Using the Weighted Rich-Club Coefficient to Explore Traffic Organization in Mobility Networks

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Abstract. The aim of a transportation system is to enable the movement of goods or persons between any two locations with the highest possible efficiency. This simple principle inspires highly complex structures in a number of real-world mobility networks of different kind that often exhibit a hierarchical organization. In this paper, we rely on a framework that has been recently introduced for the study of the management and distribution of resources in different real-world systems. This framework offers a new method for exploring the tendency of the top elements to form clubs with exclusive control over the system's resources. Such tendency is known as the weighted rich-club effect. We apply the method to three cases of mobility networks at different scales of resolution: the US air transportation network, the US counties daily commuting, and the Italian municipalities commuting datasets. In all cases, a strong weighted rich-club effect is found. We also show that a very simple model can account for part of the intrinsic features of mobility networks, while deviations found between the theoretical predictions and the empirical observations point to the presence of higher levels of organization.

Keywords: complex networks, human mobility, transportation systems.

1 Introduction

The elements of many systems, ranging from technological to economic and social ones, are often organized into hierarchies [1,2,3,4,5,6]. Investigating the nature of the interactions among the highest-ranking elements of a system can offer useful insights into the system's organization and functioning. For example, do the top elements attract and exchange among themselves the vast majority of the resources available in the system, or do they tend to distribute resources homogeneously within the system? By adopting the framework of network theory – where the system is represented in terms of nodes, corresponding to its elements, and links connecting interacting elements [7,8,9,10,11,12,13] – researchers have

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begun to study interactions among top elements by investigating whether the system's structure displays higher interconnectedness among highly connected nodes (also called *rich* nodes) than randomly expected [14]. This feature is known as the rich-club phenomenon [14,15]. By analyzing the topology of a networked system at its top hierarchical level, the rich-club phenomenon helps highlight important organizational principles of the system's structure [14,16,17,18,19].

This approach, however, assumes that the richness of a node is exclusively given by the number of connections departing from the node. In this respect, it is limited by the binary nature of links on which it draws, whereas a crucial piece of information is encoded in the strength of connections that can vary substantially across the network [6]. In infrastructure and information networks, variations in the strength of links correspond to differences in the carrying capacity of connections, measured in terms of the amount of information, energy, people, and goods that can travel along them [5,6,20,21,22]. In social networks, strong links are often found among socially embedded individuals [23,24,25,26,27,28,29,30]. A full understanding of how top nodes are organized, therefore, relies on the study not only of which other nodes they interact with, but also of the strength of their interactions. A recently introduced measure called the weighted rich-club coefficient [31] enables us to study whether and the extent to which the prominent elements of a system attract, control, and share among themselves the vast majority of the system's resources. In this paper, we apply this measure to the case of real-world mobility networks. We introduce systems with different transportation modes and of different scales of resolution, and in these systems we investigate whether and the extent to which transportation hubs manage and distribute traffic flows among themselves. Finally, we compare the empirical results with the predictions obtained from a simple model for transportation fluxes. This comparison allows us to probe which are the basic mechanisms behind the organization of the transportation datasets.

2 Mobility Networks: Air Transportation and Commuting Patterns

Transportation systems and mobility patterns of individuals can be mathematically represented as networks composed of nodes, corresponding to locations, and links describing the movement of individuals from an origin to a destination. In addition, each link connecting a node i to a node j is also characterized by a weight w_{ij} that measures the travel flux, i.e., the amount of travelers moving along that connection. Several examples of mobility networks have been analyzed, and found to exhibit skewed distributions of travel fluxes per connection, as well as large fluctuations in the traffic passing through various locations [6,20,32]. These results have been reported for different mobility types and different geographic scales, from mobility within a city [32], to commuting patterns at regional and country scales [33,34], to the worldwide air transportation network [6,20].

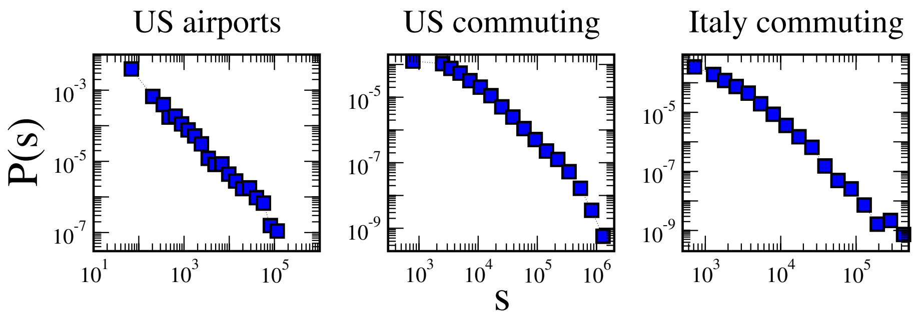

Fig. 1. Probability distribution of the traffic s of each node in the three mobility networks. Left: the US air transportation system; center: the US commuting network; right: the Italian commuting network.

