Synchronization of Complex Networks with Time-Varying Coupling Delay via Impulsive Control

Yang Dai[1], Yunze Cai[1], and Xiaoming Xu[1,2,3]

[1] Department of Automation, Shanghai Jiao Tong University, P.R. China
daiyang1980@gmail.com
[2] University of Shanghai For Science and Technology, P.R. China
[3] Shanghai Academy of Systems Science, P.R. China

Abstract. Impulsive control and exponential synchronization analysis of a class of complex networks with time-varying coupling delay is investigated in this paper. Our aim is to enhance the synchronizability of the complex networks by applying impulsive control. By introducing a comparison system and estimating the corresponding Cauchy matrix sufficient conditions on global exponential synchronization are derived. An impulsive controller is explicitly designed not only to achieve synchronized dynamics for the complex networks, but simultaneously to ensure the states of synchronous error converging with a given decay rate. A numerical example is presented to illustrate the theoretical results and proposed controller design procedure.

Keywords: Complex Networks, Impulsive Control, Synchronization, Delay.

1 Introduction

In recent years, complex networks have been extensively studied by researchers from disciplines as diverse as physics, chemistry, biology, social science, telecommunication and engineering [1,2,3,4] since the remarkable small-world networks [5] and scale-free networks [6] were put forward. With the dynamical units regarded as nodes and the interplay between them expressed by the links of nodes, network topology structure provides a powerful metaphor for describing sophisticated collaborative dynamics of many practical systems in essence.

As a collective dynamical behavior, synchronization of complex networks has been actively investigated in the literature [7,8,9,10,11,12,13]. There are different types of definition for synchronization, including complete or identical synchronization, phase synchronization, lag synchronization, generalized synchronization and anticipating synchronization. The emergence of synchronous phenomenon has a close relation with network attributes, such as network topology structure, average path length, degree distribution, betweenness and coupling delays [11,12,13,14]; hence much effort has been made to assess and compare synchronization propensity for different complex networks. On the other

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hand, some control strategies have been developed to achieve the synchronized dynamics for complex networks. In the pinning control technique [15], a small fraction of network nodes are chosen to carry out local feedback control strategy and ultimately the dynamics of the whole network was pinned to its equilibrium state. Some other control algorithms, such as adaptive control [16], state feedback control [17] and hybrid control [18], are also proposed as strategies to synchronize the interacting dynamical nodes toward an identical orbit.

As is well known, impulsive control is characterized by the abrupt changes in the system dynamics at certain instants, which is an advantage in reducing the amount of information transmission and improving the security and robustness against disturbances especially in telecommunication network and power grid [19]. In some cases, the scheme of impulsive control cannot be substituted by continuous control. For example in a financial system, with the amount of money in a market and saving rates of a central bank serving as two state variables, a monetary control policy is usually implemented at some particular instants. Considering these traits, impulsive control has been introduced into complex networks to achieve the synchronous dynamics [20,21,22]. Time delays are ubiquitous in biological and physical networks and often their variation is too significant to be ignored owing to the finite speeds of transmission [23]. However, most available literature on impulsive synchronization only take account of constant delay case for simplicity.

In this note, we investigate the impulsive exponential synchronization of complex networks with time-varying coupling delays. A dynamical network model is given and reformulated into the direct product form. By some transformations, the impulsive synchronization problem of the complex network is converted equally into the stability problem of the impulsive control system with time delays. Further, some sufficient conditions of impulsive synchronization are derived based on the comparison method and Cauchy matrix estimating. The impulsive controller is delicately designed, which can ensure the network dynamics exponentially synchronizing with a given decay rate. It is noteworthy that the purpose of introducing impulsive control is not to compete with the other control schemes, but to provide a new viewpoint to deal with the specific synchronization problem.

The rest of the paper is organized as follows. In section 2, complex network model is presented, and some relevant definitions and lemmas are presented. Section 3 deals with the synchronization analysis and gives the controller design procedure in detail. In Sectionx 4, theoretical results are verified by a numerical example to illustrate their effectiveness. Conclusions are drawn in Section 5.

