Sediment Transport Dynamics in River Networks: A Model for Higher-Water Seasons

Jie Huo[1], Xu-Ming Wang[1], Rui Hao[1], and Jin-Feng Zhang[2]

[1] School of Physics and Electric Information Sciences,
Ningxia University, Yinchuan 750021, China
wxmwang@nxu.edu.cn
[2] School of Physics and Electronic Information,
Huaibei Coal Industry Teachers' College, Huaibei 235000, China

Abstract. A dynamical model is proposed to study sediment transport in river networks in higher-water seasons. The model emphasizes the difference between the sediment-carrying capability of the stream in higher-water seasons and that in lower-water seasons. The dynamics of sediment transport shows some complexities such as the complex dependence of the sediment-carrying capability on sediment concentration, the response of the channel(via erosion or sedimentation) to the changes of discharge.

Keywords: sediment transport dynamics, fast transience, complexity, self-organization, self-adaptation.

In decades past, natural river networks have attracted a good deal of attention in physics and geophysics communities [1,2,3,4,5,6]. Three progresses can generalize the main scientific achievements. The first one is the empirical observations on natural river networks that revealed many power-law relationships between some parameters of river networks [1,2]. The second one is the modeling studies based on the local erosion rules to get deeper understandings of how these natural events occur in the evolutive processes [3,4,5,6]. The main spirit of the models embodies the nature of the water: flows downhill and changes the local height of landscape due to the erosion. Some parallel achievements are the theoretical studies based on the minimum energy dissipation [7,8], which introduce the assumption that an open system with fixed energy input tends to form a structure that minimizes the total energy dissipation in the process of the precipitated water flowing downhill. The third one, the insight into the mechanisms of what creates the scaling laws, was carried into execution by Dodds and Rothman via recurring to statistic physics method [9].

It is known that river patterns bear striking fractal forms and self-organized characteristics [2]. As stated by Dhar [8], this self-organized structure is the result of the feedback between the water flow and the landscape itself via the variation of the erosion. This implies that sedimentation can not be neglected, at least, for the purpose of investigating the evolution of the river patterns [10,11].

J. Zhou (Ed.): Complex 2009, Part I, LNICST 4, pp. 832–840, 2009.

Recently, we proposed a dynamical model to simulate sediment transport in matured river networks in lower-water season by consideration of both functions of erosion and sedimentation [12,13]. The core spirit of this model embodies the feedback mechanism between erosion and sedimentation via the adjustment of sediment-carrying capability(SCC)of runoff. A steady state predicated by the model shows scaling laws that the quantity of erosion or sedimentation(QES) distributes exponentially along the channel in the downriver direction. The result under one of the combinations of the two free model parameters implies a catastrophe due to a great deal of sedimentation or erosion in the downriver reach of the channel [13]. However, the way into the steady state will be interrupted by frequent fluctuations in runoff, caused by random precipitations. The response of the river to the abrupt change of the input shows self-organized and self-adaptive behaviors. The self-organized characteristic is represented by the observation that the variation of the SCC is opposite to that of the QES as the discharge changes, which shows that the response of the river trends to depress the increase of erosion as water flow increases and that of sedimentation as water flow decreases. This is actually incarnated by the rapid change of QES, the preceding response to the change of water flow. After this, the dynamic shows self-adaptive characteristics: the channel is forced to change its formation by sediment erosion or sedimentation via adjusting SCC of water: erosion may be enhanced(or sedimentation may be mitigated) as water flow goes up, while sedimentation may be strengthened(or erosion may be mitigated) as water flow goes down [see Ref. [13]].

This letter is aimed at a further investigation in a higher-water season to test whether a river is commonly dominated by the aforementioned dynamics. The model should give emphasis to the difference between the sediment transport in higher-water season and in lower-water season. Sediment concentration usually sharply increases in a higher-water season in comparison with a lower-water season due to the erosion of the relative more precipitation. Therefore, the SCC of water is not only adjusted by the undergone erosion or sedimentation state, but also directly driven by the sediment concentration. This means that the adjustments of SCC can be fallen into two types. The former is positive and the latter is passive.

