The Effects of Link and Node Capacity on Traffic Dynamics in Weighted Scale-Free Networks

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Abstract. The effect of link and node capacity on traffic dynamics are investigated in weighted scale-free networks by adopting a traffic routing model with local node strength information: $P_{l\to i} = \frac{S_i^\alpha}{\sum_j S_j^\alpha}$. The link bandwidth is controlled by: $B_{ij} = \max(\beta w_{ij}, 1)$, and the capacity of nodes is controlled by: $\max(\gamma s_i, 1)$. The phase transition from free flow to congestion is reproduced. The optimal routing strategy is sought out. When β increases from zero, the optimal strategy changes from preferring low-strength nodes to high-strength nodes. When $\beta \approx 1.0$, there will be two optimal routing strategies. When β is low, the system's behavior is controlled by link bandwidth, while it is controlled by node capacity when β is high. Our work may be useful for the design of modern traffic systems and communication networks.

Keywords: Weighted scale-free networks, Routing strategy, Traffic capacity.

1 Introduction

Complex networks have received much attention from physicists, mainly because a wide range of systems in nature and society could be described by complex networks [1-9]. Recently, the traffic dynamics on complex networks has attract much attention from both physical and computational societies in the past decade [1-6]. This is because of the high importance of large communication networks such as the Internet and WWW in modern society. For example, the submarine earthquake near Taiwan in December 2006 broke a few important optical cables, and after that, the information flow on the Internet were significantly delayed over many countries particularly in the Asia-Pacific region. The goal of traffic research work is to enhance traffic flow and to avoid traffic congestion on a growing large communication network. A variety of studies have been focused on developing better routing strategies [10-16]. Other models were dedicated to improve the system efficiency by changing/optimizing the topology of underlying infrastructure of the networked systems.

It is now known that the network structure plays a significant role in the dynamical process taking place on the network. In the current studies, some

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important properties have been discovered, not only in the topology but also in the weights. Empirical evidences have shown that the modern communication and transportation networks have the small-world (SW) and/or scale-free (SF) properties [1,2]. Therefore, it is natural and important to consider traffic dynamics on SW and SF networks in order to better understand and control various traffic-induced problems. Up to now, many traffic models have been proposed and studied [14-21]. The phase transition from free flow to congestion were reproduced and the point of phase transition were often used to characterize the overall capacity of the systems.

However, the network weights have not been taken into consideration in these models while in most real cases, communication networks are often associated with a large heterogeneity in the capacity and intensity of the connections. Moreover, weights have a strong correlation with the network topology [22-27] and the existing weighted features play a significant role in a variety of dynamical processes [28-30]. Therefore, a modeling approach that can capture the effects of weighted characteristics on traffic dynamics is need.

In this paper, a conceptual traffic model in which packets are routed on weighted scale-free networks is proposed and studied. The proposed model is inspired by the local routing model on un-weighted networks [21]. The present model couples the traffic flow and the weighted characteristics of the network. For traffic model on weighted SF network, we found the optimal capacity occurs at a specific value of routing strategy parameter. The overall capacity is quantified by the critical generating rate, at which a phase transition occurs from free flow to congestion. We also found that the optimal routing strategy depends strongly on the network parameters, which can generally reproduce the real observations.

The paper is organized as follows. In the following section, the traffic model is described in detail, in Sec. 3 simulation results of traffic dynamics are provided, and Sec. 4 gives the conclusion.

