Structure of Mutualistic Complex Networks

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Abstract. We consider the structures of six plant-pollinator mutualistic networks. The plants and pollinators are linked by the plant-pollinating relation. We assigned the visiting frequency of pollinators to a plant as a weight of each link. We calculated the cumulative distribution functions of the degree and strength for the networks. We observed a power-law, linear, and stretched exponential dependence of the cumulative distribution function. We also calculated the disparity and the strength of the nodes $s(k)$ with degree k. We observed that the plant-pollinator networks exhibit an disassortative behaviors and nonlinear dependence of the strength on the nodes. In mutualistic networks links with large weight are connected to the neighbors with small degrees.

Keywords: complex network, food web, mutualistic network, disparity, disassortative.

1 Introduction

Complex networks appear in many areas such as social, economic, and biological systems[1-20]. Food webs are the well known networks in ecology. Recently the complex structures of the food web are an interesting topic in the complex systems. Ecological networks are very interesting networks with weights of links. The species strongly interact with some species, but another species interact weakly with some species. We can characterize the interacting strength as a weight of link in ecological networks. Therefore, ecological network is a typical example of weighting complex networks.

In ecological network the total number of nodes is smaller than abiotic networks. The total number of nodes is less than a thousand in the reported ecological networks. These small numbers of nodes make difficulty in analysing the ecological network. Network properties of the food webs were reported for many prey-predator and mutualistic networks[21-27]. In the mutualistic network such as plant-pollinator network, the species interact with each other and the network is described by bipartite graph of species interaction[28-37]. The total number of visits by pollinators to plants consider as the strength of interaction. The pollinators have some favoring plants. They visit a plant frequently, but rarely another plants. The weight of link is assigned by the strength of the interaction. In the plant-pollinator network, pollinators prefer some plants. The

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network properties of the mutualistic networks are important to understand the dynamics and diversity of the ecological system. In this work we consider three plant-pollinator mutualistic networks. We calculate the cumulative distribution function of the degree and strength for the networks. We observed very rich functional dependence of the CDF. This article is organized as followings: In section 2 we introduce the information for data. In section 3 we present the results of the CDF for networks and the properties of the mutualistic network. In section 4 we give the concluding remarks.

2 Data of Food Web

We considered six plant-pollinator food webs. We selected these food web because they had enough nodes to analyze the network properties. We presented basic informations for networks in Table 1. Bar food web (Barrett and Helenurm 1987) was a breal forest in Canada with $N_{po} = 102$ pollinators and $N_{pl} = 12$ plants[32]. Ino food web (Inouye and Pkye 1998) belonged to montane forest which consists of $N_{po} = 91$ pollinators and $N_{pl} = 42$ plants[33]. Mem food web (Mommett 1999) had $N_{po} = 79$ pollinators and $N_{pl} = 25$ plants and it belongs to meadow food web[34]. Kat food web (Kato *et al.* 1990) was a beech forest food web and contained $N_{po} = 679$ pollinators and $N_{pl} = 93$ plants[35]. Oll food web (Ollerton et al. 2003) was an upland grassland which consists of $N_{po} = 56$ pollinator and $N_{pl} = 9$ plants[36]. Sma food web (Small 1976) was a peat bog food web and it contained $N_{po} = 34$ pollinators and $N_{pl} = 13$ plants[37]. We summarized the total number of links L and connectance $C = L/N^2$ where $N = N_{po} + N_{pl}$ is the total number of species in the food web.