Notation: The notation used throughout the paper is fairly standard. Let $\mathbb{N}$ be the set of natural number; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; Let $A \otimes B$ be the direct product of the matrices A and B. For a matrix A, the largest eigenvalue and the smallest one are denoted by $\lambda_{max}(A)$ and $\lambda_{min}(A)$, respectively; the induced matrix norm and matrix measure are

$$\|A\| = \sqrt{\lambda_{max}(A^T A)}, \quad \mu(A) = \frac{1}{2}[\lambda_{max}(A^T + A)].$$

2 Problem Formulation and Preliminaries

Consider the ensemble of N identical diffusively coupled nodes, with each one being an n-dimensional dynamical system. The proposed weighted dynamical network is described by

$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t)) + \sum_{j=1}^{N} G_{ij}\Gamma x_j(t-\tau(t)), \qquad i = 1, \ldots, N \tag{1}$$

and the initial condition function

$$x_i(\theta) = \varphi_i(\theta),\ \ \theta \in [-\bar{\tau}, 0].$$

where $t \in \mathbb{R}$ is the continuous time variable, $x_i \in \mathbb{R}^n$ is the state variable of node i, f is continuously differentiable map, $\tau(t)$ is bounded time-varying delay with $0 < \tau \leq \bar{\tau}$, A is the constant matrix, and Γ is a constant inner coupling matrix of the nodes of network at time t, $G = (G_{ij}) \in \mathbb{R}^{N\times N}$ is the outer-coupling matrix combining both configuration and weights of the entire networks. Accordingly, this is a general weighted complex network [9].

We design an impulsive control law as below to globally exponentially synchronize the complex dynamical network (1)

$$U_i(k, x_i(t_k)) = D_{ik}x_i(t_k), \qquad i = 1, \ldots, N, \quad k \in \mathbb{N} \tag{2}$$

The complex networks under impulsive control is obtained

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + f(x_i(t)) + \sum_{j=1}^{N} G_{ij}\Gamma x_j(t-\tau(t)), & t \in (t_k, t_{k+1}] \\ \triangle x_i(t) = x_i(t_k^+) - x_i(t_k^-) = D_{ik}x_i(t_k^-), & k \in \mathbb{N} \\ x_i(\theta) = \varphi_i(\theta), & \theta \in [-\bar{\tau}, 0] \end{cases} \tag{3}$$

It is assumed that $x_i(t)$ is left continuous at $t = t_k$, that is, $x_i(t_k) = x_i(t_k^-)$. Therefore, the solutions to (3) are piecewise left-hand continuous functions with discontinuities only at t_k.

Let $\mathbf{x}(t) = [x_1^T(t), \ldots, x_N^T(t)]^T$, $\mathbf{F}(\mathbf{x}(t)) = [f^T(x_1(t)), \ldots, f^T(x_N(t))]^T$, $\mathbf{D}_k = \mathrm{diag}\{D_{1k}, \ldots, D_{Nk}\}$, then the coupled dynamical network (3) can be rewritten in the compact form

$$\begin{cases} \dot{\mathbf{x}}(t) = (I_N \otimes A)\mathbf{x}(t) + \mathbf{F}(\mathbf{x}(t)) + (G\otimes\Gamma)\mathbf{x}(t-\tau(t)), & t \in (t_k, t_{k+1}] \\ \triangle\mathbf{x}(t) = \mathbf{x}(t_k^+) - \mathbf{x}(t_k) = \mathbf{D}_k\mathbf{x}(t_k), & k \in \mathbb{N} \\ \mathbf{x}(\theta) = \varphi(\theta), & \theta \in [-\bar{\tau}, 0]. \end{cases} \tag{4}$$

Our aim is to find a set of sufficient conditions on the impulsive control time sequence t_k and the constant control gain matrices $\mathbf{D}_k$, such that the closed loop complex network (4) is exponentially synchronized with a given decay rate.

In the sequel, we present some definitions and useful lemmas required throughout this paper.