Here we consider three empirical datasets involving various means of transportation and coming from diverse geographic areas: the US air transportation network [35], the commuting patterns among US counties [35], and the commuting patterns among Italian municipalities [36]. The US air transportation network is composed of 676 nodes representing the commercial airports located in the continental United States. The $3,523$ links represent direct flights between these airports, while the weights of these links indicate the average number of seats available per day in each connection [6]. The other two networks correspond to commuting patterns of two countries of different size: the US and Italy. In these networks, each node represents a location (i.e., origin or destination of the commuting), and a link corresponds to the existence of a flow of individuals commuting from an origin to a destination. The weight of each link represents the total flux of daily commuters between any two counties in the US, or any two municipalities in Italy. In both cases, the data was collected through national census surveys [35,36]. In the US, there are a total of $3,141$ counties connected by $35,340$ weighted links, whereas in Italy there are $8,101$ municipalities connected by $125,246$ weighted links.

Nodes in mobility networks can be characterized in terms of their strength or traffic s, a measure of the number of travelers passing through each node, defined as the sum of the weights of the links departing from a given node [6]:

$$s_i = \sum_{j \in \nu(i)} w_{ij} \, , \tag{1}$$

where $\nu(i)$ is the set of neighbors of node i. In the three cases under study, the traffic s_i of a node i corresponds to the number of airline travelers and the number of daily commuters passing through i in the airline transportation network and in the two commuting datasets, respectively. Figure 1 reports the probability distribution $P(s)$ of the traffic s, showing in all cases the presence of large fluctuations, and signalling that the nodes are hierarchically organized in terms of their traffic capacities.

In what follows, we consider the traffic of a node as a measure of its prominence in the system, and we measure the weighted rich-club coefficient [31] in order to investigate the extent to which the hubs at the top of the hierarchy control and share among themselves the strongest connections of the system.

3 Weighted Rich-Club Coefficient

The weighted rich-club coefficient is a measure that allows us to study the extent to which the top elements of a given system collude to secure and share resources among themselves [31]. This measure can be applied to any networked system in which ranking relationships can be established among the nodes according to a given property. This property is usually referred to as the *richness parameter* r [31], in analogy with the topological rich-club coefficient [14,15] that measures the tendency of high degree nodes (also called rich nodes) to form tight interconnected subgraphs.

The weighted rich-club coefficient builds on, and extends, the topological one into a new broader framework in which the intensity and capacity of the links are explicitly taken into account. More specifically, if we consider a richness parameter r and aim to determine the relative strength of the links connecting the rich nodes with respect to the system's total capacity, the following weighted rich-club coefficient can be defined [31]:

$$\phi^w(r) = \frac{W_{>r}}{\sum_{l=1}^{E_{>r}} w_l^{\mathrm{rank}}} \quad , \tag{2}$$

where the numerator is the sum of the weights associated to the links connecting rich nodes. Assuming that the total number of links between the rich nodes is $E_{>r}$, the denominator corresponds to the sum of the weights of the $E_{>r}$ strongest links of the graph. The term w_l^{rank} represents an order relationship established among the weights of the links in the network: $w_l^{\mathrm{rank}} \geq w_{l+1}^{\mathrm{rank}}$, with $l = 1, 2, \ldots, E$, and E being the total number of links in the graph. Thus, Eq. (2) measures the fraction of weights shared by the rich nodes compared with the total amount they could share if they were connected through the strongest links available in the network. $\phi^w(r)$ takes values ranging from 0 to 1. It is equal to 0 if there is no link connecting the rich nodes, whereas it reaches the value of 1 when the links connecting the rich nodes are the strongest available ones.

