One Kind of Network Complexity Pyramid with Universality and Diversity

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Abstract. It is based on well-known network models Euler graph, Erdös and Renyi random graph, Watts-Strogatz small-world model and Barabási-Albert scale-free networks, and combined the unified hybrid network theoretical frame. One kind of network complexity pyramid with universality and diversity is constructed, described and reviewed. It is found that most unweighted and weighted models of network science can be investigated in a unification form using four hybrid ratios (*dr,fd,gr,vg*). As a number of hybrid ratios increase, from the top level to the bottom level complexity and diversity of the pyramid is increasing but universality and simplicity is decreasing. The network complexity pyramid may have preferable understanding in complicated transition relationship between complexity-diversity and simplicity-universality.

1 Introduction

Pyramid architecture can be widely found in nature and most social fields. For example, Zoltvai and Barabási firstly proposed the life's complexity pyramid in biology science [1], and it was found that "the topologic properties of cellular networks share surprising similarities with those of natural and social networks. This suggests that universal organizing principles apply to all networks, from the cell to the World Wide Web." Based on profound analysis for network science development history [2-15] and the unified hybrid network theory frame proposed by Fang's group [15-26], we suggest and investigate one of kind network complexity pyramid with seven levels, as shown in Fig.1, so-called the network model's complexity pyramid(NMCP). The top three levels of the NMCP are Euler graph(EG, level-7)[1], Erdös and Rényi random graph (ERRG, level-6)[2], Watts-Strogatz (WS) small-world model[3] and Barabási-Albert (BA) scale-free networks (level-5)[3], respectively. These network models mark the three milestones in network science development history. The level-4 of the NMCP is the weighted evolution networks(WENM) [27,28]. The top four levels have grabbed main intrinsic quality of complex network respectively. As in depth study of network science, however, how exactly depict and fully mirror all characteristics of most real-world networks is still challenging subject because the real-world is one harmonious and unification world with both determinacy and randomness. Therefore, we have put forwarded the unified hybrid network theoretical frame with three unified hybrid network models [15-26] , which can be constructed as following three levels of the NMCP. The level-3 is the harmonious unification hybrid preferential network model (HUHPM,), the level-2 is the large unified hybrid network model(LUHNM)

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and the level-1 is the unified hybrid network model with variable speed growth (UHNM-VSG). From the top level-7 to the bottom level-1 complexity and diversity is increased but universality and simplicity is decreasing. The NMCP may have preferable understanding in complicated transformation relationship between complexity-diversity and simplicity-universality.

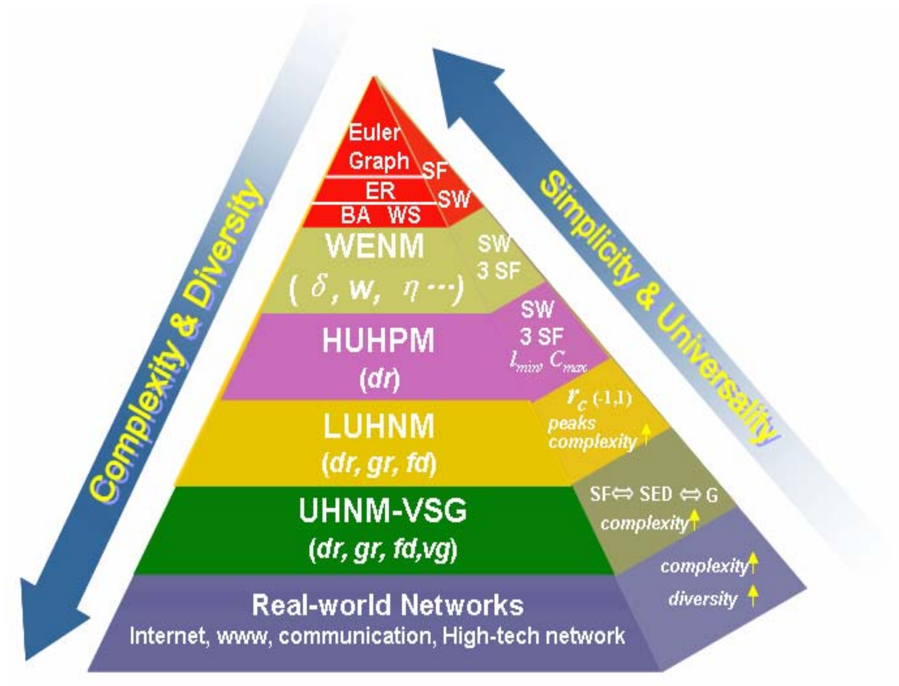

Fig. 1. Complex network model pyramid diagrams

2 The Top Three Levels of the NMCP

Retrospective network in the footsteps of scientific development, network theoretical model research has been one of the most significant issues in the network sciences. So far the history of this area has gone through three milestones which all breakthroughs from the theoretical model.