Definition 1 ([13])**.** The hyperplane

$$\mathcal{S} = \{(x_1^T, \ldots, x_N^T)^T \in \mathbb{R}^{n\times N}; x_i = x_j; i, j = 1, \ldots, N.\} \tag{5}$$

is said to be synchronization manifold of dynamical complex network (1).

Let $\mathrm{S}(t) = (s^T(t), \ldots, s^T(t))^T \in \mathcal{S}$ be the synchronized state, where $s(t)$ is the solution to $\dot{s}(t) = As(t) + f(s(t))$, which may be an equilibrium, aperiodic trajectory or a chaotic attractor of the uncoupled dynamical behavior of each node.

Definition 2. The complex network (1) is said to be globally exponentially synchronized with respect to the state $\mathrm{S}(t)$ via impulsive controller $\mathbf{U}(k, \mathbf{x}(t_k)) = \mathbf{D}_k\mathbf{x}(t_k)$ $(k \in \mathbb{N})$, if for any initial function $\varphi(\theta)$, there exist the constant $\lambda > 0$ and $M > 0$ such that the following exponential estimates hold:

$$\|\mathbf{x}(t) - \mathrm{S}(t)\| \leq Me^{-\lambda t}\|\varphi(\theta)\|, \tag{6}$$

where λ is called the decay rate of exponential synchronization.

Lemma 1 ([24])**.** Let $P \in \mathbb{R}^{n\times n}$ be a symmetric positive definite matrix and $P = Q^TQ$. For any $x, y \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n\times n}$, then

1. $x^TA^TPAx \leq \|QAQ^{-1}\|^2x^TPx$
2. $x^T(A^TP + PA)x \leq 2\mu(QAQ^{-1})x^TPx$
3. $|x^TPy| \leq \sqrt{x^TPx}\sqrt{y^TPy}$

Lemma 2 ([25])**.** For the matrices A, B and the scalar α, by the definition of direct product, the following properties are satisfied:

1. $(\alpha A) \otimes B = A \otimes (\alpha B)$
2. $(A + B) \otimes C = (A \otimes C) + (B \otimes C)$
3. $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$

3 Main Results

In this section, the global exponential synchronization conditions of complex dynamical networks are presented. Moreover, the controller design procedure is given to ensure the network dynamics converging with a given decay rate.

3.1 Synchronization Analysis

Define the network synchronization error of node i with respect to $s(t)$ as $e_i(t) = x_i(t) - s(t)$ and $\mathbf{e}(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$, then the error dynamical network is denoted by:

$$\begin{cases} \dot{\mathbf{e}}(t) = (I_N \otimes A)\mathbf{e}(t) + \mathbf{F}(\mathbf{e}(t)) + (G \otimes \Gamma)\mathbf{e}(t - \tau(t)), & t \in (t_{k-1}, t_k] \\ \mathbf{e}(t_k^+) = \mathbf{x}(t_k^+) - \mathrm{S}(t_k^+) = (E + \mathbf{D}_k)\mathbf{e}(t_k), & k \in \mathbb{N} \end{cases} \tag{7}$$

where $\mathbf{F}(\mathbf{e}(t)) = \left[f^T(x_1) - f^T(s), \ldots, f^T(x_1) - f^T(s)\right]^T$, I_N is a $N \times N$ dimension identity matrix and E is a $Nn \times Nn$ dimension one. It may be seen that the exponential stability of the impulsive time delay system (7) is equivalent to the impulsive synchronization of the complex network (1) with respect to $\mathrm{S}(t)$.

Assumption 1 (A1)**.** Suppose that there exists a nonnegative constant L_i ($i = 1, \ldots, N$) such that

$$\|f(x_i, t) - f(s, t)\| \leq L_i \|x_i(t) - s(t)\| = L_i \|e_i(t)\|, \tag{8}$$

We may obtain that

$$\|\mathbf{F}(\mathbf{e}(t))\| \leq \max\{L_1, \ldots, L_N\} \|e(t)\| = \mathbf{L} \|\mathbf{e}(t)\|, \tag{9}$$

where $\mathbf{L} = \max\{L_1, \ldots, L_N\}$ is a positive constant.