Table 1. Structural properties of six plant-pollinator food web

food web	type	pollinator	plant	link	weight	connectance
Bar	boreal forest	102	12	167	550	0.0129
Ino	montane forest	91	42	281	1459	0.0159
Kat	beech forest	679	93	1202	2384	0.0020
Mem	meadow	79	25	299	2183	0.0276
Oll	upland grassland	56	9	103	594	0.0244
Sma	peat bog	34	13	141	992	0.0638

3 Structure of Mutualistic Networks

We calculated the network properties from the mutualistic networks such as the cumulative probability distribution function(CDF) of the degree and strength, the average disparity $Y(k)$, and the average strength $s(k)$ of the node with the degree k. When we calculated the probability distribution function, we considered all species in the food web including the pollinators and plants. We assigned

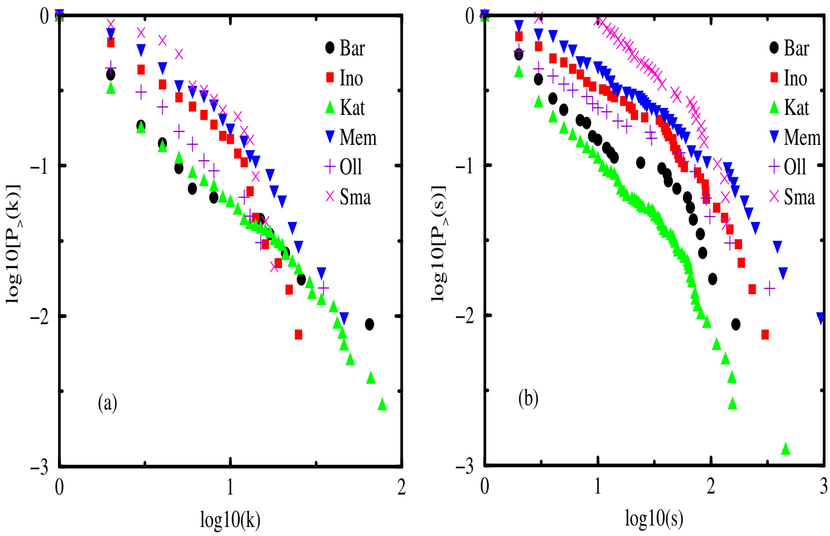

Fig. 1. The log-log plot of the cumulative probability distribution function of (a) the degree and (b) strength for the six mutualistic pollinator-plant networks. When we calculate the distribution function, we include all species in the network.

the weighting w_{ij} of the link as the visiting frequencies of the pollinators to the plants. Then, the network constructed was a weighted mutualistic network.

In Fig. 1 we present the CDF of the degree amd strength for all species in the six food webs. The degree distribution is not universal and it depends on the food webs. The CDF of the degree for all species also follows the power law except Mem and Sma food web. The stretched exponential functions are the best fitting functions in Ino, Mem and Sma food webs. Similarly, we also consider the CDF of the strength for all species. We observe the CDF of the strength for all species follows the power law. The CDF of the strength in the Mem food web,we observe two scaling regions. We summerize the functional forms of the CDF.

Table 2. The functional forms of the cumulative probability distribution function for the degree and strength in six plant-pollinator food webs. PL means the power-law and PL(2) means two regions of the power-law. SE means the stretched exponential dependence.

food web	degree	strength
Bar	PL	PL
Ino	SE	PL
Kat	PL	PL
Mem	SE	PL(2)
Oll	PL	PL
Sma	SE	PL

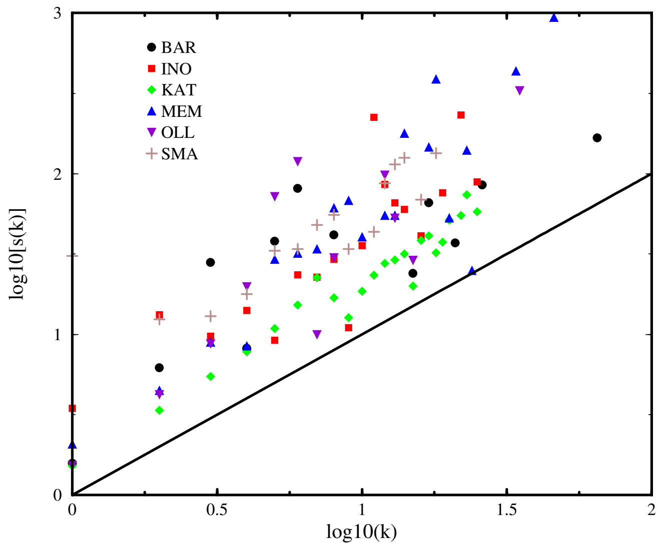

Fig. 2. The log-log plot of the average strength as a function of the degree. The solid line is a linear function $s(k) = k$.

