The Control Based on Internal Average Kinetic Energy in Complex Environment for Multi-robot System*

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Abstract. In this paper, reference trajectory is designed according to minimum energy consumed for multi-robot system, which nonlinear programming and cubic spline interpolation are adopted. The control strategy is composed of two levels, which lower-level is simple PD control and the upper-level is based on the internal average kinetic energy for multi-robot system in the complex environment with velocity damping. Simulation tests verify the effectiveness of this control strategy.

Keywords: multi-robot system, velocity damping, trajectory tracking.

1 Introduction

With the constantly expanding of the robot application areas, it is very difficult for the single robot to meet the demand on application. Multiple robots form a system and completing a more complex task through collaboration is increasingly becoming the focus problem in the robotics and intelligent science field [1-2]. The important goal of the development of the multi-robot systems is to design a basic structure of distributed control, so that the robot can implement tasks without the supervision, and it demands strong self-adaptive ability when the robot works in an unknown environment. Trajectory tracking is a comparatively effective control strategy to realize multi-robot foraging mission and map detection, and it is mainly divided into two stages: the generation of reference trajectory and trajectory tracking. Under the condition that the multi-robot model is known, the track meet dynamic stability can be generated offline. The investigation of multi-robot origins from the behavior of swarm organism in nature which can finish a complex work though coordination such as flocks of birds, school of fish. Biologists have been working on understanding and modeling of swarming behavior for a long time. There are two fundamentally different approaches that they have been considering for analysis of swarm dynamics. For one hand, statistics method is adopted for the multi-robot system from the macro aspects; for another hand, the individual-based models from the microeconomic are becoming more and more popular.

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* This work is sponsored by the National Science Foundation of China (Grant No: 60675057).
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J. Zhou (Ed.): Complex 2009, Part I, LNICST 4, pp. 607 – 617, 2009.
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In 1986, Reynolds developed a behavior model to animate the coordinated motions of a group of agents named "boids". Vicsek model is a special case of boids. Gazi(2004) established a one-order system for stability analysis of social foraging swarm[3],which is a good survey of previous work in swarm literature. He presented a continuous first-order kinematic model for swarm members, and applied the idea of virtual force to propose a decentralized controller to analyze swarm aggregation in n-dimensional space. Gazi(2004) specified a general class of attraction/repulsion functions that can be used to achieve swarm aggregation. They presented stability analysis for several cases of the functions considered to characterize swarm cohesiveness, size and ultimate motions while in a cohesive group. Above models can be used to handle different control objectives, however, the shortcoming of above models is that it require all the individuals moving with a common velocity which is not suitable for some application. Although second order dynamic model are utilized, most of them are based on kinematic swarm model. Sliding mode controller is developed to force the vehicle motions to obey dynamics of the kinematic swarm in [4]. Pedrami(2007) firstly presents control and analysis of energetic swarms in [5], in particular the internal kinetic energy is investigated in his paper, Pedrami(2007) modify the temperature definition as the internal average kinetic energy in[6].The cohesion of swarm is an important issue for multi-robot system. It is assumed that a swarm internal energy is bounded, then, a relation between the swarm size and internal average kinetic energy studied in his paper. Pedrami(2008) introduce the controller for wheeled mobile robots(WMR) In [7].

In above work, the environment of multi-robot system is ideal, which there not exist any damping. As a matter of fact, the velocity damping is ineluctable which is not investigated in above works. Another point which is worth noting is that control means energy input, e.g. when the temperature in [5] is too large, we have to offer too much energy. This is not we expect. The energy we have is limited, so how to use the limited energy for finishing the work is meaningful for the real project. Therefore nonlinear programming and cubic spline interpolation are adopted in this paper for trajectory tracking which the environment is not ideal i.e. the velocity damping is not zero.

The paper is organized as follows. In section 2 an 2-dimensional second order swarm model is presented. In section 3 the lower-level control is a simple PD control which the generation of reference trajectory is based on nonlinear programming and cubic spline interpolation. In section 4, a complete discussion of the swarm energy is performed and an upper layer internal average kinetic energy controller for multi-robot with the velocity damping is developed. It is also shown how an internal average kinetic energy controller is useful. Simulation results are presented in section 5 to verify the effectiveness of proposed control strategy. The paper ends with conclusions and future research directions in section 6.