Theorem 1. Under A1, if there exist $0 < \rho = \sup_{k \in \mathbb{N}}\{t_k - t_{k-1}\} < \infty$ and a nonsingular matrix $Q \in \mathbb{R}^{Nn \times Nn}$ such that

$$-p > q \tag{10}$$

$$\|E + Q\mathbf{D}_k Q^{-1}\| \leq \beta, \ 0 < \beta < 1 \tag{11}$$

where

$$p = \frac{2 \ln \beta}{\rho} + 2\mu\left[Q(I_N \otimes A)Q^{-1}\right] + 2\mathbf{L}\frac{\lambda_{max}(Q^T Q)}{\lambda_{min}(Q^T Q)} + 1,$$
$$q = \beta^{-2} \|Q(G \otimes \Gamma)Q^{-1}\|^2,$$

then the complex dynamical network (1) is globally synchronized via impulsive control (2) in the following sense:

$$\|\mathbf{x}(t) - \mathrm{S}(t)\| \leq M e^{-\frac{\lambda}{2}t}, \quad t \geq 0 \tag{12}$$

where $M = \frac{1}{\beta}\sqrt{\frac{\lambda_{max}(Q^T Q)}{\lambda_{min}(Q^T Q)}} \sup_{-\bar{\tau} \leq \theta \leq 0}\{\|\phi(\theta)\|\}$, $\lambda > 0$ is the solution of

$$\lambda + p + q e^{\lambda \bar{\tau}} = 0. \tag{13}$$

Proof. Choose a Lyapunov function as follows:

$$V(t) = \mathbf{e}^T(t) P \mathbf{e}(t), \tag{14}$$

where P be a symmetric positive matrix and $P = Q^T Q$.

The derivative of V along the trajectories of the error dynamical network (7) is given by

$$\begin{aligned}
\dot{V}(t) =&\mathbf{e}^T(t)\big[P(I_N\otimes A)+(I_N\otimes A)^TP\big]\mathbf{e}(t)+2\mathbf{e}^T(t)P\mathbf{F}(\mathbf{e}(t))\\
&+2\mathbf{e}^T(t)P(G\otimes\Gamma)\mathbf{e}(t-\tau(t))\\
\le&2\mu\big[Q(I_N\otimes A)Q^{-1}\big]\mathbf{e}^T(t)P\mathbf{e}(t)+2\mathbf{L}\lambda_{max}P\|\mathbf{e}(t)\|^2+2\sqrt{e^T(t)Pe(t)}\\
&\times\sqrt{e^T(t-\tau(t))(G\otimes\Gamma)^TP(G\otimes\Gamma)e(t-\tau(t))}\\
\le&2\mu\big[Q(I_N\otimes A)Q^{-1}\big]\mathbf{e}^T(t)P\mathbf{e}(t)+2\mathbf{L}\frac{\lambda_{max}(P)}{\lambda_{min}(P)}\mathbf{e}^T(t)P\mathbf{e}(t)+\mathbf{e}^T(t)P)\mathbf{e}(t)\\
&+\|Q(G\otimes\Gamma)Q^{-1}\|^2\mathbf{e}^T(t-\tau(t))P)\mathbf{e}(t-\tau(t))\\
\le&\Big(2\mu\big[Q(I_N\otimes A)Q^{-1}\big]+2\mathbf{L}\frac{\lambda_{max}(P)}{\lambda_{min}(P)}+1\Big)V(t)\\
&+\|Q(G\otimes\Gamma)Q^{-1}\|^2V(t-\tau(t)),\qquad t\in(t_{k-1},t_k]
\end{aligned}$$

And also

$$\begin{aligned}
V(t_k^+) =&\mathbf{e}^T(t_k)(E+\mathbf{D}_k)^TP(E+\mathbf{D}_k)\mathbf{e}(t_k)\\
\le&\|E+Q\mathbf{D}_kQ^{-1}\|^2V(t_k)\\
\le&\beta^2V(t_k),\qquad k\in\mathbb{N}
\end{aligned}$$

