Scaling Law between Urban Electrical Consumption and Population in China

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Abstract. The relation between the household electrical consumption Y and population N for Chinese cities in 2006 has been investigated with the power law scaling form $Y = A_0 N^{\beta}$. It is found that the Chinese cities should be divided into three categories characterized by different scaling exponent β. The first category, which includes the biggest and coastal cities of China, has the scaling exponent $\beta > 1$. The second category, which includes mostly the cities in central China, has the scaling exponent $\beta \approx 1$. The third category, which consists of the cities in northwestern China, has the scaling exponent $\beta < 1$. Using a urban growth equation, different ways of city population evolution can be obtained for different β. For $\beta < 1$, population evolutes always to a fixed point population N_f from below or above depending on the initial population. For $\beta > 1$, there is also a fixed point population N_f. If the initial population $N(0) > N_f$, the population increases very fast with time and diverges within a finite time. If the initial population $N(0) < N_f$, the population decreases with time and collapse finally. The pattern of population evolution in a city is determined by its scaling exponent and initial population.

Keywords: population, electrical consumption, scaling law, urban growth.

1 Introduction

Cities can be considered as complex systems which organize in a decentralized manner via interactions between agents, variables and the system itself [1]. An urban system is also a manifestation of human adaptation to the natural environment [2]. In recent research, cities are suggested to be complex systems that mainly grow from the bottom up and follow scaling laws [3]. With extensive body of data, L.M.A. Bettencourt et al. have shown that important demographic, socio-economic, and behavioral urban indicators are, on average,

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scaling functions of city size [4]. Taking population $N(t)$ as the measure of city size at time t, the power law scaling of material resources or measures of social activity in a city $Y(t)$ take the form [4]

$$Y(t) = A_0 N(t)^{\beta} , \tag{1}$$

where A_0 is a normalization constant. The scaling exponent β reflects the scaling behavior of urban indicator. Scaling, as a manifestation of the underlying general dynamics and geometry, exists throughout physics. It has been applied in understanding problems across the entire spectrum of science. Critical phenomena are significant examples in which scaling has illuminated important universal principles and provided responses to practical problems [8]. In the Bettencourt et al.'s researches, the data of U.S. for many decades mostly and the data of China and European countries also were used. They concluded that the scaling relation Eq.(1) exist across different nations and times.

Among the indicators of a city, we take here the household electrical consumption of cities for a more careful investigation. Electricity is the most important energy in a modern society. It is related closely with the living standard and the living style of cities. For household electrical consumption of Germany in 2002, Bettencourt et al. have found that the data follow quite well the power law scaling form with exponent $\beta = 1.00$ [4,6]. The scaling exponent of the household electrical consumption of Chinese cities in 2002 has been calculated by them also and $\beta = 1.05$ [4]. At $\beta = 1.00$, the household electrical consumption per person is equal to the constant A_0 in Eq.(1) actually. For $\beta \approx 1.00$, the household electrical consumption per person deviates a little from the constant A_0, but the deviation is small. In developed countries, like Germany, the difference of living standard between different cities is small and there is no large difference in the household electrical consumption per person. With single constant A_0, we can relate the household electrical consumption with population of cities by the power law scaling form Eq.(1). The situation for Chinese cities is different. Over the last 30 years, China has experienced rapid development both in economics and urbanization. But the development is quite heterogeneous in the different parts of China. The development speeds of province-level and coastal cities are much larger than the development speeds of the cities in inland China. In inland China, the development speed in central China is larger than the development speed in northwestern and southwestern China. Correspondingly the living standard and urbanization in different parts of China are also quite different. We give two examples to demonstrate this heterogeneity. In 2006, the population of prefecture-level and province-level cities in China is about 28 percent of the national population, but they consume about 55 percent of the national household electricity [7]. For the cities above the prefecture-level, the highest household electrical consumption per person is about 71 times of the lowest one. In many aspects, China is a multi-class society with rapid development and urbanization. In the analysis of Bettencourt et al.[4], they describe the data of Chinese cities in 2002 by a power law scaling form with the scaling exponent $\beta = 1.05$ and single coefficient constant A_0. This is possible because of the usage of logarithmic binning method, which has erased the heterogeneity of Chinese cities.