2 Traffic Model

Now, we briefly describe the traffic model. We adopt the weighted SF model proposed by Wang et al. [27] to generate the underlying network infrastructure. In this model, the power-law distributions of degrees, weights, and strengths are all in good accordance with real observations of weighted technological networks. The network model is generated with a weight-driven preferential attachment with co-evolution of weights and topology. And the weight-topology co-evolution mimics the traffic interactions of vertices. The model rules can be described as follows. Starting from m_0 nodes fully connected by links with assigned weight $w_0 = 1$, the system are driven by two mechanics: (1) the strength dynamics: the weight of each link connecting i and j is updated as $w_{ij} \rightarrow w_{ij} + 1$, with probability $P_{ij} = W \times p_{ij} = W \times \frac{s_i s_j}{\sum_{a<b} s_a s_b}$, where $s_i = \sum_{j \in \Gamma_i} w_{ij}$ is the strength of node i and Γ_i is the neighboring set of node i; (2) the topological growth: a new node n is added with m links that are randomly attached to a node i according

to the strength preferential probability: $\Pi_{n\to i} = \frac{s_i}{\sum_j s_j}$, where j runs over all existing nodes. Analysis of this model [31] shows that the outcome strength distribution follows a power law $P(s) \sim s^{-\Theta}$ with the exponent $\Theta = 2 + m/(m + 2W)$, where the exponent Θ is controlled by both the weight parameter W and the number of newly added links m. The relationship between node strength and degree follows a power law, i.e., $s \sim k^{\phi}$ with $\phi > 1$. This nonlinear correlation is in good accordance with empirical observations. Then the exponent γ of power-law degree distribution $P(k) \sim k^{-\theta}$ can be expressed as $\theta = \phi(\Theta - 1) + 1$.

The traffic dynamics is modeled on top of the network as follows. At each time step, R packets are generated homogeneously among the nodes in the system. To navigate packets, all the nodes perform a parallel local search among their immediate neighbors. If a packet's destination is found within the searched area of node l, i.e. the immediate neighbors of l, the packet will be delivered from l directly to its target and then removed from the system. Otherwise, the packet will be delivered to a neighboring node i according to the probability:

$$P_{l\to i} = \frac{S_i^{\alpha}}{\sum_j S_j^{\alpha}}, \tag{1}$$

where S_i is the strength of the neighboring node, the sum runs over the immediate neighbors of the node l, and α is an introduced tunable parameter characterizing the preferential probability in choosing neighbors to forward packets. Furthermore, the capacity (or bandwidth) of the link connecting nodes l and i is set to $B_{li} = \max(\beta w_{li}, 1)$, i.e., the link can handle at most B_{li} packets from each end per time step. When the link capacity is reached, the delivery of packets will be delayed and wait for the next time step. We treat all the nodes as both hosts and routers and assume that node i can deliver at most $C_i = \max(\gamma s_i, 1)$ packets per time step towards their destinations, where s_i denotes strength of node i. During the evolution of the system, the FIFO (first-in-first-out) rule is applied on the nodes.

3 Simulation Results and Discussions

The network overall capacity is measured by the critical generating rate R_c at which a phase transition occurs from free flow state to congestion. For this purpose, we investigate the order parameter [10]:

$$\eta(R) = \lim_{t\to\infty} \frac{1}{R} \frac{\langle \Delta N_p \rangle}{\Delta t}. \tag{2}$$

Here $\Delta N_p = N_p(t + \Delta t) - N_p(t)$, $\langle ... \rangle$ denotes taking the average over a time window of width Δt, and $N_p(t)$ is the number of packets in the system at time t. As shown in Fig.1, when $R < R_c$, $\langle \Delta N \rangle = 0$ and $\eta(R) = 0$, the numbers of added and removed packets are balanced, corresponding to the free-flow state. When $R > R_c$, $\eta(R)$ increases suddenly from zero, corresponding to a phase transition from free-flow to congestion, in which packets will accumulate in the