Let $\epsilon>0$ be an arbitrary constant. Construct the comparison system as follows:

$$\begin{cases}
\dot{\nu}(t) =\Big(2\mu\big[Q(I_N\otimes A)Q^{-1}\big]+2\mathbf{L}\dfrac{\lambda_{max}(P)}{\lambda_{min}(P)}+1\Big)\nu(t)\\
\qquad\qquad +\|Q(G\otimes\Gamma)Q^{-1}\|^2\nu(t-\tau(t))+\epsilon,\quad t\in(t_{k-1},t_k],\\
\nu(t_k^+) =\beta^2\nu(t_k),\qquad k\in\mathbb{N}\\
\nu(\theta) =\lambda_{max}(P)\|\phi(\theta)\|^2,\qquad -\bar{\tau}\le\theta\le 0
\end{cases}$$

It may be seen that $V(\theta)\le\nu(\theta)$, for $-\bar{\tau}\le\theta\le 0$. According to [24], it leads to

$$V(t)\le\nu(t),\qquad\text{for}\quad t\ge 0$$

The trivial solution of the comparison system is

$$\nu(t)=W(t,0)\nu(0)+\int_0^t W(t,s)\Big(\|Q(G\otimes\Gamma)\times Q^{-1}\|\nu(s-\tau(s))+\epsilon\Big)ds,\quad t\ge 0$$

where $W(t,s)$ is the Cauchy matrix and estimated by

$$\begin{aligned}
W(t,s) =&\beta^{2\eta(t,s)}\exp\big\{1+2\mathbf{L}\frac{\lambda_{max}(P)}{\lambda_{min}(P)}+2\mu[Q(I_N\otimes A)Q^{-1}](t-s)\big\}\\
\le&\beta^{2(\frac{t-s}{\rho}-1)}e^{(p-\frac{2\ln\beta}{\rho})(t-s)}\\
=&\beta^{-2}e^{p(t-s)}
\end{aligned}$$

in which $\eta(t,s)$ is the number of the control impulses in the interval $(s,t]$. Accordingly, for $t>0$, we have

$$\begin{aligned}\nu(t) \leq& \beta^{-2}\lambda_{max}(P)\|\phi(0)\|^2+\int_0^t \beta^{-2}e^{p(t-s)}\big(\|Q(G\otimes \Gamma)Q^{-1}\|^2\nu(s-\tau(s))+\epsilon\big)ds\\ \leq& \gamma e^{pt}+\int_0^t e^{p(t-s)}\big(q\nu(s-\tau(s))+\epsilon\big)ds\end{aligned} \tag{15}$$

where $\gamma=\beta^{-2}\lambda_{max}(P)\sup_{-\bar{\tau}\leq s\leq 0}\|\phi(s)\|^2$.

In the following, we utilize the method of reduction to absurdity to verify that

$$\nu(t)\leq \gamma e^{-\lambda t}-\frac{\epsilon}{\beta^2(p+q)}, \qquad t\geq 0 \tag{16}$$

Since $\epsilon>0$ and $-p>q$, then we have $\frac{\epsilon}{\beta^2(p+q)}<0$. Firstly, it is assumed that there exists a $t^*>0$ such that

$$\nu(t^*)\geq \gamma e^{-\lambda t^*}-\frac{\epsilon}{\beta^2(p+q)}, \tag{17}$$

$$\nu(t)< \gamma e^{-\lambda t}-\frac{\epsilon}{\beta^2(p+q)}, \qquad t<t^* \tag{18}$$

