Spiral Waves Emergence in a Cyclic Predator-Prey Model

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Abstract. Based on a cyclic predator-prey model of three species, spiral waves on global level of the system are obtained. It is found that the predation intensity greatly affects on the behaviors of spiral waves. The wavelength of spiral waves alter with the mobility in the form of $\lambda \sim D^{\theta}$. Values of θ are determined by predation rates between species. It indicates the behaviors of spiral waves varying with mobility are universal at the same predation rate which reveals competition of resources among species.

Keywords: cyclic predator-prey model, pattern formation, spiral waves.

1 Introduction

Spatial distribution of individuals is a common feature of ecological systems. Recently, spatial heterogeneity of species has attracted much attention because it is closely related with the stability and coexistence of species in ecological and evolutionary systems [1,2,3]. Two factors concern spatial heterogeneity as well as spatial patterns in which populations distribute spatially and individuals interact locally. The first is internal noise which induce spatio-temporal pattern of species in concerning range [4,5]. The second is predation intensity of species [6]. In this paper, we systemically investigate spiral waves emergence on global level of the system concerning the two factors.

Cyclical interactions of species which dominate each other in a cyclic manner emerge widely in nature such as rodents in the hight-Arctic tundra in Greenland [7], lizards in the inner Coast Range of California [8], and microbial populations of E.coli [9,10]. Recent experiment reveals that cyclical interactions promote biodiversity of three strains of E.coli [10]. Reichenbach *et al.* have found that noise induced by mobility of individuals greatly affects the biodiversity and spatial heterogeneity [4,5]. However, the intensity of interaction is seldom concerned.

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In community food webs, each species have different predation intensity which exhibits as the intensity of interaction among species. Predation intensity has been confirmed to be strong coupled with diversity in ecological systems [6] and invoked as an evolutionary force [11,12]. However, the effect of predation intensity on spatial heterogeneity is not very clear when diversity is promoted.

In this paper, we investigate the effect of predation intensity on spiral waves induced by mobility of individuals. It is found that predation rates affect the local interactions of species which display global effect via mobility of individuals. Spiral waves emerge when global oscillations are achieved. And the wavelength of spiral waves is satisfied power law with the mobility of individuals, $\lambda \sim D^{\theta}$. The value of θ is determined by the predation rate. It is confirmed that the behaviors of wavelength altering with mobility of individuals is universal at the same value of predation rate. In addition, preying rates are related with vacancy resources in the systems. Our work provides basic understanding of effects of predation intensity on the spatial heterogeneity of species as well as pattern formation.

2 Model

Based on the previous work of Reichenbach *et al.* [4,5], we introduce a cyclic predator-prey model: nodes of spatial lattice present mobile individuals of three species (marked by 1, 2, 3)in the microcosmic bacteria system. Each node can be located at most one individual of a species or a vacancy (denoted by V) which presents resource. There are three interactions, namely predation, reproduction, and exchange which only occur between neighboring nodes. *Predation.*— 1 beats 2 at a selection rate α and 2 becomes a vacancy, in the same way, 2 beats 3, and 3 beats 1. *Reproduction.*— An individual can reproduce an offspring to a neighboring V node at a rate of β. *Exchange.*— An individual could exchange positions with one of its neighbors at a rate γ due to its mobility.

Unlike the deterministic approach which regards the time evolution as a continuous process, here, the applied stochastic approach regards the time evolution as a kind of random-walk process. A standard algorithm for stochastic approach simulation was developed by Gillespie [13,14]. In this model, reactions occur in a random manner: preying happens with probability of $\alpha/(\alpha+\beta+\gamma)$, reproducing with probability of $\beta/(\alpha+\beta+\gamma)$, and moving with probability of $\gamma/(\alpha+\beta+\gamma)$.

3 Results

The initial condition are shown in Fig. 1. It is worth noting that nodes out of these three area are copied by vacancies which present spatial resources to reproduce new individuals. A node in a regular lattice can be occupied at most by an individual or a vacancy. By using an efficient algorithm of Gillespie introduced in section 2, we simulate the evolving process with Monte Carlo (MC) method. At each simulation step, a randomly chosen individual interact with one of its four nearest neighbors which is randomly determined. One step of Mote Carlo time is defined as all the individuals having been chosen once on average. We set