2.1 The Level-7: Euler (Regular) Graphs

A graph that has an Euler circuit is called an Eulerian graph. The first milestone was Euler graphs born in the 1736 [1], which attributed to the graph father. Euler has done many pioneering work, such as he first solved the famous Konigsberg Seven Bridge problem and the many facets of the Euler theorem [1]. The Euler's theorem is that (a)If a graph has more than two vertices of odd degree then it cannot have an Euler path. (b)If a graph is connected and has just two vertices of odd degree, then it at least has one Euler path. Any such path must start at one of the odd-vertices and end at the other odd vertex.The Euler graphs have been studied for longest period since then. The regular EG theory has laid the foundation of the graph theory development and should be at the top level-7 of the pyramid.

2.2 The Level-6: ER Random Graph

In graph theory, the Erdös-Rényi model[2], so-called ER random graph theory, named for Paul Erdős and Alfréd Rényi, is either of two models, $G(n, p)$ and $G(n, M)$, for generating random graphs, including one that sets an edge between each pair of nodes with equal probability, independently of the other edges. It can be used in the probabilistic method to prove the existence of graphs satisfying various properties, or to provide a rigorous definition of what it means for a property to hold for almost all graphs. The $G(n, p)$ model was first introduced by Edgar Gilbert in a 1959 paper which studied the connectivity threshold. The $G(n, M)$ model was introduced by Erdös and Rényi in their 1959 paper. As with Gilbert, their first investigations were as to the connectivity of $G(n,M)$, with the more detailed analysis following in 1960. The ER theory impact graph theory for 40 years long. Erdös is known as the 20th century Euler, and obtained Wolf Award in 1984. The ER random graph obeys the Poisson degree distribution, and has a smaller average path length and smaller clusters coefficient. After the ER model, from the late 1950s to late 1990s, large-scale networks with no clear design principles primarily uses this simple and easy random graph topology, which is accepted by the majority of people. Many mathematicians give random graph theory strict mathematical proof, and obtain many similar and accurate results. Properties of $G(n, p)$ are as follows. A graph from $G(n, p)$ has on average $\binom{n}{2}p$ edges. The distribution of the degree of any particular vertex is binomial: $P(\deg(v)=k)=\binom{n-1}{k}p^k(1-p)^{n-1-k}$, where n is the total number of vertices in the graph. In a 1960 paper, Erdös and Rényi described the behavior of $G(n,p)$ very precisely for various values of p. Their results included that:

(a) If $np < 1$, then a graph in $G(n,p)$ will almost surely have no connected components of size larger than O(logn).

(b) If $np = 1$, then a graph in $G(n,p)$ will almost surely have largest component whose size is of order $n^{2/3}$.

(c) If np tends to a constant $c > 1$, then a graph in $G(n, p)$ will almost surely have a unique "giant" component containing a positive fraction of the vertices. No other component will contain more than O(logn) vertices.

(d) If $p < \frac{(1-\varepsilon)\ln n}{n}$, then a graph in $G(n, p)$ will almost surely not be connected.

(e) If $p > \frac{(1-\varepsilon)\ln n}{n}$, then a graph in $G(n, p)$ will almost surely be connected.

Thus $\frac{\ln n}{n}$ is a sharp threshold for the connectivity of $G(n, p)$.

So far the ER random graph has succeeded in revealing the emergence of certain structural properties and multi threshold function and so on. Thus it should be at the level-6 of the pyramid.

2.3 The Level-5: Small-World Network and Scale-Free Models

In the level-6, both of the two major assumptions of the *G(n, p)* model (that edges are independent and that each edge is equally likely) may be unrealistic in modeling real situations. In particular, an Erdös-Rényi graph will likely not be scale-free like many real networks. Therefore the Watts and Strogatz model attempts to correct this limitation.