According to (13), (15) and (18), it holds

$$\begin{aligned}\nu(t^*) \leq& \gamma e^{pt^*}+\int_0^{t^*} e^{p(t^*-s)}[q\nu(s-\tau(s))+\epsilon]ds\\ <& e^{pt^*}\{\gamma-\frac{\epsilon}{\beta^2(p+q)}+\int_0^{t^*} e^{-ps}[\gamma q e^{-\lambda(s-\tau(s))}-\frac{\epsilon q}{\beta^2(p+q)}+\frac{\epsilon}{\beta^2}]ds\}\\ \leq& e^{pt^*}\{\gamma-\frac{\epsilon}{\beta^2(p+q)}+\gamma q e^{\lambda\bar{\tau}}\int_0^{t^*} e^{-(p+\lambda)s}ds+\frac{\epsilon p}{\beta^2(p+q)}\int_0^{t^*} e^{-ps}ds\}\\ \leq& e^{pt^*}\{\gamma-\frac{\epsilon}{\beta^2(p+q)}+\gamma[e^{-(p+\lambda)t^*}-1]-\frac{\epsilon}{\beta^2(p+q)}(e^{-pt^*}-1)\}\\ =& \gamma e^{-\lambda t^*}-\frac{\epsilon}{\beta^2(p+q)}\end{aligned}$$

This contradicts (17), thus the assumption is not tenable and the estimate (16) holds. Let $\epsilon\to 0$, then

$$V(t)\leq \nu(t)\leq \gamma e^{-\lambda t}, \qquad t\geq 0 \tag{19}$$

Moreover,

$$V(t)\geq \lambda_{min}(I_N\otimes P)\|\mathbf{e}(t)\|^2, \qquad t\geq 0 \tag{20}$$

Combining the inequality of Eq. (19) and Eq. (20),

$$\|\mathbf{e}(t)\|\leq \frac{1}{\beta}\sqrt{\frac{\lambda_{max}(P)}{\lambda_{min}(P)}}\sup_{-\bar{\tau}\leq\theta\leq 0}\{\|\phi(\theta)\|\}e^{-\frac{\lambda}{2}t}$$

which implies the conclusion (12) and this completes the proof. □

Due to the limited space, similar results for the case $\beta\geq 1$ is omitted.

3.2 Impulsive Controller Design

In the sequel, a design procedure of impulsive controller is provided based on Theorem 1. For a given scalar $\lambda_0 > 0$, we shall provide a set of steps such that the complex network (1) may be synchronized under the impulsive control (2) with an exponential decay rate $\lambda \geq \lambda_0$.

Design procedure:

1. Calculate the parameters $\mathbf{L}$,$\bar{\tau}$;
2. Choose a symmetric positive definite matrix P, which is factorized as $P = Q^T Q$. Select the matrices series $\{\mathbf{D}_k\}$ such that $\|E + Q\mathbf{D}_k Q^{-1}\| \leq \beta$;
3. For a given λ_0, determine the set of the impulsive control instants $\{t_k\}$, $t \in \mathbb{N}$ as below: let $\Theta := 2\mu\big[Q(I_N \otimes A)Q^{-1}\big] + 2\mathbf{L}\frac{\lambda_{max}(P)}{\lambda_{min}(P)} + 1 + \beta^{-2}\|Q(G \otimes \Gamma)Q^{-1}\|^2\big(1 + e^{\lambda_0 \bar{\tau}}\big) > 0$, then the upper bound of control intervals can be taken as $\rho = \sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\} = -(ln\beta)/\Theta$.

4 Application to the Network of Coupled Lorenz Oscillators

In this section, a numerical example is presented to illustrate the effectiveness of derived results.

For the sake of simplicity, consider a dynamical network consisting of 6 identical Chen systems. A single Chen system is described by

$$\begin{pmatrix} \dot{x}_{i1} \\ \dot{x}_{i2} \\ \dot{x}_{i3} \end{pmatrix} = \begin{pmatrix} a(x_{i2} - x_{i1}) \\ (c - a)x_{i1} - x_{i1}x_{i3} + cx_{i2} \\ x_{i1}x_{i2} - bx_{i3} \end{pmatrix}, \tag{21}$$

with $a = 35, b = 3, c = 28$. For these parameter settings the dynamics of the system has a chaotic attractor as shown in Fig.1.