2 Swarm Model

Consider a swarm of M members moving in 2-dimensional space. For the multi-robot system we have assumptions as follow

Assumption 1

(1) We ignore each robot's dimension and treat it as a point mass.
(2) All robots move synchronously, i.e., there is no communication delay between each other.
(3) All robots are homogenous, i.e., there is no difference in essence.

Previous theoretical and modeling efforts fall into a number of categories. Many of the original ideas about group spacing were formulated qualitatively, or using simple formulate. For each robot, we can establish the model according to the classical Newtonian mechanics law as follow

$$\dot{x}_i = v_i \tag{1}$$

$$m_i \dot{v}_i = u_i^{ext} + u_i^{in} - b_i v_i \tag{2}$$

where $i = 1,2,3...,M$, $x_i \in \Re^2$ is the position of the i th robot, $v_i \in \Re^2$ is the velocity of the i th robot, $m_i = m$ is the mass of the i th robot, u_i^{ext} is input which we impose. u_i^{in} is the total force on the robot i as a result of inter-robot interaction, obviously, we have $\sum_{i=1}^{M} u_i^{in} = 0$ because the internal interaction offset. The term u_i^{in} is for the cohesion of swarm and is of the form

$$u_i^{in} = -\sum_{j=1, j\neq i}^{M} [g_a(\|x^i - x^i\|) - g_r(\|x^i - x^i\|)](x^i - x^i) \tag{3}$$

where $g_r : \Re^+ \to \Re^+$ and $g_a : \Re^+ \to \Re^+$ represents respectively the magnitude of repulsion force and the attraction force. $-b_i v_i$ represents the velocity damping from the environment. It is worth noting that $b_i = b \neq 0$ in this paper.

Assumption 2. There exist corresponding functions $J_a : \Re^+ \to \Re$ and $J_r : \Re^+ \to \Re$ such that for any $y \in \Re^2$

$$\nabla_y J_a(\|y\|) = y g_a(\|y\|), \nabla_y J_r(\|y\|) = y g_r(\|y\|) \tag{4}$$

Definition 1. The swarm center $\bar{x} \in \Re^2$ is defined by

$$\bar{x} = \frac{1}{M}\sum_{i=1}^{M} x_i \tag{5}$$

Therefore, the velocity of the swarm center $\bar{v} \in \Re^2$ is derived by time differentiation

$$\bar{v} = \frac{1}{M}\sum_{i=1}^{M} v_i \tag{6}$$

The position and velocity can be denoted as follows

$$x = (x^{1^T}, ..., x^{M^T})^T, v = (v^{1^T}, ..., v^{M^T})^T$$

Then, the system can be expressed as

$$\begin{pmatrix} \dot{x}^i \\ \dot{v}^i \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & \text{-b} \end{pmatrix} \begin{pmatrix} x^i \\ v^i \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} (u^i{}_{ext} + u^i{}_{in}) \tag{7}$$

We have assumption that $u_i^{ext} = u^{ext}$ because of (4) and $\sum_{i=1}^{M} u_i^{in} = 0$, we have

$$m\ddot{\overline{x}} + b\dot{\overline{x}} = u^{ext} \tag{8}$$

3 The Generation of Reference Trajectory Based on Minimum of Energy Consumption

In the field of robotics control, it has always been a goal pursued that multi-robot system can finish a complex mission with minimum of energy consumption. It is of great significance for space exploration which multi-robot system does. The engineering background of this paper can be understand that the multi-robot system moves from an initial position to a position designated in advance in order to achieve the assignment of map detection, foraging and so on. In this paper, the performance index is the minimum of energy consumption by means of nonlinear programming; the optimal position is achieved, then the reference trajectory gained according to cubic spline interpolation. So the reference trajectory can be tracked though PD controller.