For the convenience of discussions, the passive adjustment of SCC is incarnated in the relation between the transported sediment S and the discharge Q as well as the sediment concentration Q_s(which is actually sediment concentration of the nearest neighboring upriver stream for the discussed segment). Then we have

$$S = kQ^{\alpha}Q_s^{\beta}. \tag{1}$$

Where k denotes a part of SCC, which corresponds to the positive adjustment of SCC based on the undergone erosion or sedimentation. α and β are indices, which indicate, without losing generality, that the transported sediment S nonlinearly depends on both of Q and Q_s.

In our discussions, the ordering scheme of river is designed by differing the lateral and not lateral between two injecting streams as the confluence takes place. If one goes against a stream, say rank h, at each of nodes the laterally injecting stream belongs to the branches of rank $h+1$, but the non laterally injecting stream is actually one segment of this stream. So the rank of a stream can be uniquely determined by the times of its laterally injecting confluences as it drains to the sea. And the segments of a stream can be also numbered with the times of its non-laterally injecting confluences as it drains to the sea. The first segment is at the upper end, which is actually taken as the source. And then the segment number increases one by one as branches of rank $i+$ 1 inject into the stream in turn. For convenience, we denote the stream flowing on a channel segment by the segment itself.

Let's firstly consider the confluence on the segments in a time interval, $t \to t+\delta t$, which can be simply denoted by t(one of the discrete time variables, i.e., $t = 1, 2, \cdots$). So the confluence, in the tth time step, can be directly expressed as

$$Q_h^l(i+1,t) = Q_h^l(i,t) + Q_{h+1}^i(i_{last},t). \tag{2}$$

Where $h = 0, 1, 2, \cdots$, $i, l = 1, 2, \cdots$, and $i = 1$ denotes the source of the stream. The character Q denotes the stream flow. The left side of Eq. (1) represents the outflow of the i+1th segment of lth branch of rank h, the right side is the inflows respectively coming from the ith segment of this branch and the ith branch of rank h+1(which is in fact denoted by its last, i_{last}, segment). The outflow balances the inflows. It implies that the change of water can only be attributed to the confluence on condition that the rainfall in the basin is denoted by the change of water from headstreams.

Generally, the sediment carried by the outflow may not be equal to that by the inflows. So the erosion or sedimentation will occur in these segments. Based on the confluence expressed by Eq. (1) and the definition presented above, the QES can be determined by equation,

$$\Delta S_h^l(i+1,t) = S_h^l(i+1,t) - S_h^l(i,t) - S_{h+1}^i(i_{last},t). \tag{3}$$

On the right side of the equation, the first term denotes the sediment carried by the outflow, the second and third denote that carried by the inflows, respectively from the upper segment and the injecting branch of higher rank. $\Delta S_h^l(i+1,t) > 0$ indicates that the i+1th segment is in a scouring state in the tth time interval, while $\Delta S_h^l(i+1,t) < 0$ implies that it is in a deposition state.

According to the above discussions, the positive adjustment of SCC via the self-variation of k can be conducted by the QES on a segment and that on its neighboring segments. This is similar to that of lower-water season.

(1) For a natural river, the sedimentation implies that both the deposited and the suspended sediment particles are relatively smaller, and the viscosity intensity of fluid is enhanced, so that the friction of river-bed against the flow will increase and the stream can carry more sediment. On the contrary, the erosion implies the deposited material over the river-bed and the suspended particles become coarser, the friction will decrease, thus the stream can transport lesser sediment. It is the adjustment of k of a segment conducted by QES on itself.

(2) Whereas, the QES on the upriver neighboring segment will influence k of the discussed segment, in comparison with what has been presented above,

in the reversed way. If a segment is in scouring state, the increasing sediment may cause k of its downriver neighboring segment to increase so that the stream can discharge more sediment, while if the segment is in deposited state, the decreasing sediment may let k of the downriver neighboring segment to decrease. Naturally, the QES on a higher rank branch may affect k in the same way.