It is to be expected that China will experience further rapid urbanization in the next 20 years. During this period, the urban population in China will increase from about 40 percent at moment to 80 percent. To know the situation and development of China, it is extremely essential for us to investigate the global properties of cities quantitatively. In the investigation of Chinese cities, it is necessary to consider heterogeneity of Chinese cities. It needs to be checked if Chinese cities satisfy still the power law scaling form after considering their heterogeneity.

In this paper, we choose the province-level and prefecture-level cities of China to explore the relation between household electrical consumption and population. In section 2, we discuss the data of urban population and household electrical consumption of the cities in 2006. In section 3, we analyze the data of the cities with the power law scaling form. It will be found that the cities should be divided into three categories, which follow the power law scaling with different scaling exponent β and coefficient constant A_0. In Section 4, we discuss the population evolution of the cities in different categories with a urban growth equation. In section 5 we make some conclusions.

2 Data of Urban Population and Household Electrical Consumption

The data of household electricity for Chinese cities in 2006 are from Chinese Urban Statistic Year Book 2007 [7]. The prefecture-level and province-level cities contribute the main part of Chinese economics, even though their population share is only about 30% of the whole nation. In 2006, there are 268 prefecture-level cities , 4 provincial cities and 15 sub-provincial cities. Here sub-provincial city is the city which has a little bit higher position in administration than normal prefecture-level city. A provincial city is equal to a province in administration and has more autonomy than a prefecture-level city. In the Chinese Urban Statistic Year Book 2007, the data of cities Tibet and Wuzhou have been missed. So there are only data of 285 cities for the discussion in the following. Provincial-level and prefecture-level cities in China have not only urban area, but also rural area. We will use only population and household electrical consumption of urban area for discussion.

Table 1. Descriptive statistics of the data

	N	Min	Max	Sum	Mean	Max/Min
Population of the city(ten thousand)	285	14.93	1510.99	36652.79	128.1566	101.2
Household Electrical Consumption(ten thousand kWh)	285	2010	1223700	17784259	62400.91	608.8
Household Electrical Consumption per person(kWh/person)	285	42	2964	– –	395.593	70.6

The statistics of data is given in Table 1. The largest city has a population of 15109.9 thousand. The smallest city has a population of 149.3 thousand. The population of the largest city is about 101 times of the smallest city's population. Among 285 cities, the minimum of household electrical consumption is 20100 thousand kWh and the maximum is 12237000 thousand kWh, which is 608 times of the minimum. The household electrical consumption per person has the minimum 42 kWh/person and the maximum 2964 kWh/person, which is about 71 times of the minimum.

3 Data Analysis

3.1 Power Law Scaling Form

We use the power law scaling form Eq.(1), introduced by Bettencourt et al. [4], to investigate the relation between urban population and household electrical consumption of 285 Chinese cities in 2006. With a double-logarithmic representation, the power law scaling form can be rewritten as

$$\ln Y(t) = \ln A_0 + \beta \ln N(t). \tag{2}$$

Using this representation, the data of Chinese cities are shown in Fig. 1. A fitting line with slope β has been plotted in Fig.1. The scaling exponent β is equal to 1.182 with lower limit 1.086 and upper limit 1.278 at 95% confidence. The constant $\ln A_0$ is equal to 4.961 with lower limit 4.522 and upper limit 5.4 at 95% confidence. The R square of the linear fitting is equal to 0.674, which means that the fitting has not caught the essential points of the data and is not a good description. It can be seen in Fig. 1 that the data have been scattered in a broad range, which is related to the very large difference of the household electrical consumption per person in China.

Considering the large heterogeneity of cities in China, it is natural for us to divide the 285 cities into different categories. For the cities in each category, we can analyze the relation between household electrical consumption and population with the power law scaling form.

3.2 Categorization of Cities in Term of Power Law Scaling

To improve the linear fitting, One of the methods is the logarithmic binning [9]. The logarithmic binning can reduce the stochastic noise when dealing with experimental data. In the fitting, the data are averaged in the bins. The value of R square for "binned" data can be remarkably better than original data in the linear fitting.