In 1998, Watts and Strogatz proposed small world (SW) network mode[3].They revealed that the SW effect of the complex network is a kind of hybrid results of determinacy and randomness. Soon Newman and Watts and others made some improvements for the SW models [6-12]. The degree distribution of ER random model and the WS model are not completely in line with many networks in reality and have certain limitations. Many empirical graphs are well modeled by small-world networks. Social networks, the connectivity of the Internet, and gene networks all exhibit small-world network characteristics. A certain category of small-world networks were identified as a class of random graphs by Watts and Strogatz. They noted that graphs could be classified according to two independent structural features, namely the clustering coefficient and average node-to-node distance, the latter also known as average shortest path length. Purely random graphs, built according to the ER model, exhibit a small average shortest path length (varying typically as the logarithm of the number of nodes) along with a small clustering coefficient. Watts and Strogatz measured that in fact many real-world networks have a small average shortest path length, but also a clustering coefficient significantly higher than expected by random chance. Watts and Strogatz then proposed a novel graph model, now currently named the WS model, with (i) a small average shortest path length, and (ii) a large clustering coefficient. The first description of the crossover in the WS model between a "large world" (such as a lattice) and a small-world was described by Barthelemy and Amaral in 1999. This work was followed by a large number of studies including exact results.

In 1999, Barabási and Albert (BA) proposed a scale-free (SF) network model[4] and found the power-law nature of the complex networks, i.e. degree distribution follows $p(k) \sim k^{-\gamma}$. Two discoveries of the SW and the SF networks mark the third milestone of network development[5] and network sciences was born [5-12]. The formation mechanism of the SF network is based on two rules: growth and preferential attachment in accordance with the degree of nodes. The BA model is the first model of a random network with the SF property. Further, network with complex topology describe as diverse as the cell, the WWW or society. One of the most surprising finding is that despite their apparent differences and sharing the same large-scale topology, each having a SF structure. Subsequently, it was found that the formation mechanisms of the SF are also as diverse as replication, nearest neighbour connections, hybrid preferential linking and local connective information. In summary, main feature is the evolution of complex network is driven by self-organizing processes that are governed by simple but generic scaling laws. Many subsequent empirical research of real-world networks(RWN) have demonstrated that the RWNs is neither regular nor random, but they belong to a large class of hybrid network both determinacy and randomness, and commonly possess both the SW and the SF properties, as well as the statistical property which is completely different from the level-7 regular graph and the level-6 random graph.

3 The Level-4: Weighted Evolving Network Models

Up to now three milestones above from the level-7 to the level-5 are all un-weighted networks. They reflect most of topological properties and dynamical behavior between network nodes and connectivity but they could not describe different role of nodes and all characteristics of the RWNs completely since almost RWNs belong to weighted networks. Only weighted evolving networks can carefully portray the nodes connection and mutual interaction. Thus it is natural boost that from the un-weighted network models above toward weighted evolving network models (WENM) , which has became the level-4 of the NMCP. Along with more and more empirical studies on weighted networks, fresh properties related link weight are obtained by some typical WENMs [27,28]. In the level-4 there are a lot of preferential driving mechanisms: (1) Node strength; (2)Edged weight; (3)Both strength and edged weight; (4)Both weight and fitness; (5) Both topological growth and strength driving; (6) Geographical link of position neighborhood; (7) Local information or both local world and weight driving; (8) Topological growth with strengths' driving, and so on. In the level-4 the WENMs have revealed some common characteristics: the SW as well as the three SF (node degree, strength and weight distributions), i.e., all obey the power-law property with different exponents.

4 The Level-3: HUHPNM

It is noted that all WENMs in the level-4 belong to generalized random networks, which always ignored deterministic linking. They are useful for theoretical analysis easily and reproduce main topological properties for the RWNs. But based on the foundational observation fact for a unifying world in natural and social networks, one cannot ignore anyone of order and random since their interactions in real world are neither completely regular nor completely random and lying between the extremes of order and randomness.

To overcome weak point of the level-4, the unified hybrid network model frame [15-26, 31-32] with trilogy was proposed and can be constructed as following three levels of the NMCP. The level-3 is the harmonious unifying hybrid preferential network model (HUHPNM), in which one total hybrid ratio is introduced by

$$dr = \frac{d}{r} = \frac{DPA}{RPA} \tag{1}$$

where d is a number of time intervals (step) for deterministic preferential attachment (DPA), and r is a number of random preferential attachment (RPA). It was found in the level-3 that some universal topological properties, including the exponents γ of the three power-laws (degree, node strength, and edged weight) are highly sensitive to total hybrid ratio the *dr*. A threshold of the exponent is at *d/r* = 1/1.

