Scaling in Modulated Systems

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Abstract. How can we understand a system that is too complicated to be simulated and in which some important quantities cannot be determined from observation? This is a question we were confronted with when we analyzed experimentally observed magnetic patterns in ultrathin ferromagnetic films. We have now, for this specific case, found a method that gives a good qualitative understanding of a surprising reentrance of order observed experimentally. This method is based on scaling arguments and may prove useful in the study of other complex systems.

Keywords: frustrated systems, modulated systems, long-range interactions, competing interactions, scaling, pattern formation.

Pattern formation is found in a wide range of physical and chemical systems and often the result of competing interactions. Such a competition leads the basic units to form modulated structures. This self-organization expresses itself in geometrical configurations such as stripes, labyrinths or bubbles that can extend over length scales much larger than the basic units [1]. In such a situation, the basic interactions cannot be completely satisfied individually and the system is in a frustrated state. The difference in (free) energy of geometrically distinct configurations can be very small. Nevertheless, the occurrence of geometrically distinct configurations is deterministic and transitions between such configurations can be triggered by varying an external parameter such as the temperature. Normally, order tends to decrease with the increased excitation energy at higher temperatures [2]. This, however odd it may seem, is not universal [3]. There are rare instances where a system displays a lower symmetry (i.e. higher geometrical order) at a higher temperature even though the overall entropy has to increase. It is a challenge to understand the mechanisms that are responsible for such a peculiar and unexpected behaviour.

We use a scaling analysis to investigate the experimentally observed reentrant order in ultrathin ferromagnetic films [4,5]. Reentrance of order means that a less symmetric pattern (here: stripes) that is present at lower temperatures reoccurs at higher temperatures after a more symmetric intermediate state (labyrinth), see Fig. 1. The basic units in this system are electrons that interact with each other via the quantum-mechanical exchange interaction that favours a parallel alignment of their magnetic moments and via the magnetostatic interaction between their magnetic moments that favours an antiparallel alignment. In principle, it is possible to study this system with Monte-Carlo simulations and test the appropriateness of various models, but so far no single simulation has succeeded

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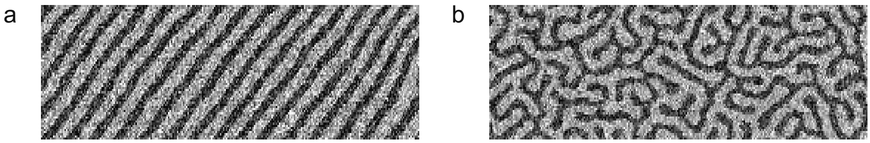

Fig. 1. Images of magnetic patterns in ultrathin iron films on Cu(001) taken with SEMPA (Scanning Electron Microscope with Polarization Analysis). The magnetic moments in this system are perpendicular to the plane of the film. Areas where the magnetic moments point downward are represented by dark grey levels, areas where the magnetic moments point upward are marked by light grey levels. a Stripe pattern. b Labyrinthine pattern. The width of the images is 92 μm.

in reproducing the experimentally observed behaviour [6,7,8]. We find that the system sizes required to observe the phenomenon of reentrance in simulations are beyond present-day computational capabilities, the main problem being the long-range character of the magnetostatic interaction. Nonetheless, we have been able to obtain a good qualitative understanding of the system by analytically reducing this problem in two spatial dimensions to an effectively one-dimensional problem that retains important properties of the original system. In a mean-field approach [9], we find a highly anomalous temperature dependence of an elastic constant. By means of a scaling analysis, we can relate this experimentally inaccessible elastic constant to experimentally measurable quantities [10]. Comparison with experiment suggests that the driving force for the reentrance of order is indeed the strongly anomalous behaviour of this elastic constant.

Our scaling analysis is based on one basic assumption for the effect of small deviations from the equilibrium state and correctly predicts anomalous behaviour. We already have indications that our scaling approach adequately describes further phenomena in ultrathin magnetic films and anticipate that a similar approach may prove fruitful in the analysis of other systems where system-size requirements prohibit numerical simulations and interesting quantities are not accessible experimentally, as in the observation of phenomena that cannot be influenced at will.

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