In analogy with the topological rich-club coefficient [14], Eq. (2) in itself is not informative and has to be compared to an appropriate null model [14,37]. In fact, even random graphs can show a non-zero value in Eq. (2). To properly evaluate the weighted rich-club phenomenon, we therefore need to assess it against a null model that is random, but at the same time comparable to the real network. In particular, our choice of an appropriate null model reflects the need to discount for associations between weights and topology. To this end, the null model must meet three main requirements. First, it must have the same number of nodes and links as the original network. Second, it must have the same weight distribution $P(w)$ (i.e., the probability that a given link has weight w) – a crucial constraint since we are looking for non-trivial intensity of interactions among rich nodes. Third, the nodes in the rich club must be the same as in the real network, which also preserves the richness distribution $P(r)$ (i.e., the probability that a given node has richness r) of the real network. A null model that does not fulfill the above three requirements cannot be compared to the real network, and thus does not allow for a proper weighted rich-club assessment (for a full discussion and a comparison with other proposed methods, see Ref. [31]).

In the context of mobility networks, we want to explore the tendency of highly trafficked locations to attract the majority of the passenger fluxes circulating on a system. By defining the richness parameter in terms of the traffic passing through each node, we need a null model that is able to keep the node traffic fixed while destroying all associations between links and weights observed in the real network. In [31], we introduced a procedure to generate null models that keep the value of node strength unchanged. We called this procedure Directed Weight Reshuffle because it is based on the randomization of directed networks that preserves not only the topology and $P(w)$, but also the out-strength distribution $P(s_{out})$ (i.e., the probability that the sum of weights of the outgoing links of a node is s_{out}) of the real network [38]. In the Directed Weight Reshuffle null model, the weights are locally reshuffled for each node across its outgoing links (see Ref. [31] for details). This procedure is applicable to directed graphs but can be easily extended also to the undirected case by duplicating each undirected link into two directed links, one in each direction.

It is now possible to assess the weighted rich-club effect by measuring the ratio:

$$\rho^w(r) = \frac{\phi^w(r)}{\phi^w_{\text{null}}(r)}, \tag{3}$$

where the denominator is the weighted rich-club coefficient measured on the null model. When ρ^w is larger than one, the network displays a positive weighted rich-club ordering, with rich nodes concentrating a disproportionately large amount of their efforts towards other rich nodes compared with what happens in the random null model. Conversely, if it is smaller than one, the links among the members of the club are weaker than randomly expected.

4 Results

We measured Eq. (3) in the three empirical datasets. As shown in Fig. 2, we found that all mobility networks under study, while displaying a relatively mild topological ordering, are characterized by a strong weighted rich-club effect. This result clearly points to the presence of large backbones of travel fluxes associated with the connections that link locations with very high traffic. Not only do busy airports direct routes to one another, but they also secure control over travel fluxes by channeling on those routes a larger proportion of passengers than randomly expected [31].

In analogy with what was found in the airport network, also commuting flows between highly trafficked locations are much stronger in terms of number of

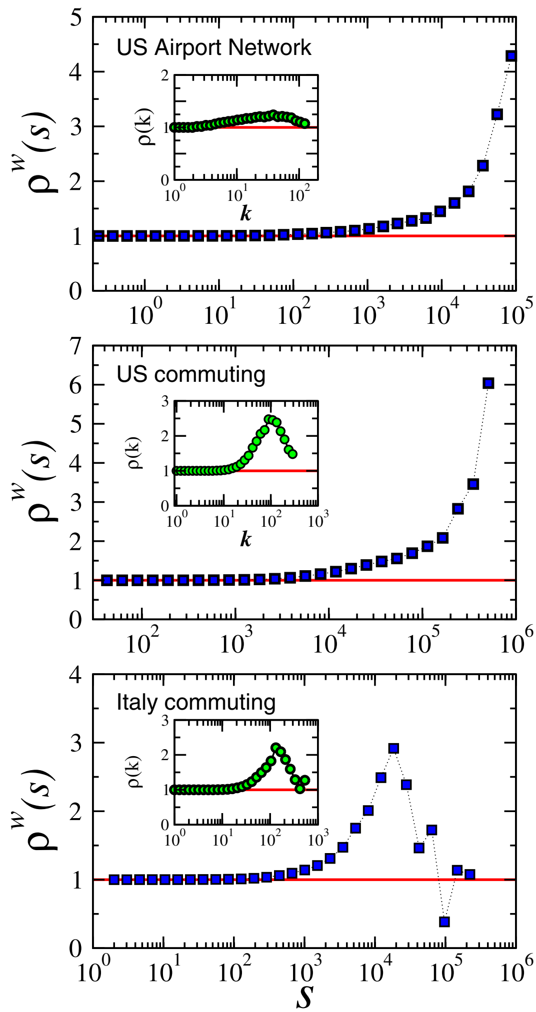

Fig. 2. Weighted rich-club coefficient for the US airport network (top), the US commuting network (center) and the Italian commuting network (bottom), where richness is measured in terms of node traffic. The insets refer to the topological rich-club coefficient.