3.1 The Generation of Reference Trajectory

For (8), let $\overline{x} = (\overline{s}_x, \overline{d}_x)$, then $\overline{s}_x$ can be expressed as polynomial of degree m over $\overline{d}_x$ as follows

$$\overline{s}_x = p_0 + p_1 d_x + p_2 d_x{}^2 + \cdots + p_m d_x{}^m \tag{9}$$

Then,

$$\dot{\overline{s}}_x = p_1 + 2 \cdot p_2 d_x + \cdots + m \cdot p_m d_x{}^{m-1}$$
$$\ddot{\overline{s}}_x = 2 \cdot p_2 + \cdots + m \cdot (m-1) p_m d_x{}^{m-2}$$

where $p_0, p_1 \cdots p_m$ are undetermined coefficients; suppose that the initial point is $\overline{x}(0) = (0,0)$; the target point is $\overline{x}(T)$; the tracking time T are unknown. $[\overline{x}(0), x(T)]$ is divided into N parts average according to $\Delta t = x(T)/N$. Take minimum energy as the objective function, the state of initiation point and target point are the constraint conditions. Then according to the definition of definite integral, we have

$$\min J = \int_0^{x(T)} u(t) \cdot u^T(t) dx = \sum_{k=0}^{N-1} u(k) \cdot u^T(k) \cdot \frac{x(T)}{N} \tag{10}$$

Furthermore, optimal trajectory is achieved taking the form as follow

$$\overline{s}_x = p_0 + p_1 d_x + p_2 d_x{}^2 + \cdots + p_m d_x{}^m$$

3.2 Numerical Experiment

The dynamic model for multi-robot system can be expressed in form of (8). $X=(x,y)$ is position of robot in 2-D plane. The structural parameter values in this paper are respectively the quality $m=1$, the number of robots $M=5$, $b=0.2$; the initial position $\overline{x}(0)=(0,0)$;the target position $\overline{x}(T)=(10,10)$ .The reference trajectory based on minimum of energy consumption is obtained through Matlab numerical experiment:

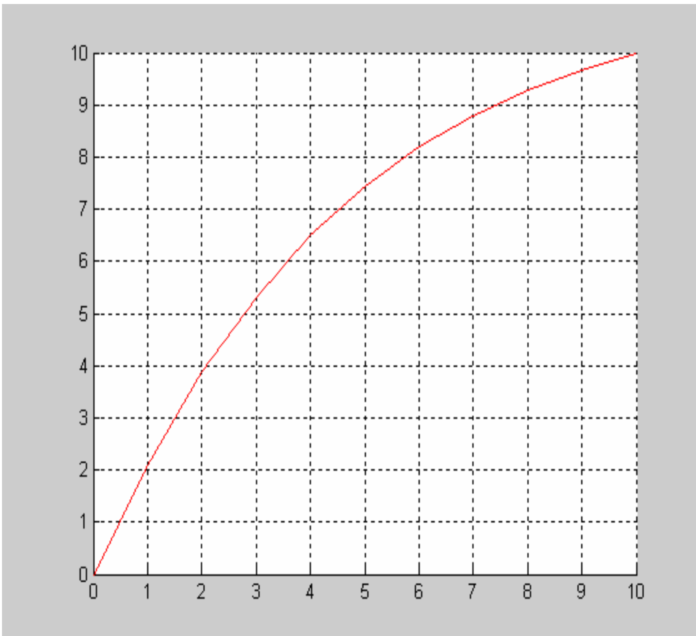

Fig. 1. Reference trajectory

4 Internal Average Kinetic Energy Controller

In this section, a controller based on internal average kinetic energy is introduced. Design procedures include two different steps. The first step is to design the controller such that the swarm center arrives at the specified position as soon as possible. The second step is to modify the control input designed in the first step. Any combination of individual input u_i^{ext} that satisfies (16) guarantees the optimal control for swarm center. However, it does not guarantee the cohesion of swarm. Various types of energy are present firstly. Total potential energy of swarm

$$J(x)=\sum_{i=1}^{M-1}\sum_{j=i+1}^{M}[J_a(\|x^i-x^i\|)-J_r(\|x^i-x^i\|)] \tag{11}$$