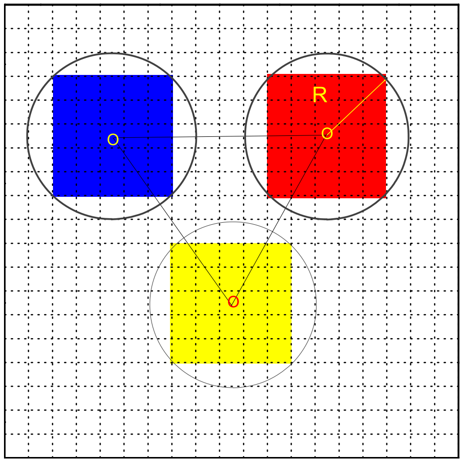

Fig. 1. Illustration of the initial condition. Here, R is set as 3.5, and the lattice completely inside the circle with radius R consist of the species' initially locating area, and three colors correspond three species. In this paper, the injected radius R is fixed at 10.5.

reproducing rate β=1, and the size of system $N = 1024 \times 1024$. The mobility of individuals D is defined as follow[4,5]:

$$D = 2\gamma/N, \tag{1}$$

As shown in Fig. 2, pattern formations depend on predation rate. All patterns in this paper are obtained after the system reach a stationary state of non-equilibrium. Low predation rate seems to promote spiral waves of global level. In the top of Fig. 2 with $D = 5.0\times10^{-6}$, spiral waves format at $\alpha = 0.01$, while edges of spiral waves break up at $\alpha = 0.1$, and spiral waves break into fragmentation at $\alpha = 1.0$ and $\alpha = 5.0$. The α becoming larger makes internal noise larger, which induce spiral waves breaking into fragmentation. The wavelength of spiral waves is defines as $\lambda = X/L$, where X is length of a spiral waves as shown in Fig. 2(a). In the bottom of Fig. 2, the wavelength of spiral waves decreases as increasing of α, and the arm of spiral waves becomes rough, which falls to pieces in the end, as shown in Fig. 2(h). In conditions of the same ability of individuals, predation rate determines sizes of spirals' arm which change into small one at higher predation rate.

Fig. 3 shows oscillation of species' percentage evolving with time for the same parameter of Fig. 1. When the global oscillations of species' percentage are achieved, spiral waves emerge in the systems. It is interesting to find that amplitudes of oscillation increase with the values of D, which is quite different from cases in target waves. In conditions of the same mobility of individuals, the average percentage of species decrease with the increasing of predation rate. It is confirmed that with the increasing of predation rate make the increasing of vacancies the same mobility, which induces strong internal noise to destroy

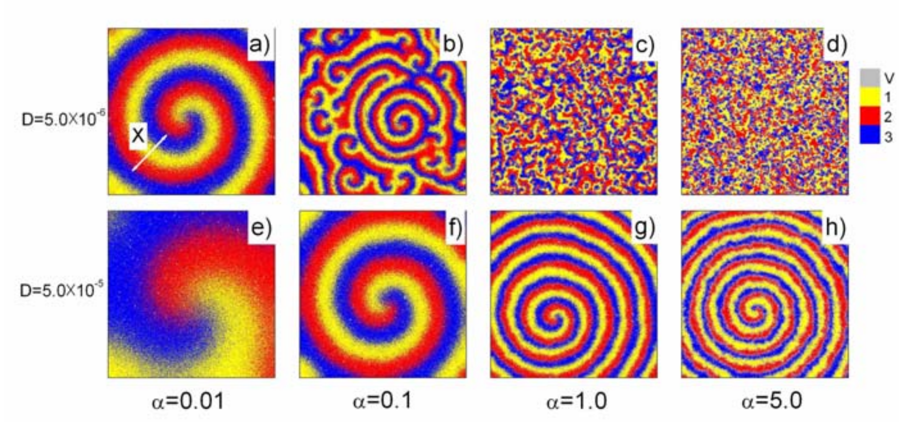

Fig. 2. Pattern formation for different value of α and D. (a) and (e) for $\alpha = 0.01$,(b) and (f) for $\alpha = 0.1$, (c) and (g) for $\alpha = 1.0$, (d) and (h) for $\alpha = 5.0$, at the same time (a),(b),(c),(d) for $D = 5.0 \times 10^{-6}$ and (e),(f),(g),(h) for $D = 5.0 \times 10^{-5}$.