commuters than would be expected in a fully random model. However, some differences between the commuting patterns in the US and in Italy can be found. If we compare the highest value reached by $\rho^w(s)$ in the two cases, the weighted rich-club effect is indeed twice as strong in the US network as it is in the Italian one. Moreover, the strong fluctuations observed for very large values of s in the Italian network, which bring the value of the ratio $\rho^w(s)$ down to 1, are not observed in the US case, characterized instead by an increasing trend of $\rho^w(s)$ for the whole range of traffic values. This behavior seems to uncover a difference in

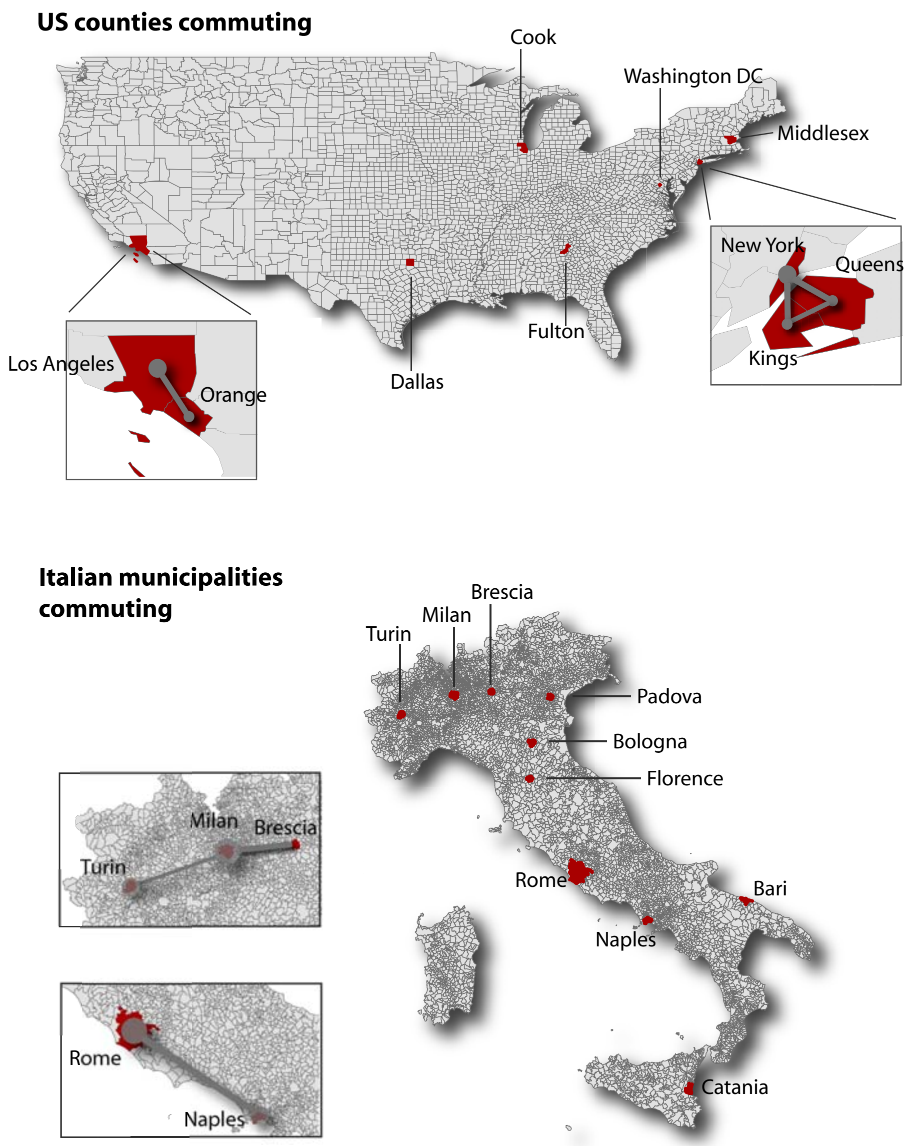

Fig. 3. Map of the ten top locations, counties or municipalities, in the ranking of commuting traffic, top for the US and bottom for Italy. In the plot only the strongest connections in number of passengers are shown.

the commuting patterns between the largest traffic hubs within the two countries under study. In the US, the larger the commuting traffic passing through two locations, the larger the travel flux that connects these two locations. In the Italian case, this is true only up to a certain traffic value, approximately equal to $2 \cdot 10^4$ commuters. Above this threshold, the weighted rich-club ratio decreases, thus showing that the largest commuting hubs in the country are more likely to channel a large proportion of passengers toward less trafficked locations than toward other hubs.