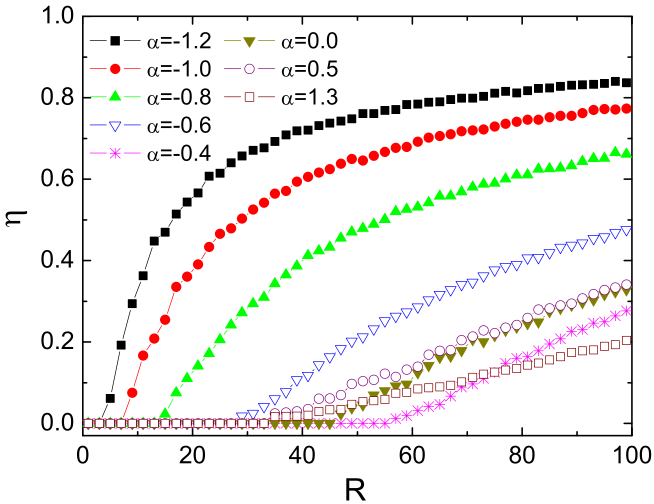

Fig. 1. (Color online) Typical variation of order parameter η versus R for weighted scale-free networks with different routing parameter α. Other parameters are $N = 1000$, $m0 = m = 5$, $W = 2$, $\beta = 1.0$ and $\gamma = 0.5$.

system. Hence, R_c is the maximum packet generating rate under which the system operates effectively. The system's overall handling and delivering capacity can be measured by the critical value of R_c. In Fig.1, one can also see that R_c is different for different α values. When $\alpha = -0.4$, R_c reaches a maximum value of $R_c^{max} \approx 57$. When $\alpha = 0.5$ and 1.3, R_c takes almost the same value. Thus we can investigate the variation of R_c with routing parameters to find an optimal navigation strategy.

Figure 2 and 3 shows the variation of R_c with α for different value of bandwidth parameter β and fix $\gamma = 0.5$, which shows the effect of α and β on the system capacity. One can see that there is a maximum value of R_c at some optimal α_c, which corresponds to the optimal routing strategy. And β has a strong influence on the value of α_c. When β is low, α_c remains at -0.4. With the increment of β, α_c increases from -0.4 to -0.3. When $1.0 \leq \beta \leq 1.5$, there are two peak values of R_c^{max} in the curves. The second peak appears at around $\alpha_c = 1.3$, but with a lower value of R_c^{max}.

When $\beta > 1.5$, the second peak vanishes and the optimal value of α_c increases further from negative to positive. This means that the optimal navigation strategy changes from preferring the low-strength nodes to preferring the high-strength nodes. The system's maximal capacity will increase with β until a threshold is reached at around $\beta = 5$. Then the maximal value of R_c will not increase when $\beta > 5.0$. And α_c remains at 0.3 when $\beta > 5$. This means that the system capacity will not be improved only by increasing the bandwidth of the links. It is because when β is large, all links are operating efficiently under their maximum capacity, so that the network capacity is mainly controlled by node capacity. One should think about some other ways to improve the system

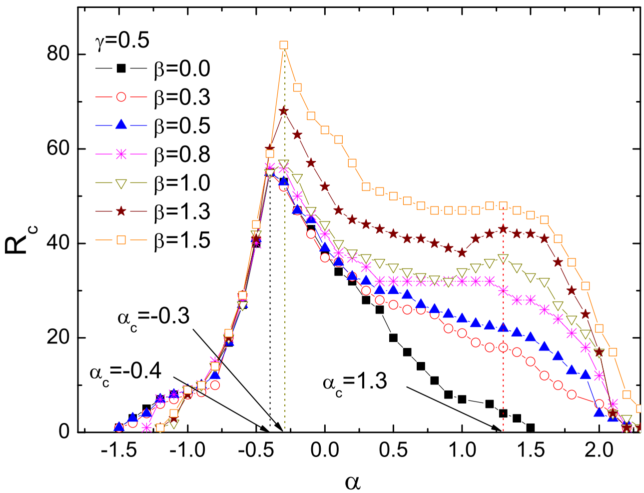

Fig. 2. (Color online) R_c vs α with different value of β. The results are obtained by averaging R_c over ten network realizations. Other network parameters are $N = 1000$, $m0 = m = 5$, $W = 2$.