In fact, k of a segment should be determined by the integration of aforementioned factors. This integration may imply competition between the two opposite effects, increasing and decreasing k of a segment. So the expression that presents the above-mentioned self-adjustment mechanism can be given

$$\begin{aligned} k_h^l(i+1,t+1) = k_h^l(i+1,t) - A_1 \Delta S_h^l(i+1,t)/(Q_h^l(i+1,t+1))^\alpha (Q_{sh}^l(i,t+1) \\ + Q_{sh+1}^i(i_{last},t+1))^\beta + A_2(\Delta S_h^l(i,t) \\ + \Delta S_{h+1}^i(i_{last},t))/(Q_h^l(i+1,t+1))^\alpha (Q_{sh}^l(i,t+1) \\ + Q_{sh+1}^i(i_{last},t+1))^\beta. \end{aligned} \tag{4}$$

Where A_1 and A_2 are free parameters of the model. They denote the adjustment strength of k executed by the QES on a segment and its nearest upriver neighbors, respectively.

In our model, any one upriver segment can be chosen as the most top one, and also treated as the source of the stream. The regular or irregular changes, $\xi(t)$, of the discharge and that of sediment concentration, $\zeta(t)$, may represent the rainfalls. Then we have

$$Q_h^l(1,t) = Q_{h_0}^l + \xi(t), \tag{5}$$

$$Q_{sh}^l(1,t) = Q_{sh_0}^l + \zeta(t), \tag{6}$$

and

$$\Delta S_h^l(1,t) = \Delta S_{h_0}^l \tag{7}$$

for all possible h, l and t. Where $\Delta S_{h_0}^l$ may have a fixed value for the simpleness.

The core spirit of the model consists in the description of self-adjustability of a river in the sediment transport process, which is manifested by the positive adjustment of SCC that is expressed by Eq.(4). Similar self-adjustability often dominates the dynamics of a completely open system. The simulation work may not only help us to understand the nature of sediment transport on river networks but also provide us with some illumination for general dynamics of self-adaptive systems else.

Before carrying out the simulation with the model, it is necessary to test it. The database about the Yellow River can provide us with the observed data to perform this test [14]. However, a problem may occur: how to conduct the test? Two objects to compare are not math in time scale: the observed data is the average over a month, but the simulating results are instantaneous if we follow the real scene in the river network(random rainfalls). So we have to introduce random fluctuation of discharge to mimic the rainfalls confined by the condition that the average of the discharge is equal to the average observed. The results calculated by this scheme are presented in Fig. 1 and show that they are in good

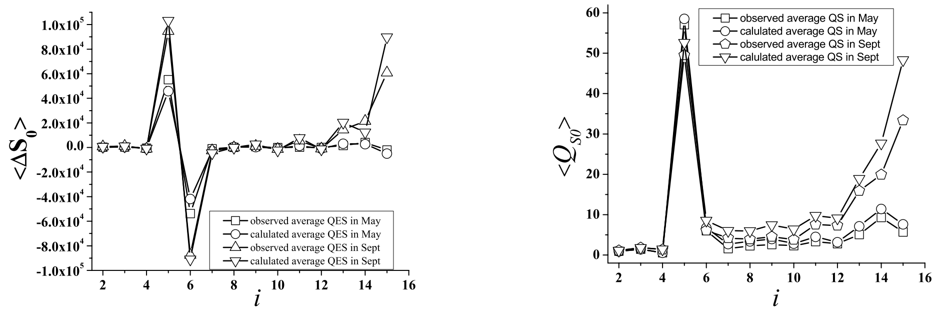

Fig. 1. The comparisons of (a) the calculated average QES($< S_0 >$) and (b) sediment concentration $< Q_{s0} >$, with that of the observed on 15 segments from Tangnaihai to Sanmenxia reach of the Yellow River in 1982. The formers are obtained by introducing the random fluctuation each other 50 time steps, and taking average over 500 time steps.