Total kinetic energy of swarm

$$E_k(v) = \frac{1}{2}\sum_{i=1}^{M} m_i \left\| v^i \right\|^2 \tag{12}$$

Average kinetic energy of swarm

$$E_b(v) = \frac{1}{2}(\sum_{i=1}^{M} m_i)\left\| \overline{v} \right\|^2 \tag{13}$$

Definition 2. Internal average kinetic energy T is defined as follow

$$T = 1/M[E_k(v) - E_b(v)] \tag{14}$$

where T is directly related to velocity distribution. Velocity distribution is useful to study a coverage path planning. There exist some types of equivalent form

$$T(v) = \frac{m}{2M}(\sum_{i=1}^{M} \left\| v^i \right\|^2 - M\left\| \overline{v} \right\|^2) = \frac{m}{2M^2}\{\sum_{i=1}^{M-1}\sum_{j=i+1}^{M} \left\| v^i - v^j \right\|^2\}$$

According to assumption 2, the time differentiation of T is given by

$$\dot{T}(v) = \phi + \sigma + \psi$$

where ϕ, σ, ψ are

$$\phi = \frac{1}{M}\{\sum_{i=1}^{M}[(u_{ext}^i)^T v^i] - \overline{v}^T(\sum_{i=1}^{M} u_{ext}^i)\}$$

$$\sigma = \frac{1}{M}\{\sum_{i=1}^{M}[(u_{in}^i)^T v^i]\}$$

$$\psi = \frac{1}{M}\{\overline{v}^T(\sum_{i=1}^{M} b_i v^i) - \sum_{i=1}^{M}[(v^i)^T b_i v^i]\}$$

Proposition 1. Consider the following controller in viscous environment

$$u_i^{ext} = u^*(\overline{x}, \overline{v}) + u_i^T$$

where the extra control is given by

$$u_i^T = -\sum_{i=1}^{M} \alpha_{ij}(v^i - v^j) \tag{15}$$

The α_{ij} is the control parameter and developed as

$$\alpha_{ij} = -\frac{(x^i - x^j)\beta_{ij}(v^i - v^j)}{\left\| v^i - v^j \right\|^2} + (\lambda - b/M)\frac{(\left\| v^i - v^j \right\|^2 - k)}{\left\| v^i - v^j \right\|^2}$$

where λ is a positive constant and k is a parameter for control; β_{ij} is given as

$$\beta_{ij} = g_a(\left\| x^i - x^i \right\|) - g_r(\left\| x^i - x^i \right\|)$$

It can be shown that T is convergent.

Proof. For optimal control, it is required to investigate the input. From (20) the input is calculated as

$$\sum_{i=1}^{M} u^{i}{}_{ext} = Mu^{*}(\bar{x},\bar{v}) + \sum_{i=1}^{M} u^{i}{}_{T}$$

To keep optimal characteristic of above control, the extra input $u^{i}{}_{T}$ should satisfy the following condition $\sum_{i=1}^{M} u_i^T = 0$. This condition holds since control parameters α_{ij} satisfies the symmetry, e.g., $\alpha_{ij} = \alpha_{ji}$, so

$$\sum_{i=1}^{M} u_i^T = -\sum_{i=1}^{M-1} \sum_{j=i+1}^{M} [\alpha_{ij}(v^i - v^j) + \alpha_{ji}(v^j - v^i)] = 0$$

This means that the extra control input u_i^T can be viewed as an internal interaction. Furthermore, we have