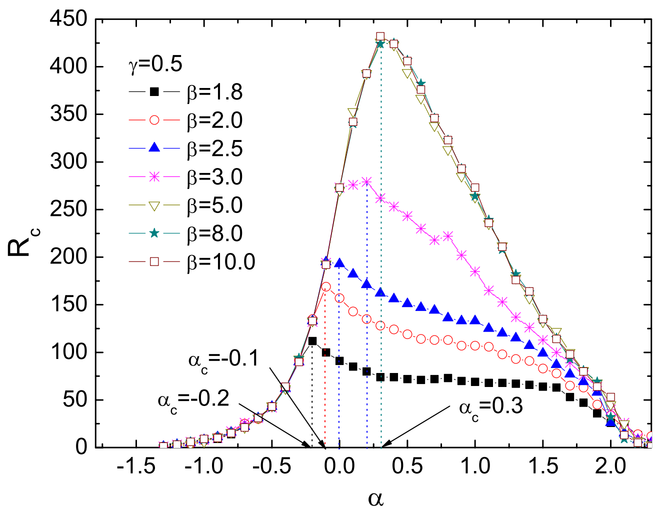

Fig. 3. (Color online) R_c vs α with varying β. Parameters are the same as in Fig.2.

capacity, such as increasing the node capacity, changing the network topology, or developing more efficient routing strategy, and so on.

Further simulations with bigger values of γ show that the system capacity will be greatly improved only when $\beta \geq 5.0$. As shown in Fig.4 and Fig.5, when $\beta < 5.0$, the dynamic behavior of the system remains almost the same. When $\beta \geq 5.0$, R_c^{max} is improved. And the value of R_c^{max} is doubled when $\beta = 8$ and 10.

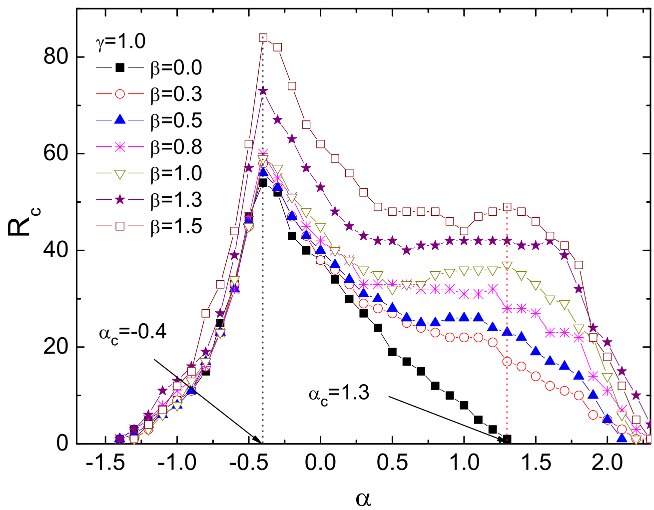

Fig. 4. (Color online) R_c vs α with different value of β and $\gamma = 1.0$. The results are obtained by averaging R_c over ten network realizations. Other network parameters are $N = 1000$, $m0 = m = 5$, $W = 2$.

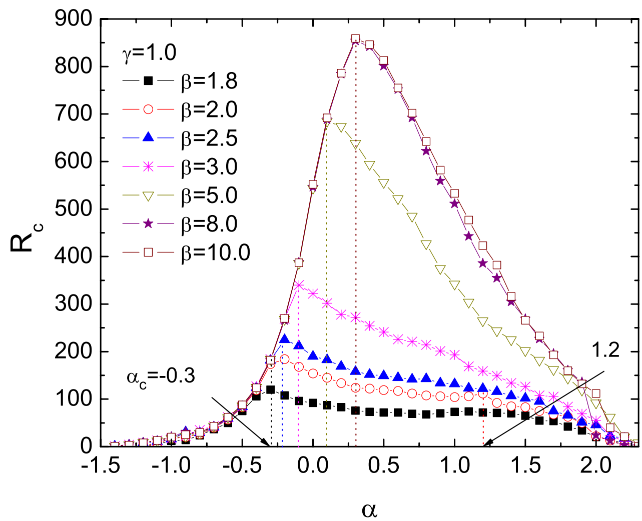

Fig. 5. (Color online) R_c vs α with different value of β and $\gamma = 1.0$. Parameters are the same as in Fig.5.