Geographic distances, population and area sizes associated to counties and municipalities, and the costs associated with commuting, can partly explain the variations in the flux of passengers traveling between locations seen in Italy and in the US. Economic considerations come into play also as a result of the likely change of means of transportation as distances between locations increase. While short travels can be faced by means with similar low cost such as train, metro, car or buses, longer travels require the use of planes. This observation finds support especially if we look at the value of $\rho^w(s)$ at the very end of the traffic range in the Italian commuting network. This result shows that the connections between the largest Italian centers of commuting (Milan and Rome) share a travel flow of individuals that does not considerably deviate from the random value.

On the other hand, the commuting fluxes between neighboring counties, such as those forming LA or New York (see Figure 3) strongly enhance the signal observed for $\rho^w(s)$ in Figure 2 at very high values of s. American large cities indeed occupy on average larger surface areas than their European counterparts, therefore typically including several counties, among which there might be a larger commuting flow than among counties which are found at larger distances. This behavior is not observed in Italy where the largest commuting hubs are not found within close distance.

Additional sociological and cultural considerations can help explain the results. Demographic studies have shown that, while people in the US exhibit a pronounced proclivity toward mobility not constrained by distances, not only for enhancing their social and economic status, but also for raising residential satisfaction [39], Italians are typically characterized by a stronger attachment to the places where they started their career [40]. Thus, for cultural reasons, people in Italy might not tend to look for jobs far away from where they live, even when economic opportunities do not abound locally.

In order to gain a better understanding of the mechanisms that are responsible for the observed rich-club effect, in the next section we introduce a simple traffic model based on some statistical laws found in the empirical data, and compare the predictions obtained from that model with the results we found in our datasets.

5 Comparison with a Simple Traffic Model

A very peculiar feature observed in many transportation networks is that there exists a relationship between the weight w_{ij} of a connection from node i to node j and the product of the degrees of the two nodes, $k_i k_j$ [6,41,42,43,44,45,46]:

$$\langle w_{ij} \rangle = (k_i \, k_j)^{\theta} \ . \tag{4}$$

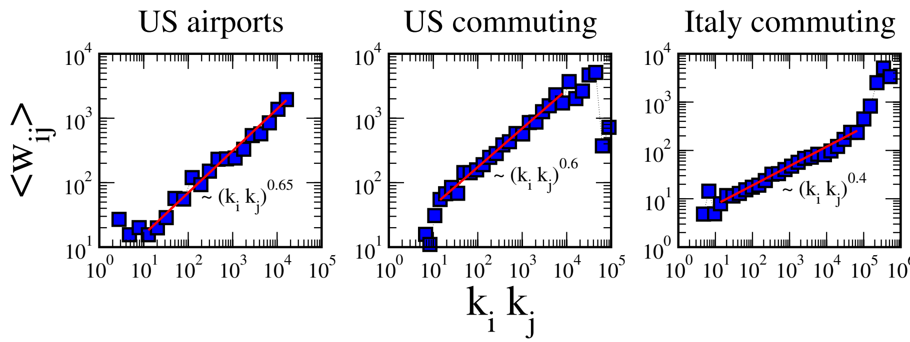

Fig. 4. Relationship between the weight w_{ij} of a link connecting node i to node j and the product of the degrees of the two nodes, $k_i k_j$, in the US airport network (left), the US commuting network (center), and the Italian commuting network (right). The red lines are fits to the empirical data.

This relationship was found in transportation networks of individuals at various scales [6,44] and also in systems of freight and mail transportation [44]. The exponent θ is typically found in the range $0.4-0.6$. Figure 4 shows this relationship for the three networks here investigated, supporting previous results observed in other settings.

This finding leads us to compare the real-world networks under study with a simple traffic model for the location of the strong connections in a network. The idea is to maintain the same structure as in the real network, and to generate new weights for the links based on Eq. (4) [41,47,48]. We then check whether the weighted rich-club effect observed in the previous section is just a byproduct of the way weights are created through Eq. (4). It is important to note that, when generating the new weights, we lose some of the variability in the location of the strong connections since the value of the weight of each link only depends on the degree of the two connected nodes. However, this simple traffic model is in itself sufficient to test if the observed weighted rich-club ordering results simply from the *first order* relationship captured by Eq. (4).