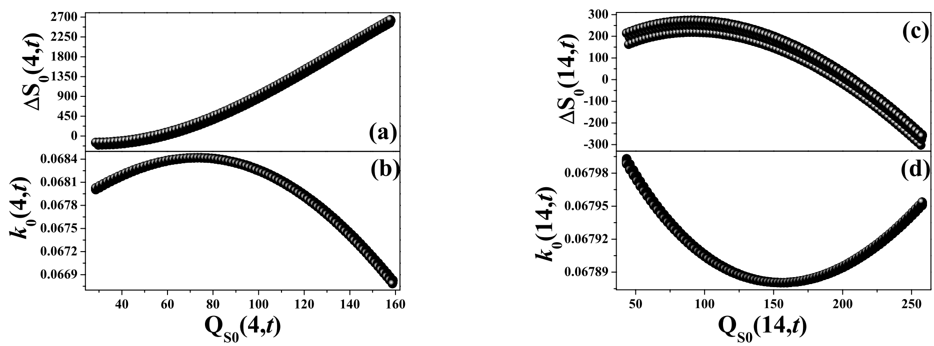

Fig. 2. The calculated results to show the dependence of (a) ΔS_0 and (b) k_0 on Q_{s0} of segment 4 and that of segment 14 showed respectively by (c) and (d). The details are presented in the text.

agreement with the average observed QES and sediment concentration. These indicate that our model can describe the sediment transport in river networks in higher-water seasons. The model can be employed to mimic the process in a natural river network. It is necessary to point out that data in the database are only about the main stream and its first rank branches, we have to change our model by neglecting the adjustment of k made by the first rank branches. So $\Delta S^i_{h+1}(i_{last}, t)$ in Eq. (4) can be deleted, and the superscripts of the terms can be canceled.

Let's firstly see about the influence of sediment concentration on the positive self-adjustable part of SCC, k, in the simplest case, that is, only the injections of the first rank branches into the main stream are considered. The result as sediment concentration of the first rank branches, Q_{s1}, increases is presented by Fig. 2, which is obtained as the related variables take the following values: $\Delta S_0(1,t) = 50$, $Q_{s0}(1,t) = 100$, $k_0(i,1) = 5.7 \times 10^{-2}$, $Q_0(i,1) = RN(180,350,N_1)(RN(180,350,N_1)$ denotes N_1 random numbers between 180 and 350), $Q_{s0}(i,1) = RN(30,108,N_1)$, $Q_1(i,t) = 40$, $A_1 = 0.6 \times 10^{-4}$, $A_2 = 0.9 \times 10^{-5}$, $\alpha = 1.3$, $\beta = 0.98$, $Q_{s1}(i,t) = 50 + \zeta(t)(\zeta(t) = 0.1\frac{t}{260}$ for $mod(t,260) = 0$, which implies that Q_{s1} will increase 0.1 at each regular time-step interval, 260). The number of the first rank branches, $N_1 = 30$. As shown in Fig. 2 (evolves for 23400 time steps), the influence of Q_{s1} on k_0 or ΔS_0 bears complex behaviors.