Thus we can see, when β is low, the system dynamic behavior is mainly controlled by the bandwidth of links. When β is high, it is controlled by the node capacity.

These findings are different from the results on un-weighted scale-free networks. Previous studies with a traffic model routing with node degree information have found that the maximum traffic capacity appears at $\alpha_c = -1.0$ when the nodes' capacity are equal: $C = const$ [21]. This means to repel the packets from the central nodes and to make them move along the periphery of the network. When considering the heterogeneity of node capacity, it is found that $\alpha_c = 0.0$ when the capacity of node is equal to its degree: $C = k$. This means to prefer random walk in the system. Different from previous results, here it is shown that the traffic dynamic behavior strongly depends on the value of β and γ. This finding is valuable since most real networks are weighted, and the traffic will probably be affected by the link bandwidth.

Here we give a heuristic explanation for the peaks and α_c. We note that when $\alpha = 0.0$, if we neglect the effect of link bandwidth, all packets perform randomlike walk in the system. A well-known result in random walk theory valid for our analysis is that the time the particle spends at a given node is proportional to the degree of such a node in the limit of long times [32]. Thus one can see that the probability of sending packets to a given node averaging over a period of time is proportional to the degree of that node. Now we consider the constriction of link bandwidth. When $\beta = 0$, all links have the same bandwidth of 1. The high-strength nodes may cause congestion at the links connecting to them. So that α_c should be negative to avoid sending packets to high-strength nodes. The same effect appears when β is low. On the other hand, when β is very large, the bandwidth of links will not trigger congestion. For the case, because the strength of node is proportional to but bigger than its degree, α_c should be positive to direct more packet to high-strength nodes so that the capacity of these nodes can be fully used. Therefore, it is easy to understand that α_c will gradually change from negative to positive when β increases from zero to some large value.

For the second peak, we note that when $\beta = 1.0$, all links are operated with a bandwidth equaling to their weight. And $\alpha = 1.0$ means that the probability of delivering packets to a given node is proportional to its strength, which is the sum of weight of links connecting to the node. Thus there should be an optimal configuration at around $\beta \approx 1.0$ and $\alpha \approx 1.0$ in order to fully make use of the link bandwidth and node strength together. This is confirmed by previously presented simulation results.

Finally, we briefly introduce the effect of link weight growth rate W on the packet traffic capacity. Since W is just a multiplicative factor, the qualitative behavior is not affected by varying W. In general, the system's overall capacity will increase with the increase of W, but the optimal value of α_c remains the same.

4 Conclusion

In summary, the traffic dynamics on weighted scale-free networks is studied with a local routing strategy. Simulation results show that the link bandwidth, the node capacity and the routing strategy of packets have different effects on the system's dynamic behavior. We found that when the bandwidth is low, it is better to make use of low-strength nodes in the routing strategy. And the optimal routing strategy will change from low-strength nodes to high-strength nodes with the increment of link bandwidth. But the system's capacity will not be improved by merely increasing link bandwidth. This behavior is in agreement with empirical practice. Moreover, we found a second optimal routing strategy when both the link bandwidth and node strength could be fully used.

Our study shows that the traffic dynamics on weighted scale-free network has many new characteristics. It is also different from traffic dynamics on well-organized lattice, on regular or random networks [15,16]. Our work may have practical implications for optimizing some modern traffic and communication systems in the real world.

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