Figure 5 reports the rich-club ordering obtained from the model and the one observed in the empirical datasets. In all three cases, the traffic model based on Eq. (4) is able to reproduce approximately the whole range of values of traffic per node s. However, it seems to fail to replicate some of the values for the weighted rich-club effect observed in the empirical networks. In the US airport network and in the US commuting network, the increasing trend of $\rho^w(s)$ is captured by the model but the observed values in the empirical datasets for the most trafficked nodes are approximately $1.5-2$ times larger than the ones theoretically predicted. In the Italian commuting network, the model correctly produces the magnitude of the weighted rich-club effect. However, it fails to reproduce the relative strength and capacity of the links connecting the highly trafficked nodes. This result, therefore, highlights a deviation of the trend of commuting flows between the Italian municipalities where the top locations do not share a

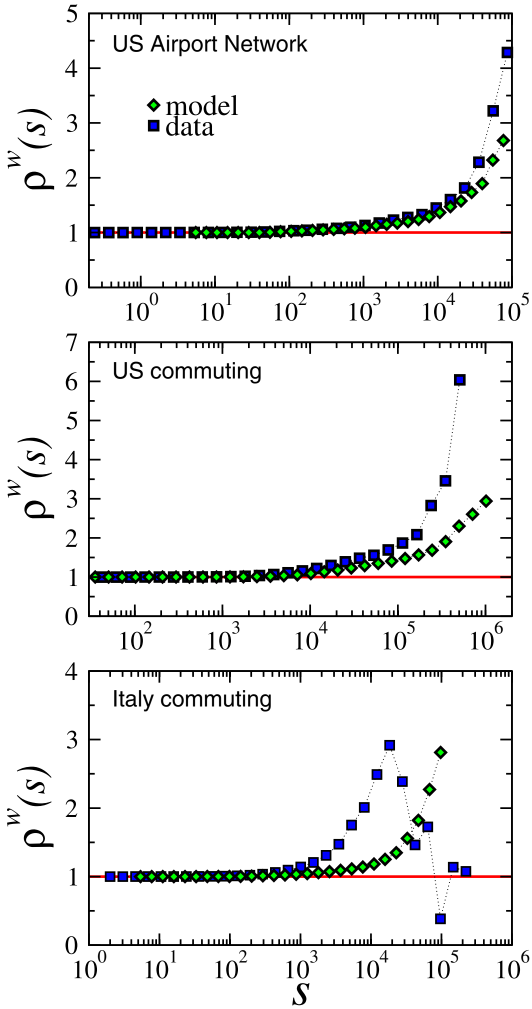

Fig. 5. Comparison of the weighted rich-club effect observed in the mobility networks (squares) and the one obtained by using the traffic model (diamonds). From top to bottom: the US airport network, the US commuting network, and the Italian commuting network.

considerably large amount of commuters, as instead predicted by Eq. (4). In order to explain this deviation and the differences between the two commuting patterns under study, other features ought to be taken into account. Among these are the geographic distance between locations, transportation costs, problems of traffic congestion, geographic concentration and availability of business opportunities, as well as possible cultural discrepancies between the habits, views, and values of the populations on the two sides of the Atlantic.

6 Conclusions

The study of how the prominent elements of a complex system interact with one another has fundamental implications on our understanding of the organization and functioning of the system in a variety of empirical applications and from a number of disciplinary perspectives. In this paper, we investigated transportation networks by relying on a method that has been recently introduced in [31]: the weighted rich-club effect. The empirical datasets considered here are the US air transportation network, the US counties commuting network, and the Italian municipalities commuting network. In the US networks, we found a strong weighted rich-club ordering, showing that the most trafficked locations control and share among themselves the vast majority of the overall traffic in the system. The Italian commuting network displays deviations from this behavior at the very top hierarchical level. We also considered a simple model for the generation of travel flows in a transportation network, and tested the extent to which this model can account for the observed weighted rich-club ordering in the real networks. Interestingly, the deviations between the traffic model and the real networks appear to be stronger for the commuting systems than for the airport network. These results pave the way toward new avenues of investigation concerned with the role played by additional factors (e.g., geographic distance, congestion problems, cultural habits) in shaping the weighted rich-club ordering in mobility networks.

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