Two strong impressions of the figures on one may be the varying trends of k_0 and ΔS_0 as Q_{s1} increases. The first one is that the variation of k_0 is almost opposite to that of ΔS_0 on the same segment, which implies that the self-adjustment of the SCC tends, as a whole, to depress large increase of the erosion or sedimentation. The second one consists in that the variation of k_0 on a upriver segment, say 4, are opposite to that of a downriver segment, say, 14. This show a complex response mechanism of k_0 to Q_{s0}. Firstly, it demonstrates a complex dependence of k_0 on Q_{s0}, and basically agrees with the fact that the SCC will weaken incipiently and then strengthen when sediment concentration monotonously increases from a relative smaller initial value. The experimental results [15] indicated that the suspension of sediment is mainly attributed to the eddy motion of a stream with smaller value of Q_s. However, the eddy motion will be damped as Q_s gets larger. Therefore, as Q_s increases to a threshold the SCC of the stream will strengthen since the suspension of sediment particles is governed now by the buoyancy force of water instead of the eddy motion. Secondly, the complexity of the dynamics is also indicated by the dependence of the varying trend of SCC on the initial state, erosion or sedimentation. As the core idea of our model, the SCC of stream is apt to go down when the segment is in a scoring state, while SCC of stream is inclined to go up when the segment is in a deposited state. Obviously, Fig. 2 represents the above-mentioned complexities. Fig. 2(a) shows that segment 4 is initially in deposited state the SCC should go up(Fig. 2(b)), meanwhile the stream has lower sediment concentration, the SCC should go down. The competition of these two varying trends makes the SCC to transform from increase to decrease at threshold, about 70, of Q_{s0}. Fig. 2(c) and (d) indicates that both of the initial erosion of segment 14 and the increasing sediment concentration of the stream can lead the SCC to decrease. At threshold, about 160, of Q_{s0}, the curve has an inflexion, which means that adjustment mechanism of SCC dominated by the eddy motion is replaced by that governed by the increasing buoyancy force due to the increase of sediment concentration.

Now let's discuss the transient characteristic of sediment transport dynamics in the scouring or deposited process. It can be presented by the dependence of QES on discharge. To show more related details, we calculate the QES by considering the influences of the second order branches on the dynamics of the mainstream in the case of the parameters and variables are set with the following values: for all possible i, i' and t, $\Delta S_0(1,t) = 50$, $Q_0(i,1) = RN(60,120,N_1 + 1)$,$Q_{s0}(i,1) = RN(25,90,N_1 + 1)$,$k_0(i,1) = 5.7 \times 10^{-2}$, $Q_1(i',1) = RN(25,90,$

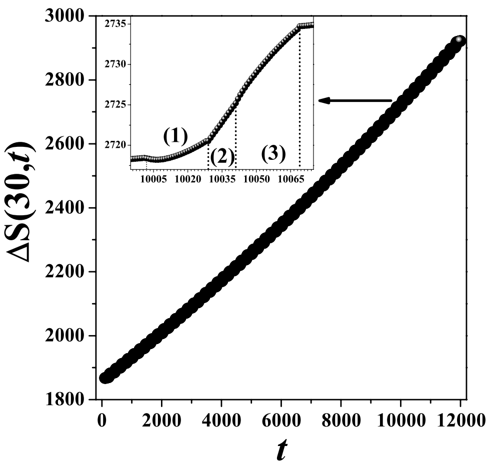

Fig. 3. The transient QES on segment 30 as the confluence of the second rank branches is considered. The inset is its partial magnification for showing the three SFTs. The details are presented in the text.

$N_2+1)$, $Q_{s1}(i^{'},1) = RN(50,180,N_2+1)$, $k_1(i^{'},1) = 2.0\times10^{-1}$, A_1, A_2, α and β are the same of Fig. 2. $Q_2(i^{''},t) = RN(0.8,1.6,N_2)+\xi(t)(\xi(t) = 0.005\frac{t}{200}$ for $mod(t,200)=0)$. The number of the first and the second rank branches is $N_1 = 30$ and $N_2 = 40$, respectively.

The calculated results shown in Fig. 3 is similar to the characteristics of transient dynamics, in a lower-water season, of which the response of QES to the abrupt changes of discharge can actually be fallen into the faster and the slower phases. The former is the rapid response of river to the change and will last for the stream flow getting a new stationary value, the latter can lead the dynamics to the steady state if the discharge keeps changeless. The partial magnification, the insets, of Fig. 3, show that the faster transience can be divided into three sub-phases, three so-called sub-faster-transiencies(SFTs). Ref.[13] has reported the mechanism that SFT (1) is caused by the changing branches(in the water flux) of first rank injecting, one by one, into this segment. It will last for all of the changing stream from its upriver branches of the first rank reach the segment; SFT (2) is induced by the more and more changed streams from the branches of rank 2 reach the segment. This SFT will go on up to its nearest upriver stream of rank 1 becoming stationary; then the transience enters into SFT (3), and comes to the end time step at which the uppermost stream of rank 1 also get stationary. The situation is opposite when the discharge of rank 2 stream decreases.