The strength s_i of the node is defined by

$$s_i = \sum_{j=N_i} w_{ij} \tag{1}$$

where N_i is the nearest neighbors of the species i. In the homogeneous linear weighted network, the average strength $s(k)$ is proportional to the degree k. In Fig. 2 we present the average strength as a function of the degree. All mutualistic network shows the nonlinear dependence on the degree. The average strength has the functional form like $s(k) \sim k^{\beta}$. We calculate the exponents β by the least square fits. We obtain the exponents, β =0.9(2)(Bar), 1.1(2)(Ino), 1.11(5)(Kat), 1.43(7)(Mem), 1.3(2)(Oll), and 0.74(16)(Sma).

The disparity of a node i is defined by

$$Y_i = \sum_{j=N_i} \left(\frac{w_{ij}}{s_i}\right)^2 \tag{2}$$

If the links have the comparatively same weights, then the average disparity is inversely proportional to the degree, $Y(k) \sim 1/k$. If the weight of a link or a few link are dominated, the average disparity is constant, $Y(k) \sim 1$. In general the average disparity shows the power law, $Y(k) \sim k^{-\delta}$. We observe that the mutualistic network exhibits an disassortative properties. Larger weight links are connected to the neighbor with smaller degree, or vice verse. In Fig. 3 we show the average disparity as a function of the degree. The data points are above the

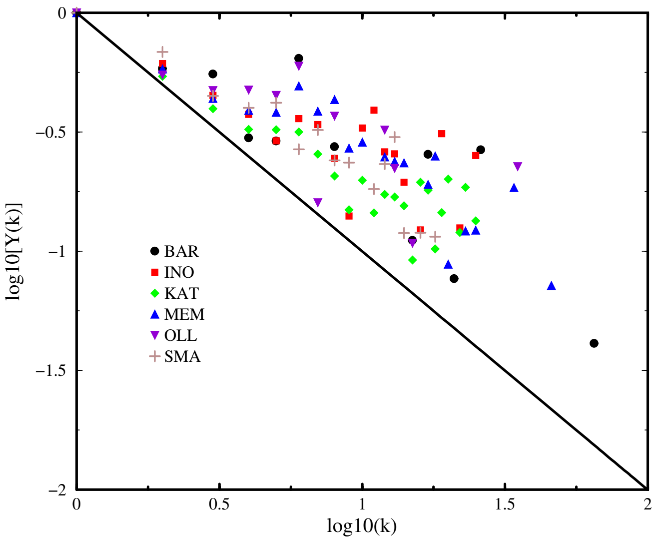

Fig. 3. The log-log plot of the disparity $Y(k)$ as a function of the node. The solid line is a inverse function $Y(k) = 1/k$.

$1/k$ function. We obtain the exponents δ =0.7(1) (Bar), 0.49(9) (Ino), 0.58(6) (Kat), 0.62(7) (Mem), 0.5(1) (Olll), and 0.71(7) (Sma).

In the mutualistic network, the network is strongly influenced by the competitions between the pollinators to get the pollens and the conditions of the environment. The plant prefers to connect to many pollinators to disperse their pollens. However, the pollinators favor plants to collect their foods easily. These competing interactions determine the structure of the networks.

4 Conclusions

We consider six mutualistic pollinators-plant food webs. The CDF of the degree is classified into the power-law and stretched exponential functions. The CDF of the strength follows the power law in all food webs. We calculate the average strength as a function of the degree and the average disparity. The average strength shows a nonlinear dependence on the degree. From the average disparity we observed that the pollinator-plant mutualistic network exhibits a disassortative behavior.

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