Here we do not use the logarithmic binning because of the features of Chinese cities. The strong scattering of the data in Fig. 1 is not only because of the stochastic noise, but mostly because of the large difference of the household electrical consumption per person for cities with similar sizes. If we use logarithmic binning, we will not only reduce the stochastic noises of data, but lost also

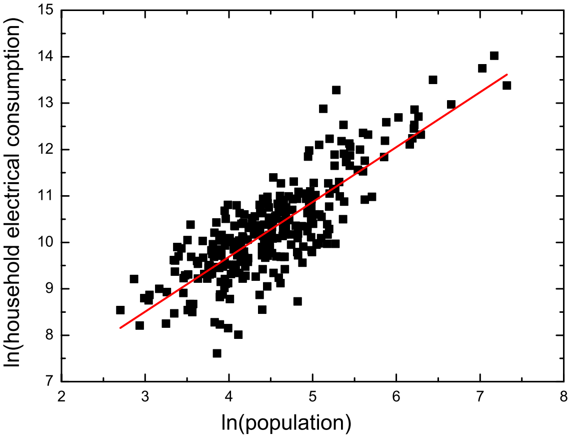

Fig. 1. Double-logarithmic representation of household electrical consumption as a function of the population size of cities in China

the detailed information about the heterogeneity of Chinese cities. For cities in a developed country, like Germany, the living standard is similar and the scattering of data are related mostly to the stochastic noises of data. The logarithmic binning can increase the accuracy of scaling exponent β substantially.

As mentioned before, the household electrical consumptions per person in different cities of China are quite different. Even for cities above the prefecture-level, the difference is almost of two order of magnitude. So we will divide the 285 cities into different category. For each category "j", we introduce the power law scaling form

$$Y(t) = A_j N(t)^{\beta_j}, \tag{3}$$

where the scaling exponent β_j and the constant A_j can be taken as the characterization of category "j".

There are many methods for defining category, such as cluster analysis. Clustering is the classification of objects into different groups, or more precisely, the partitioning of a data set into subsets (clusters). In this way, the data in each subset share some common trait - often proximity according to some defined distance measure. In our study here, it is hard for us to define a distance when using cluster analysis. Here we need to find a different way.

We will define a category by requiring the R square of cities belonging to this category is larger than a value. The R square is the criterion which characterizes the quality of a fitting. For population of cities et al., there is always stochastic inaccuracy in their data. So the value of R square is always smaller than 1 and limited. At the same time, the value of R square should not be too small. Otherwise the fitting cannot describe the data correctly.

At first, we choose the R square to be at least 0.90 for cities in a category. For the first category, we begin from the city with the highest household electrical