Through theoretical analysis for the HUHPNM, we obtain the complicated function relationship of power exponent γ with *d/r* for some weighted HUHPNM which are quite coincide with the numerical curves. Moreover, for all three BA, BBV and TDE models their γ has quite complicated relation with the weighted parameters (δ, *w)* and the total hybrid ratio *d/r*. Their complicated function relationship of power exponent γ with *d/r* for the BA and BBV are [16]:

$$\gamma_{BA} = \frac{1}{\beta} + 1 = A_1 e^{-\left(\frac{d/r}{A_2}\right)^{A_3}} + A_4 \cdot \tag{2}$$

where $\gamma_0 = 3$.

$$\gamma_{BBV} = \frac{4\delta + A_1 e^{-\left(\frac{d/r}{A_2}\right)^{A_3}} + A_4}{2\delta + 1} \cdot \tag{3}$$

where A_i is parameter, *i*=1, 2, 3, 4.

This reflects both mutual competition and harmonious unification. The level-3 has both the SF and the SW properties. It was found that the HUHPNM-BA is of the shortest average path length(APL) and largest average clustering coefficient (*ACC*).

5 The Level-2 : LUHNM

In fact, the level-3 HUHPNM does not completely reflect the actual network links of the diversity and complexity. To describe diverse complex networks and improve the HUHPNM, we have extended the HUHPNM toward a large unifying hybrid network model (LUHNM)[19-26], which become the level-2 of the NMCP. Two new hybrid ratios: determinist hybrid ratio *fd* and random hybrid ratio *gr* are introduced respectively by

$$fd = \frac{f}{d} = \frac{HPA}{DA} \tag{4}$$

and

$$gr = \frac{g}{r} = \frac{GRA}{RA} \tag{5}$$

where HPA is helping poverty attachment, and GRA is general random attachment, thus we have DA = HPA + DPA; RA = GRA + RPA. In the level-2, it is found that much more complex relation of topological properties depending on three hybrid ratios (*dr, fd, gr*). The degree-degree correlation r_c (the assortative coefficient) is one of interesting quantity. The level-2 exhibits two fresh transition features of the r_c from negative 1 to positive 1 in the both un-weighted and weighted LUHNM. Firstly, only if the *fd* $\geq$0.9/1, whatever the *gr* value is, the r_c curves appear multiple peaks phenomena as (*dr, fd, gr*) change. As *dr* increases, the r_c increases and can reach largest positive 1. The *fd* =0.9/1 plays a key role for the transition features of the r_c depending on the matched sense of 3 different hybrid ratios (*dr, fd, gr*). The LUHNM can have a better understanding the r_c change in different hybrid rations. Obviously, the results in the level-2 are more closer to the RWNs and can give a reasonable answer concerned question: why social networks are mostly positive degree-degree correlation but biological and technological networks tend to be negative degree-degree correlation. The LUHNM can further increase additional hybrid ratio according to actual need, and makes it more flexible and potential application.

6 The Level-1: UHNM-VSG

Further comparison to the RWNs and in-depth analysis, it is obvious that, even so, in the level-2 is still not fully reflect the actual network growth situation, because actual networks usually display variable speed growing process, such as high-tech network, the Internet, the WWW, human social networks, communication networks and so on. Therefore it is necessary to introduce a variable growth hybrid ratio, vg, which is defined by

$$vg = \frac{DVG}{RVG} \tag{6}$$

where *DVG* is time intervals of deterministic variable speed growth, and *RVG* is time intervals of random variable speed growth. Thus we propose and construct the unified hybrid network model with various speed growing (UHNM-VSG)[31-32] as the level-1 of the NMCP[31-33]. The level-1 has two variable growth pictures: deterministic and random growth, for example, one may take a growing format as follows [13-14]:

$$m(t) = p(N(t))^{\alpha} \tag{7}$$

where *m(t)* is *t* time to increase the number of nodes connected edge, *N(t)* is the number of nodes at the network at *t* time, α is growth index, p is a constant for deterministic growth; but for random growth the linking probability is $0 < p(t) < 1$. According to the value of the variable speed index α we have normal(α=0), deceleration(α<0), acceleration(0< α <1) and super-accelerated situation(α >1). Therefore the UHNM-VSG is of flexible and includes most current important kinds of network models. The level-1 has rich fresh features as follows.