Rewrite (21) in the Lur'e form as

$$x_i = Ax_i + f(x_i),$$

where

$$A = \begin{pmatrix} -a & a & 0 \\ c - a & c & 0 \\ 0 & 0 & -b \end{pmatrix}, \quad f(x_i) = \begin{pmatrix} 0 \\ -x_{i1}x_{i3} \\ x_{i1}x_{i2} \end{pmatrix},$$

As shown in Fig.1, the trajectory of Lorenz attractor is restricted in a bounded region $\Omega \in \mathbb{R}^3$. An estimation of the upper bound for a chaotic system is $R = 30.23$, such that $|x_{ij}| < R$, $s_j < R$ $(i = 1, \ldots, N;\ j = 1, 2, 3)$. Thus

$$\|f(x_i) - f(s)\| < 2R\|e_i\|.$$

Since each node has the same dynamics and the identical bound of trajectory, then we have $\mathbf{L} = 2R = 60.46$.

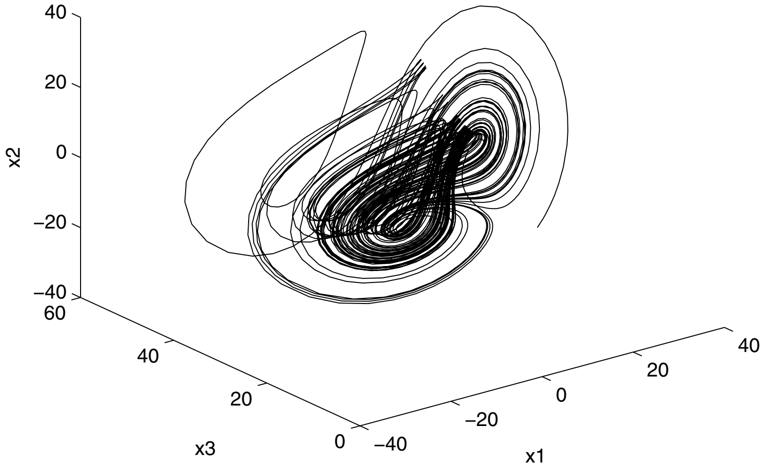

Fig. 1. Dynamical behavior of Chen chaotic system.$(a = 35, b = 3, c = 28)$

Without loss of generality, we assume that the asymmetric outer-coupling matrix is

$$G = \begin{pmatrix} -3 & 1 & 1 & 0 & 0 & 1 \\ 3 & -4 & 0 & 1 & 0 & 0 \\ 1 & 0 & -5 & 2 & 0 & 2 \\ 1 & 1 & 2 & -5 & 1 & 0 \\ 0 & 0 & 0 & 1 & -3 & 2 \\ 2 & 0 & 2 & 0 & 2 & -6 \end{pmatrix},$$

and the inner-coupling matrix is

$$\Gamma = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

Let $\tau(t) = 0.05 \sin t$, which is bounded by $\bar{\tau} = 0.05$. Take $Q = 0.1E$ and $P = Q^T Q$. Select the impulsive feedback controller gain as $D_{ik} = \text{diag}\{-0.8, -0.8, -0.8\}$, then $\beta = \|E + \mathbf{D}_k\| = 0.2 < 1$. Let the decay rate $\lambda_0 = 50$, then $\Theta = 224.4997$. Accordingly, the upper bound of the impulsive interval is $\rho = \sup_{k\in\mathbb{N}}\{t_k - t_{k-1}\} = -\ln\beta/\Theta = 7.2 \times 10^{-3}$.

We implement the equidistant impulsive control and denote $\Delta T = t_{k+1} - t_k = 5 \times 10^{-3}$, $k \in \mathbb{N}$. Define the i-th element of the synchronous state error between node 1 and node i as $e_{ji} = x_{j1} - x_{ji} (j = 1, 2, 3$ and $i = 2, \ldots, 6)$, then the results of the dynamical network under impulsive control law $\{t_k, \mathbf{D}_k\mathbf{x}(t_k)\}$ is shown in Fig. 2. For comparison, state error of original complex networks without controller is visualized in Fig.3.