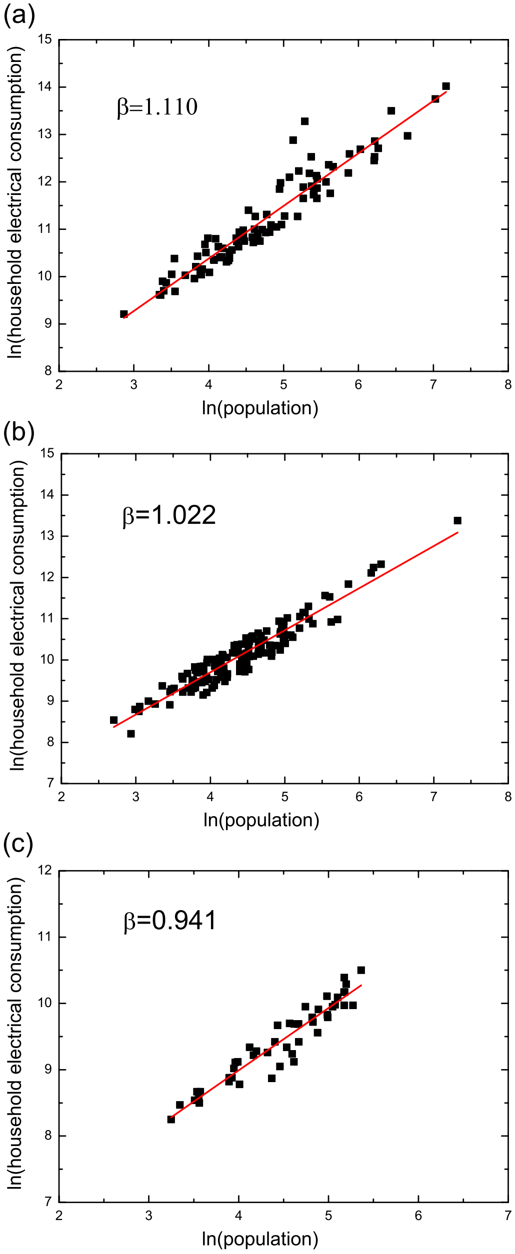

Fig. 2. Double-logarithmic representation of household electrical consumption as a function of the population size of cities in China. (a) the first category with household electrical consumption per person $\geq$ 430kWh/person; (b) the second category with household electrical consumption per person between 189kWh/person and 430kWh/person; (c) the third category with household electrical consumption per person between 89kWh/person and 189kWh/person.

Table 2. Categories of household electrical consumption per person of Chinese cities

	Observations	Max(kWh)	Min(kWh)
Whole	285	2964(Shenzhen)	42(Yulin)
Category 1	90	2964(Shenzhen)	430(Huzhou)
Category 2	140	428(Chongqing)	189(Chaohu)
Category 3	48	188(Xuancheng)	91(Zhaotong)
Left	7	85(Dingxi)	42(Yulin)

Table 3. Scaling exponents for urban electrical consumption vs. population with $R^2 = 0.90$

	Observations	β	$\ln A_j$	95% CI	R^2
Whole samples	285	1.18	4.961	[1.086, 1.278]	0.674
Category 1	90	1.110	5.943	[1.031,1.188]	0.90
Category 2	140	1.022	5.611	[0.964,1.079]	0.90
Category 3	48	0.941	5.227	[0.849,1.032]	0.90

consumption per person, which is the city Shengzhen with 2962 kWh/person in 2006. Then we add other cities with lower household electrical consumption per person, such as Dongguan, Changsha, Guangzhou, etc. For the cities included, we make a fitting to their data using the power law scaling form Eq.(3) with a suitable scaling exponent β_1 and a suitable constant A_1. The R square decreases when more cities are included. It reaches the value 0.90 after the city Huzhou is taken into account. There are totally 90 cities in the first category. The data of the cities in the first category are shown in Fig. 2a with a linear fitting, which has the scaling exponent $\beta_1 = 1.110$ and $\ln A_1 = 5.943$.

In this way, we can determine further the cities in the second and third category. There are 140 cities in the second category whose household electrical consumption is between 428 kWh/person and 189 kWh/person. The scaling exponent in the second category is very near 1 with $\beta_2 = 1.022$ and the corresponding coefficient constant is $\ln A_2 = 5.611$. The data and the fitting line are shown in Fig. 2b. In the third category, there are 48 cities whose household electrical consumption is between 188 kWh/person and 89 kWh/person. Its scaling exponent is less than 1 with $\beta_3 = 0.941$ and its coefficient constant is $\ln A_3 = 5.227$. We show the data and the fitting line of the third category in Fig. 2c. There are still 7 cities left which cannot be described by a power law scaling form.

The scaling exponents of different categories are summarize in Table 3, where the lower and upper limit of the scaling exponents at 95% confidence interval are given also.

In general, the category with higher household electrical consumption has larger scaling exponent. The scaling exponent of the first category is definitely larger than 1. The exponent of the second category is almost equal to 1. The exponent β of the third category is obviously smaller than 1.

Table 4. Scaling exponents of different categories divided by $R^2 = 0.90, 0.91, 0.89$

R^2	Category	Observations	β
0.674	all cities	285	1.18
0.90	Category 1	90	1.110
	Category 2	140	1.022
	Category 3	48	0.941
	Left	7	– –
0.91	Category 1	78	1.108
	Category 2	140	1.033
	Category 3	58	0.960
	Left	9	– –
0.89	Category 1	103	1.083
	Category 2	125	0.983
	Category 3	50	0.942
	Left	7	– –

To understand our results, we identify the locations of the cities in different categories. It is found that the first category includes the province-level and coastal cities. The cities of the second category are in central China and the cities of the third category are in northwestern China.