6.1 Transition of *P(k)* from Single Scale to Broad Scale as α Changes

It is found in the level-1 that transition of the cumulative degree distribution *P(k)* can be changed from single SF to double stretched exponential (SED) as growth index α increases. Fig.2(a)-(c) shows comparison of the *P(k)* under different α for fixed *fd*=0/1and *gr*=0/1 case. In Fig.2(a) α =0, no matter *dr* how change, *P(k)* follows the power-law distribution:

$$p(k) \approx k^{-\gamma} \tag{8}$$

The power exponent γ increases in nonlinear way as α increases, the change of γ is sensitive to *dr* increase, this is consistency with the level-3. However, if α is not equal to zero, for instance, when α =0.3 and 0.6 under different *dr,* the *P(k)* not only emerges the SF but can change to double SED, which is expressed by [33] :

$$P(k) = e^{-(\frac{k}{k_0})^c} \tag{9}$$

where k_0 is a parameters, c is stretched exponent. It is found that *P(k)* obeys the SED and there exits a transition point near at *dr*=1/1, at two sides of the transition point *P(k)* has two different SED, named as first SED and second SED. The topological properties can be changed from the SF to double SED as α>0.3, as shown in Fig.2, and depend on the hybrid ratios. It implies the relation of stretched exponent *c* with *dr* and α is much more complex.

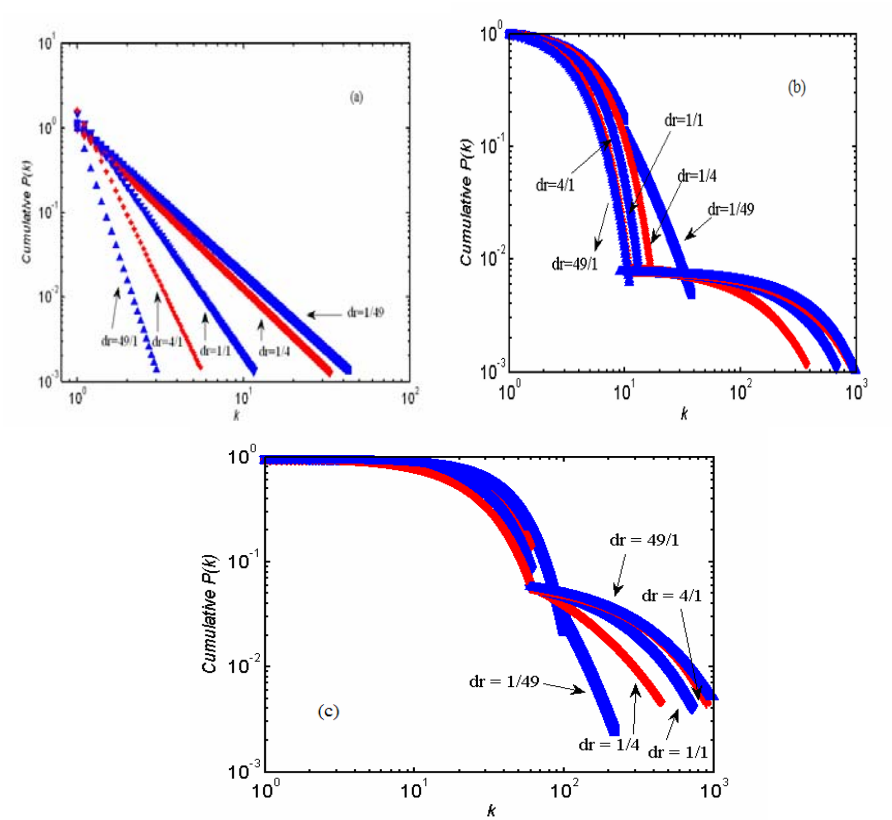

Fig. 2. For *fd*=0/1 and *gr*=0/1 cumulative degree distribution *P(k)*VS *k*. (a) α=0; (b) α=0.3, (c) α=0.6.

6.2 Transition of *P(k)* from Single Scale to Broad Scale as *vg* Changes

The hybrid growth ratio *vg* is another key control parameter in the level-1 and effects on topological properties largely. It is found that all cumulative degree distribution *p(k)* display two kinds of distribution. For random prevailing (*dr*=1/49) case first half curve *p(k)* follows delayed exponential distribution:

$$p(k) = A_1 e^{-\frac{k}{t_1}} + y_0 \tag{10}$$

where A_1, y_0 and t_1 are three parameters but second half curved *p(k)* obeyed the power-law $p(k) \approx k^{-\gamma}$.

