yearbook 2007*. In this paper,  $\alpha = 0.3$ ,  $\theta = 1.5$ ,  $a_1 = 0.5$ ,  $a_2 = 0.5$ ,  $\pi_x = 11759.45$ , and  $\pi_y = 3382.87$  which uses national capita disposable income and national housing

price as the threshold of  $x$  and  $y$  respectively. This paper uses formula (1),(2) and (3) to calculate  $d_x, d_y$  and  $D$  whose results are illustrated in table 1.

**Table 1.** The deprivation of 31 provinces in China

Province	$y$	$x$	$d_y$	$d_x$	$D$
Beijing	8279.64	19977.52	1.448	0.000	0.973
Shanghai	7196.07	20667.91	1.127	0.000	0.902
Guangdong	4877.41	16015.58	0.442	0.000	0.681
Zhejiang	4802.15	18265.10	0.420	0.000	0.671
Tianjin	4773.76	14283.09	0.411	0.000	0.667
Fujian	3994.16	13753.28	0.181	0.000	0.521
Hainan	3632.17	9395.13	0.074	0.201	0.560
Jiangsu	3588.69	14084.26	0.061	0.000	0.376
Liaoning	3073.42	10369.61	0.000	0.118	0.459
Shandong	2603.23	12192.24	0.000	0.000	0.000
Hubei	2555.80	9802.65	0.000	0.166	0.508
Shanxi	2463.24	9267.70	0.000	0.212	0.547
Yunnan	2345.52	10069.89	0.000	0.144	0.486
Anhui	2322.12	9771.05	0.000	0.169	0.511
Chongqing	2258.09	11569.74	0.000	0.016	0.252
Sichuan	2228.05	9350.11	0.000	0.205	0.541
Heilongjiang	2195.32	9182.31	0.000	0.219	0.552
Xizang	2170.09	8941.08	0.000	0.240	0.567
Hebei	2100.02	10304.56	0.000	0.124	0.465
Ningxia	2063.70	9177.26	0.000	0.220	0.552
Jilin	2028.64	9775.07	0.000	0.169	0.510
Sanxi	1997.13	10027.70	0.000	0.147	0.490
Henan	1982.05	9810.26	0.000	0.166	0.508
Qinghai	1973.57	9000.35	0.000	0.235	0.564
Hunan	1928.67	10504.67	0.000	0.107	0.445
Xinjiang	1843.75	8871.27	0.000	0.246	0.571
Neimenggu	1817.15	10357.99	0.000	0.119	0.460
Guizhou	1805.94	9116.61	0.000	0.225	0.556
Gansu	1778.86	8920.59	0.000	0.241	0.568
Jiangxi	1731.23	9551.12	0.000	0.188	0.527
Guangxi	2195.41	9898.75	0.000	0.158	0.501

The results show that the top five of the overall deprivation are Beijing, Shanghai, Guangdong, Zhejiang and Tianjin and the last five are Liaoning, Hunan, Jiangsu, Chongqing and Shandong. The deprivation of Beijing and Shanghai are the largest which are 0.973 and 0.902 respectively. The deprivation of Shandong province is 0, which indicates that there aren't deprivation in two dimensions. Only Hainan province suffers double deprivation and the deprivation of 17 provinces lies in the interval (0.5, 0.6). Interestingly, the top five provinces of deprivation belong to the eastern region, indicating that despite the high per capita income in these provinces, the housing price is far beyond the limit of tolerance.

## 4 Conclusion

Firstly, this paper qualitatively describes the deprivation under different coupling situations of two-dimensional indicators. And then, this paper establishes a two-dimensional coupling model on social deprivation through using the welfare function method to establish the deprivation model of sub-indicator and Foster-Greer-Thorbecke  $P_\alpha$  method to aggregate sub-deprivation and obtain an overall deprivation. Finally, this paper uses the model to evaluate the social deprivation of 31 provinces in China under the coupling state of capita disposable income and housing price. Empirical results indicate that (1) only Shandong province is zero deprivation; (2) only Hainan suffers with double deprivation; (3) the provinces whose deprivation are the largest belongs to the eastern region. The next work mainly includes the researches on aggregating method of overall deprivation, multi-dimensional deprivation model and the simulation of social deprivation.

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