The province-level and coastal cities has $\beta_1 > 1$. This means that the household electrical consumption of these cities increases faster than their population. The larger cities in this category consume super-proportionally more household electricity. The cities in northwestern China has $\beta_3 < 1$. The larger cities in the third category consume sub-proportionally less household electricity. For cities in central China, $\beta_2 \simeq 1$ and the cities in this category consume household electricity in proportion with their population.

The classification of categories above is obtained by requiring that the R square of each category is at least 0.90. To verify the robustness of our classification, we need to use different values of R square to classify the category of the cities. As we have discussed before, the R square should has a reasonable value which cannot be too large or too small. So we have chosen the R square to be 0.89 and 0.91. The results of different R square are given in Table 3. Of course, the city numbers of different categories and their scaling exponents will be modified with the change of R square. But their dependence is weak. For the three values of R square, we have always three categories of cities which satisfy the power law scaling form and a few cities left, which do not satisfy. The scaling exponents of different categories are kept to be almost the same. So our classification of the categories of cities is robust and reliable.

4 Growth of City in Different Categories

As we have demonstrated in the last section, the cities of China can be divided into three categories which follow the power law scaling form with different scaling exponents. We are interested in the urban growth of the cities of different categories. Related to household electrical consumption and population, Bettencourt et al. have introduced a simple resource balance equation [4]

$$Y = r_0 N + e_0 (dN/dt), \tag{4}$$

where r_0 is the quantity per unit time to maintain an individual of the population and e_0 is the quantity to add a new individual to the population. This equation can be rewritten into a general growth equation

$$\frac{dN(t)}{dt} = \frac{A_0}{e_0} N(t)^\beta - \frac{r_0}{e_0} N(t). \tag{5}$$

The solution of this equation [4] is

$$N(t) = \left[A_0/r_0 + \left(N(0)^{(1-\beta)} - A_0/r_0 \right) \exp\left[-(1-\beta)\frac{r_0}{e_0} t \right] \right]^{1/(1-\beta)}. \tag{6}$$

In the following, we will discuss the solutions in for different β in detail.

(1) $\beta = 1$:
The solution reduces to an exponential form

$$N(t) = N(0) e^{(A_0 - r_0)t/e_0}. \tag{7}$$

Because r_0 is the electrical consumption per unit time to maintain an individual, therefore

$$A_0 = \frac{Y}{N^\beta} = r_0 + \frac{e_0}{N}(dN/dt) > r_0. \tag{8}$$

In Fig. 3a, the population growth in this case is shown. From this result, we can conclude that the cities in the second category with $\beta \simeq 1$ can have a rapid population increase.

For scaling exponent $\beta \neq 1$, the solution Eq. (6) has a fixed point $N_f \equiv (A_0/r_0)^{1/(1-\beta)}$. If the initial population $N(0) = N_f$, the population keeps to be constant.

With the fixed point population N_f we can rewrite Eq.(6) as

$$N(t) = \left[N_f^{1-\beta} + \left(N(0)^{(1-\beta)} - N_f^{1-\beta} \right) \exp\left[-(1-\beta)\frac{r_0}{e_0} t \right] \right]^{1/(1-\beta)}. \tag{9}$$

If the initial population $N(0) \neq N_f$, the population evolutes then with time. The evolution at $\beta < 1$ is quite different from the evolution at $\beta > 1$.

(2) $\beta > 1$:

When $N(0) > N_f$, the population increases with time very fast and becomes divergent in a finite time t_c given by

$$t_c = -\frac{e_0}{(\beta - 1) r_0} \ln \left[1 - (N(0)/N_f)^{(1-\beta)} \right]. \tag{10}$$