Obviously, SFT(1) embodies self-organized characteristics of the sediment transport process: a river trends to depress a mass of erosion when the stream flow increases, while control a vast sum of sedimentation when the stream flow decreases. However, when the input of a river changes, it may be forced to adjust its state to acclimatize itself to the new conditions. SFT(2) and SFT(3) are just the case. They indicate that the QES has to increase as the discharge increases, but decrease as the discharge decreases. This may be a common characteristic of self-adaptive dynamics for a kind of completely open systems. These dynamic characteristics may have engineering significance. If the majority of random fluctuations of discharge takes place within the faster transient phase, especially within SFT(1), the gross of erosion trends to balance that of sedimentation, so the channel can hold relative steadier. If we can control the discharge increasing(decreasing) at regular time-step intervals within SFT(2) or SFT(3)(the latter may be better), eroding(accumulating) can go on to improve the conditions of river channels for transporting sediment.

Although the natural fluctuation of water flux is attributed to all of the upriver branches of higher rank with random precipitation. This may weaken or erase the divisions of the three SFTs. However, according to the above discussions, engineers can take use some man-made-flood-peaks to control sediment transport processes for different engineering purposes. The detailed descriptions of this research and the other understandings to sediment transport in river networks are in process.

This study is supported by the National Natural Science Foundation of China under Grant No. 10565002 and the Program for New Century Excellent Talents in University of China under Grant No. NCET-06-0914.

References

1. Leopold, L.B.: Downstream Change of Velocity in Rivers. Am. J. Sci. 251, 606 (1953)
2. Rodríguez-Iturbe, I., Rinaldo, A.: Fractal River Basins: Chance and Self-Organization. Cambridge University Press, Cambridge (1997)
3. Banavar, J.R., Colaiori, F., Flammini, A., Giacometti, A., Maritan, A., Rinaldo, A.: Sculpting of a Fractal River Basin. Phys. Rev. Lett. 78, 4522–4525 (1997)
4. Manna, S.S.: Branched Tree Structures: from Polymers to River Networks. Physica A 254, 190 (1998)
5. Maritan, A., Colaiori, F., Flammini, A., Cieplak, M., Banavar, J.R.: Universality Classes of Optimal Channel Networks. Science 272, 984 (1996)
6. Giacometti, A.: Local Minimal Energy Landscapes in River Networks. Phys. Rev. E 62, 6042–6051 (2000)
7. Sun, T., Meakin, P., Jøssang, T.: A Minimum Energy Dissipation Model for Drainage Basins that Explicitly Differentiates Between Channel Networks and Hillslopes. Physica A 210, 24 (1994)
8. Dhar, D.: Theoretical Studies of Self-organized Criticality. Physica A 369, 29–70 (2006)
9. Dodds, P.S., Rothman, D.H.: Geometry of River Networks. I. Scaling, Fluctuations, and Deviations. Phys. Rev. E 63, 016115–016117 (2000)
10. Murray, A.B., Paola, C.: A Cellular Model of Braided Rivers. Nature 371, 54–57 (1994)
11. Meakin, P., Sun, T., Jossang, T., Schwarz, K.: A Simulation Model for Meandering Rivers and Their Associated Sedimentary Environments. Physica A 233, 606–618 (1996)

12. Hao, R., Wang, X.-M., Huo, J., Zhang, J.-F.: Simulating Sediment Transport on River Networks. Mod. Phys. Lett. B 22, 127–137 (2008)
13. Wang, X.-M., Hao, R., Huo, J., Zhang, J.-F.: Modeling Sediment Transport in River Networks. Physica A 387, 6421–6430 (2008)
14. http://www.loess.csdb.cn/hyd/user/index.jsp
15. Cao, Z., Egashira, S., Carling, P.A.: Role of Suspended-Sediment Particle Size in Modifying Velocity Profiles in Open Channel Flows. Water Resour. Res. 39, 1029 (2003)