$$\begin{aligned}
\dot{T}(v) &= \phi+\sigma+\psi=\sigma+\psi=\frac{1}{M}\{\sum_{i=1}^{M}[(u_{in}^{i}+u_{i}^{T})^{T}v^{i}]\}+\frac{1}{M}\{\bar{v}^{T}(\sum_{i=1}^{M}b_{i}v^{i})-\sum_{i=1}^{M}[(v^{i})^{T}b_{i}v^{i}]\} \\
&=1/M\{\sum_{i=1}^{M}[(-\sum_{j=1,j\neq i}^{M}\beta_{ij}(x^{i}-x^{i})-\sum_{i=1}^{M}\alpha_{ij}(v^{i}-v^{j}))^{T}v^{i}]\}+1/M[\bar{v}^{T}bM\bar{v}-b\sum_{i=1}^{M}\left\|v^{i}\right\|^{2}] \\
&=-(1/M)\{\sum_{i=1}^{M-1}\sum_{j=i+1}^{M}\beta_{ij}(x^{i}-x^{i})^{T}(v^{i}-v^{j})+\sum_{i=1}^{M-1}\sum_{j=i+1}^{M}\alpha_{ij}(x^{i}-x^{i})^{T}(v^{i}-v^{j})\}+1/M(-2TMb/m) \\
&=-(1/M)[(\lambda-b/M)\sum_{i=1}^{M-1}\sum_{j=i+1}^{M}(\left\|v^{i}-v^{j}\right\|^{2}-k)+(2TM/m)b] \\
&=-(1/M)[(\lambda-b/M)T-(\lambda-b/M)Mk(M-1)/2+(2TMb)/m] \\
&=-\lambda(2M/m)T-k(M-1)(b/M-\lambda)/2
\end{aligned}$$

In other words, we have

$$\dot{T}(v) + \lambda\frac{2M}{m}T(v) + \frac{k(M-1)(b/M-\lambda)}{2} = 0$$

The differential equation is stable since $\lambda > 0$. This completes the proof.

5 Simulation Results

In this section, simulation will verify the validity of the controller proposed above. The goal of the experimental tasks is that swarm robots system need to arrive at the specified prior location so as to implement rescue as soon as possible, at the same time, the swarm system need to keep cohesion. The swarm system has 5 robots and moving in 2-D space. The attraction and repulsion functions are expressed by