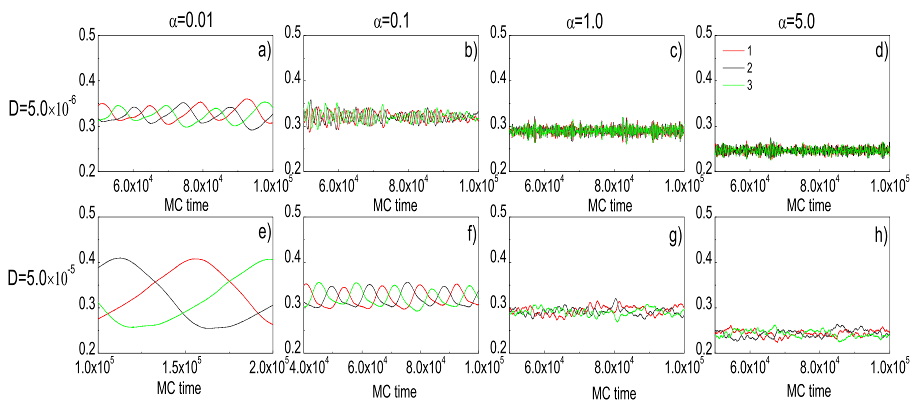

Fig. 3. Oscillation of species' percentage evolves with time for the same parameters of Fig. 2

global oscillation. Therefore, global oscillation can not be promoted at high predation rate, and the system comes into spatiotemporal chaos, as shown in Fig. 3(c) and Fig. 3(d) corresponds patterns in Fig. 2(c) and Fig. 2(d).

We systemically study the effect of predation rate on spiral waves. As shown in Fig. 4(a), for the same value of α, wavelengths of spiral waves vary with the mobility D in the form of $\lambda \sim D^{\theta}$. The exponent θ decrease with the increasing of α. It means that the behaviors of wavelengths varying with the mobility are universal at the same predation rate. $\lambda \sim \sqrt{D}$ at $\alpha = 1.0$, which confirms the results of Reichenbach *at el* [4]. When α deviates from 1.0, the exponent θ depart from $\frac{1}{2}$. At $\alpha = 1.0$ the system can be described by stochastic partial equation in which the mobility of individuals can be seen as diffusion. While

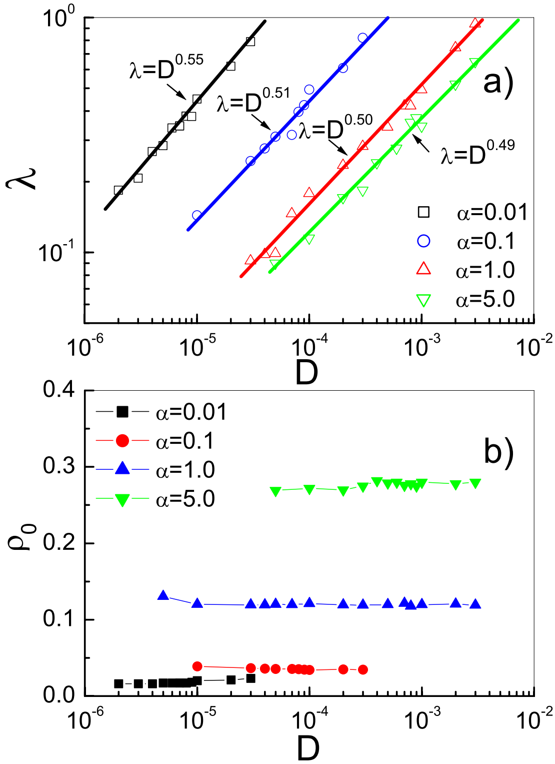

Fig. 4. The top: wavelength of spiral waves as function of mobility D. The functions satisfy $\lambda = D^{0.55}$,$\lambda = D^{0.51}$,$\lambda = D^{1/2}$, and $\lambda = D^{0.49}$ for $\alpha = 0.01$, $\alpha = 0.1$, $\alpha = 1.0$, $\alpha = 5.0$ respectively. The bottom: the percentage of vacancies ρ_0 as functions of mobility D for different value of α.

the system can be described by stochastic partial equation when α deviates from 1.0. To study why behaviors of wavelengths varying with the mobility are determined by predation rates, we define ρ_0 as percentage of vacancies which shows variation of resources. In the Fig. 4(b), one can find that ρ_0 keeps the same value ranging different mobility of individuals at a certain predation rate. It is confirmed that percentage of vacancies control kinds of university in spiral waves' formation. Because all individuals reproduce via vacancies, the number of vacancies determines properties of spiral waves' propagation.

4 Conclusion and Discussion

To conclusion, we study emergence of spiral waves affected by predation intensity. Spiral waves emerge when global oscillations are achieved. And the wavelength of spiral waves is satisfied power law with the mobility of individuals, $\lambda \sim D^{\theta}$. The value of θ is determined by the predation rate. It is confirmed that the behaviors of wavelength altering with mobility of individuals is universal at the same value of predation rate. In addition, predation density is related with vacancy resources in the systems. Our work provides basic understanding of effects of predation intensity on the spatial heterogeneity of species as well as pattern formation [15].

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