However, for *dr*=1/1 case first half curve obeys the Gaussian distribution defined by

$$P(x) = y_0 + \frac{A}{w\sqrt{\frac{\pi}{2}}} e^{-2(\frac{x-x_c}{w})^2} \tag{11}$$

where y_0 ,x_c ,w and A are the Gaussian associated parameters, but second half curve *p(k)* is the SDE. For determinacy prevailing (*dr*=49/1) mode first half curve *p(k)* follows delayed exponential distribution above but second half curve *p(k)* is the SED. The different *p(k)* and transition relations depend on four hybrid ratios. Therefore the level-1 has more complex topological properties comparing with the other levels of the pyramid.

6.3 The r_c Versus Variable Hybrid Growth Ratio *vg*

Degree-degree correlation coefficient r_c is another important characteristic quantity of complex network. In the level-1 one of main features is that always display complicated nonlinear relation. Only if the *vg* greater than 1 then the r_c always appears multiple peaks with *vg* changes,, no matter what *dr* work mode, the relation of the r_c with four hybrid ratios (*vg, dr, fd, gr)* are more complex wave crests. For example, if *dr*=49/1, great change of topological property is taken place in the level-1 as *vg* changes. Under the same parameters, if the *vg* is approach to 1/1, then the r_c appears a r_c maximum value. As the *vg* changes, the r_c value is changed largely within a range of [-1,0.4]. For the 3-dimension picture there are complicated relations of the r_c with log(*vg*) and log(*dr*), generally, many wave crest and trough of oscillation ups and downs crisscross depending on four hybrid ratios can be observed. In additional, there also exits complex relation of the average clustering coefficient (ACC) with (*fd, gr, dr,* α), as α increases the ACC increases in nonlinear way depending on all hybrid ratios. The mystery of the UHNM-VSG has implication relation in their special cases.

7 Comparison and Summary

It is seen from the level-1 the UHNM-VSG model can provide much more information about topological properties and their transition relations of the cumulative degree distribution *P(k)* between the SF and the SED by controlling four hybrid ratios (*vg, dr, fd, gr)* for two kinds of growth pictures in Eq.(7). Three-dimensional relationships for the *ACC* and the r_c with the hybrid ratios (*dr, fd, gr, vg,* α) are much more complicated. The *ACC* can be changed from 0 to 1 in nonlinear fashion and appear a variety of wave crests and troughs. The r_c is changed between +1 and -1 only depending various matching of the 4 hybrid ratios. Compared with the level-3 the HUHPM and the level-2 the LUHNM, the level-1 the UHNM-VSG can include most current network models and approaches to real-world networks in a close range as demonstrated in Ref. [14]. The SED can provide a better description for economical networks [14] and high technology networks [19-22].

In short, we suggest, describe and review one kind of network model complexity pyramid with seven levels. Table-1 gives a comparison and summary for various levels of the pyramid under the different hybrid ratios.

It is seen from Fig.1 and Table-1 that all models of the pyramid levels can be very well studied in unification form by the 4 hybrid ratios (*dr, fd, gr, vg*). It is found that from the bottom level-1 to the top level-7 of the pyramid universality-simplicity is increasing but complexity-diversity is decreasing. On the other hand, from the top level-7 to the bottom level-1 of the pyramid universality-simplicity is reducing but complexity-diversity is strengthening. All properties and changes between seven levels of the pyramid depend on matching of four hybrid ratios (*dr, fd, gr, vg*)

Table 1. Comparison of the pyramid levels under the different hybrid ratios

Hybrid ratios / *Model*	dr	gr	*fd*	*Vg*	Properties	Pyramid Level
EG	1/0	0/0	0/0	0/0	Simple	7
ER	0/1	1/0	0/0	0/0	Emergence	6
WS	1/0.1 (a few)	1/0	0/1	0/0	Small World Simplicity↑	5
BA	0/1	0/1	0/0	0/0	SF, universality↑	
	0/1	0/1	0/0	0/0	Scale-free(SF)	
BB,BBP,BBV, , TDE etc	The	same as	BA model	above	3-power-law SF,SW	4
HUHPM	tunable	0/1	0/1	0/0	Complexity Diversity SF,SW	3
LUHNM	tunable	tunable	tunable	0/0	Complexity↑ Diversity↑ simplicity↓	2
LUHNM-VSG	tunable	tunable	tunable	tunable	Complexity↑↑ simplicity↓↓ SF↔SED	1

strongly. The network model complexity pyramid has preferable understanding in complicated transition relationship among various characteristics as well as the change features. The NMCP is of universal, self-adapting and flexibility and can be extended to study many real-world networks. However, more exact theoretical work of complex network pyramid is still open and a very challenging for researchers.

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