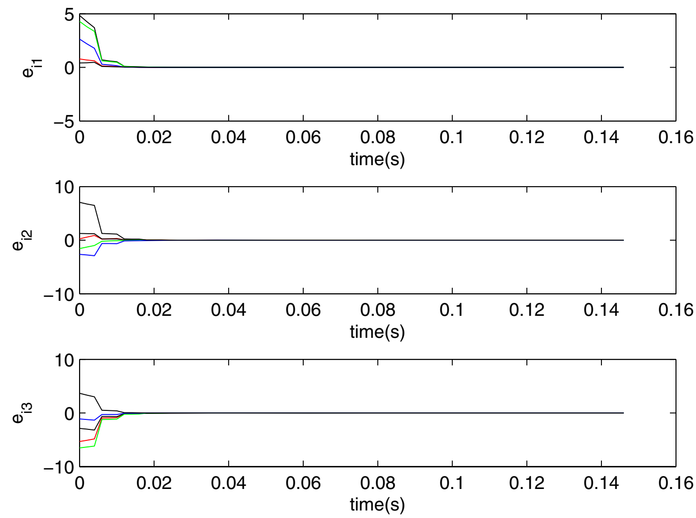

Fig. 2. Synchronous state error for the complex networks via impulsive control. ($\lambda_0 = 50$, $\Delta T = t_{k+1} - t_k = 5 \times 10^{-3}$, $D_{ik} = \mathrm{diag}\{-0.8, -0.8, -0.8\}$).

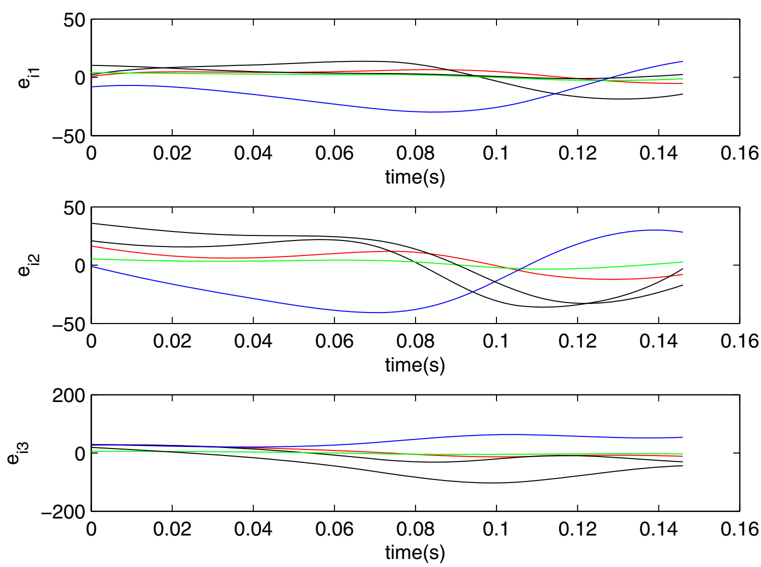

Fig. 3. Synchronous state error for the complex networks without control

From the Fig. 2 and Fig. 3, we can see that synchronizability of the complex networks has been significantly improved by introducing impulsive controller strategy. On the other hand, control actions are performed for the dynamical network at every 0.005 interval, which is prominent in reducing information exchange in the circumstance of large scale network.

5 Conclusion

In this paper, the global exponential synchronization of complex dynamical networks via impulsive control is investigated. A general model of network consisting of time-varying coupling delays has been formulated and the synchronous sufficient conditions have been established. An impulsive controller is designed and analyzed, which may ensure the dynamical networks achieve synchronization with a given decay rate. Compared with the other control methods in the literature, our control scheme is efficient and practical in dealing with synchronization problems particularly in the mass data transmission circumstances. As application, the numerical simulations of coupled Chen systems are given to demonstrate the usefulness and practicability of proposed theoretical results.

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