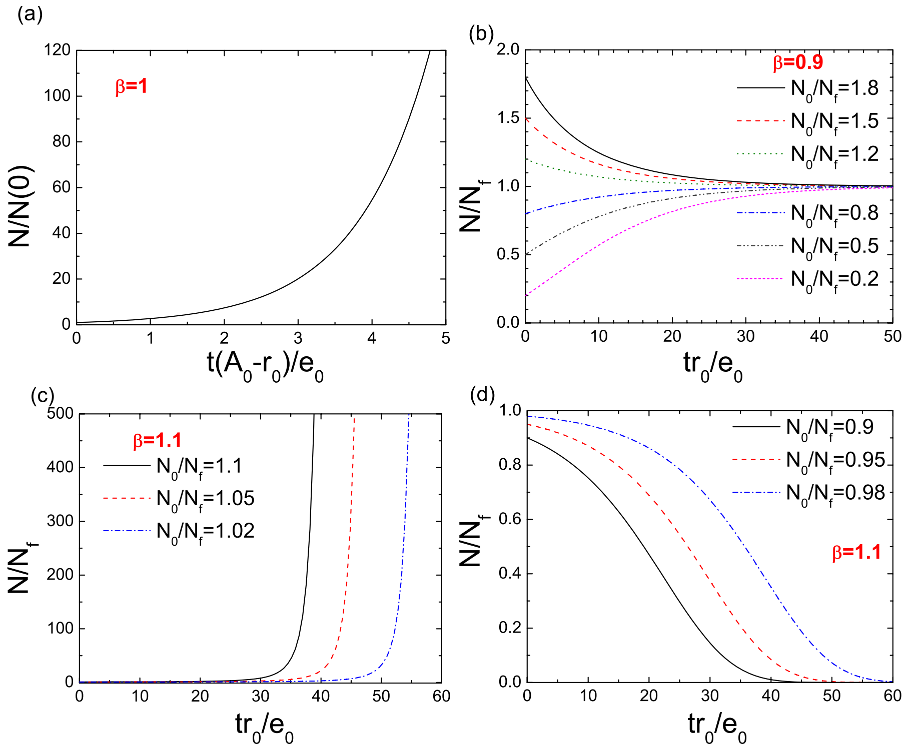

Fig. 3. The growth of population: (a) $\beta = 1$; (b) $\beta < 1$; (c) $\beta > 1$ with $N(0) > N_f$; (d) $\beta > 1$ with $N(0) < N_f$

In Fig. 3c, we show the evolution of population for different initial populations $N(0) = 1.1N_f, 1.05N_f, 1.02N_f$. When $N(0) < N_f$, the population decreases with time and collapse finally. The evolutions of population for different initial populations $N(0) = 0.98N_f, 0.95N_f, 0.9N_f$ are shown in Fig. 3d.

(3) $\beta < 1$:

If $N(0) > N_f$, the population decreases with time and reaches the fixed point N_f finally. If $N(0) < N_f$, the population increases with time and approaches N_f. The evolutions of population with $\beta = 0.9$ for initial population $N(0) = 1.8N_f, 1.5N_f, 1.2N_f, 0.8N_f, 0.5N_f, 0.2N_f$ have been demonstrated in Fig. 3b.

In the analysis of Bettencourt et al. [4], the dependence of population evolution on initial population in cases 2) and 3) has been ignored.

5 Conclusions

Using the power law scaling form introduced by Bettencourt et al. [4] for material resources and population of cities, we have investigated the relation of household electrical consumption and population of Chinese province-level and prefecture-level cities in 2006. In our discussion, the heterogeneity of Chinese cities has been taken into account, which has been ignored in the investigation of Chinese cities by Bettencourt et al. with the logarithmic binning [4]. From the data of 285 cities of China without the logarithmic binning, we find that the Chinese cities can be divided into three categories which follow the power law scaling form with different scaling exponents. The first category includes the province-level and coastal cities and has the scaling exponent larger than 1. The second category, which includes the cities of central China, has the scaling exponent $\beta_2 \approx 1$. In the third category, the scaling exponent is less than 1 and the cities of this category are in northwestern China.

With a urban growth equation, we have studied the population evolution of cities in different categories with different scaling exponents and initial population. We find that the population evolution depends essentially on the scaling exponent and the initial population. For $\beta < 1$, the population will increase or decrease with time to a fixed value N_f. When $\beta > 1$, the population will be divergent in a finite time if initial population $N(0) > N_f$ and collapse if $N(0) < N_f$.

Electricity is just one kind of material resources of city. To get more understanding about Chinese cities, we need to explore the relation between other kinds of material resources and population. Further it is very interesting to take also individual needs and social activity of Chinese cities into account. In these further investigations about Chinese cities, our results here have demonstrated that the heterogeneity of Chinese cities must be considered. The Chinese cities should be divided into different categories for discussion. In this way, we will work on a quantitative description of the global properties of Chinese cities.

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