$$g_a(y) = A exp(-\frac{y}{a}); g_r(y) = \mathrm{R}\, exp(-\frac{y}{r})$$

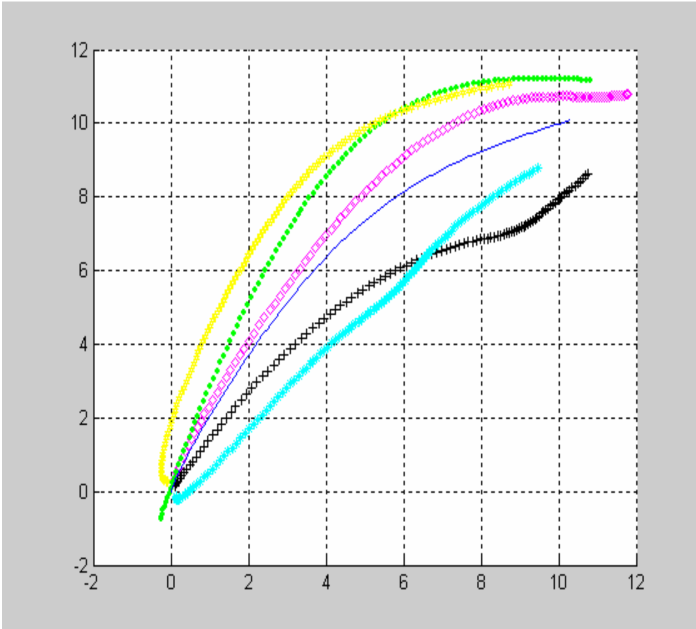

Fig. 2. Each robot trajectory for group 1

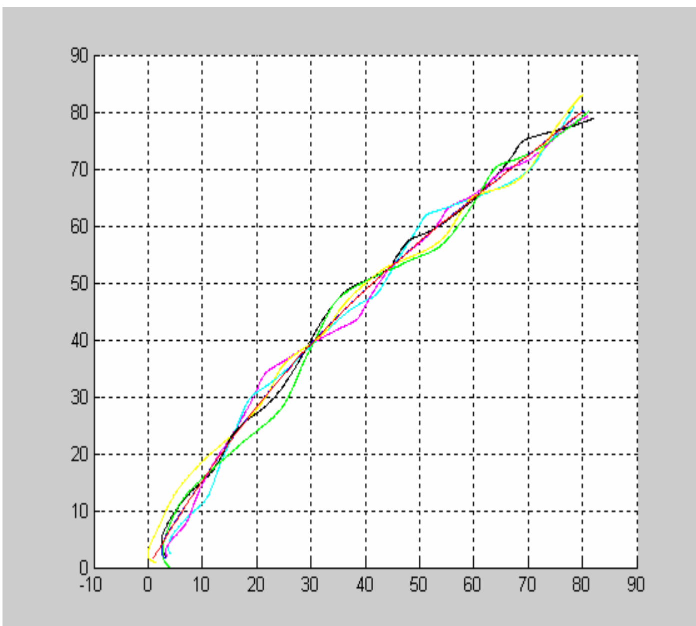

Fig. 3. Each robot trajectory for group 2

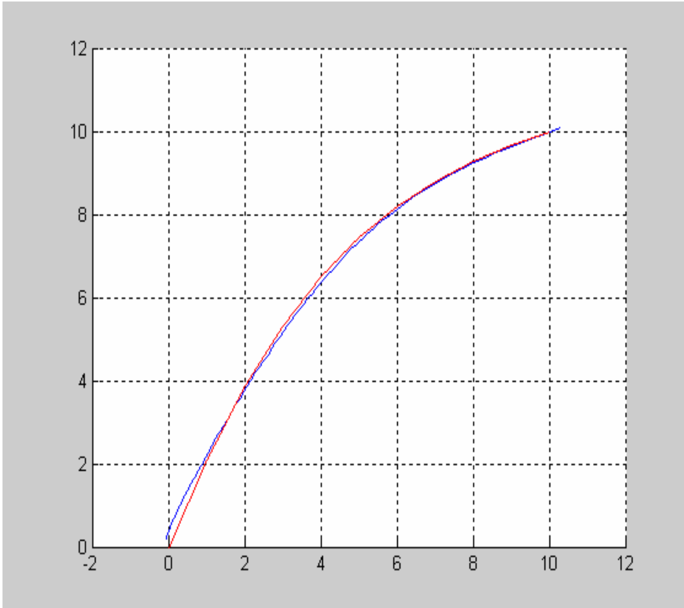

Fig. 4. Reference trajectory and average trajectory

where A is the magnitude of attraction and a is the spatial range of the attraction, while R the magnitude of repulsion and r is the spatial range of the repulsion. In order to keep cohesion, $R > A$ *and* $a > r$ is necessary which is the short-ranged repulsion and long-ranged attraction case. There are 2 groups of experiments for the proposed controller, for group 1, we select the design constants to be $\overline{x}(0) = (0,0)$, $\overline{x}(T) = (10,10)$, $b = 0.2$, $m = 1$, $M = 5$ .For group 2, $\overline{x}(T) = (80,80)$.

Fig.2 and Fig.3 show all the trajectories for the robots keep cohesion. And the average trajectory is from $\overline{x}(0) = (0,0)$ to $\overline{x}(T) = (10,10)$ and $\overline{x}(T) = (80,80)$ respectively. Different colors represent different robots and the blue line is the average trajectory of the swarm system. Fig.4 is the effect of PD controller between reference trajectory and average trajectory.

6 Conclusion and Future Work

In this paper, we showed our unique strength in dealing with distributed control for multi-robot in complex environments with velocity damping. Swarm model is built based on the Newton second law. Reference trajectory is designed according to minimum energy consumed for multi-robot system, which nonlinear programming and cubic spline interpolation are adopted. The control strategy is composed of two levels, which lower-level is simple PD control which can track the reference trajectory and the upper-level is based on the internal average kinetic energy for multi-robot system which can control the size of swarm. For the upper-level, convergence proof is presented for the velocity damping environments. The simulation results testify the effectiveness of the control algorithm for the velocity damping environments.

For future work, we are planning to investigate other approaches which are concentrated on local information exchange for the swarm robot system, and the robustness of the system will be researched.

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