Modelling of Population Migration to Reproduce Rank-Size Distribution of Cities in Japan

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Abstract. We investigate the rank-size distribution of cities in Japan by data analysis and computer simulation. From our previous data analysis of the census data after World War II, it has been clarified that the power exponent of the rank-size distribution of cities changes with time and Zipf's law holds only for a restricted period. We show that Zipf's law broke down owing to the great mergers and recovered by investigating the time evolution of the rank-size distribution of cities without mergers.

Keywords: population, lognormal distribution, power-law distribution, Zipf's law, agent-based modelling.

1 Introduction

Many empirical data that obey the power-law distribution can be observed in both natural and social phenomena. Among them, we often find the special case that the power exponent becomes unity in various phenomena [1]. Generally, we call the empirical law as Zipf's law [2].

In Japan, the rank-size distribution of cities shows the power-law distribution while the rank-size distributions of other municipalities, such as towns and villages, can be approximated much better by the lognormal distribution [4]. Auerbach [3] first reported that the rank-size distribution of the population of cities obeys the power-law distribution which is described by

$$\log R(x) = a - b \log x, \tag{1}$$

where $R(x)$ is the rank of the population x, and both a and b are fitting parameters. Afterward, Zipf proposed that the power exponent b empirically becomes unity in the rank-size distribution of cities [2].

However, the power exponent of the rank-size distribution of cities can easily change due to various factors such as population migration, a change in birth rate, economic situation, etc.[5]. In the case of Japan, after the end of World War II, the rank-size distribution of cities has been changed and strongly affected by the two great mergers of municipalities: the Showa (from 1955 to 1960) and Heisei (from 2000 to now) great mergers [6]. Our previous data analysis and simulation have clarified that the power exponent of the rank-size distribution

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of cities changed under the influence of those great mergers and approached to unity after the Showa great merger [6].

In this paper, we investigate the time-evolution of the rank-size distribution of cities without great mergers of municipalities. Our data analysis is based on the census data from 1950 to 2006, which were obtained from the Statistics Bureau, Ministry of International Affairs and Communications, Japan [7], and data book from Japan Statistical Association [8].

2 Data Analysis

Figure 1(a) shows the time evolution of the rank-size distribution of cities in Japan from 1950 to 2000 [6]. We can find that the head and tail parts of each distribution can be fitted by discrete power-law distribution functions. In addition, between 1950 and 1960, we can find a remarkable increase in the number of cities by 307.

Plus points in Fig. 2 show the time evolution of the power exponent of the rank-size distribution of cities [6]. Between 1950 and 1960, we can find a remarkable increase of the power exponent from the value around unity, which is mainly due to the increase in the number of cities caused by the great Showa merger. In addition, during this period, some towns might be promoted to cities when they fulfilled the promotion condition, one of which is that the population within a town is larger than 50,000 people. These factors caused the increase of the power exponent. After the increase, the power exponent shows a monotonic decrease approaching unity, which means that Zipf's law is recovered. Thus, we can suppose that the power exponent might show monotonic decrease approaching unity without the increase of the number of cities.

To exclude the effect of the great mergers of municipalities and the promotion to cities, we trace the rank-size distribution of the cities as of October 1, 2000 back to 1950. Figure 1(b) shows the time evolution of the rank-size distribution of the 672 cities as of October 1, 2000. The power exponent for each year is

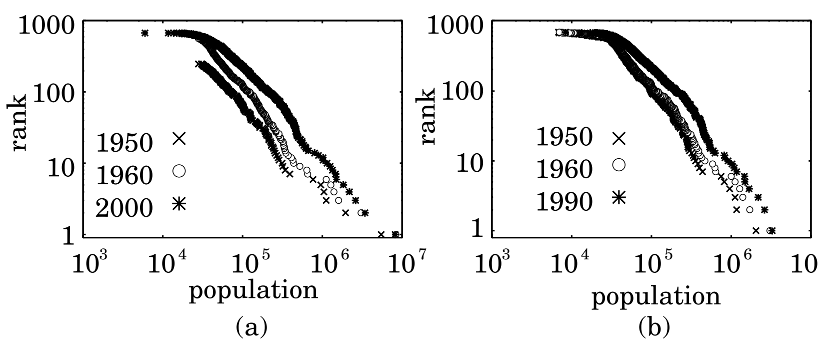

Fig. 1. Time evolution of rank-size distribution of cities in Japan both (a) with and (b) without mergers of municipalities

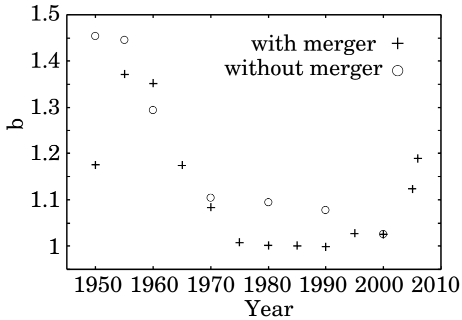

Fig. 2. Time evolution of power exponents of rank-size distribution of cities both with and without mergers

shown as the open circles in Fig.2, where we find the monotonic decrease of the power exponent approaching unity although the values of b are larger than those with mergers in 1980 and 1990. We need to investigate the time evolution of the rank-size distribution of the cities at the instant of 1950 until 2000.

3 Simulation

Here we introduce our model of the population migration to reproduce the increase in b due to the merger of municipalities and its convergence to unity after the merger [6]. Our model is based on an agent-based model that consists of 3500 sites corresponding to all the municipalities. Each site has a uniform random number between 0 and 1 as the initial population. The basic procedure of one simulation steps is summarized as follows:

1. We randomly choose a source site m with the population N_m.
2. We choose a group of sites, $G_{N<N_m}$ or $G_{N>N_m}$, whose populations N are less and more than N_m, respectively. The probability to choose $G_{N<N_m}$ is α (migration parameter), while that to choose $G_{N>N_m}$ is $1-\alpha$.
3. Among the group of sites chosen in the previous step, we randomly choose the destination site n for migration.
4. The P_{mn} percent of N_m is transferred to the site n, so that the populations of sites m and n vary in quantity as $N_m - P_{mn}N_m$ and $N_n + P_{mn}N_m$, respectively.

In the second step, the migration parameter α is introduced to describe the tendency that people migrate to less populated area from large cities, which was evident after the high economic growth from 1960 to the early 1970s [9]. In addition, P_{mn} is randomly chosen in the range from 0 to 20. We iterate this procedure 10^6 times in our simulation. A sample average is taken over 10 different initial population distributions for all the sites.

When the population of a given site becomes larger than 0.95, we regard the site as a city. Once a site is promoted to a city, the site will not be demoted to a smaller municipality such as towns and villages. This rule corresponds to part of the Local Autonomy Law of Japan that municipalities must have a population of 50,000 or more to be promoted to cities [10]. Our model does not distinguish between towns and cities. Thus, if a site does not belong to cities, we henceforth call the site as a "town".

After the first migration of 10^6 simulation steps, we merge some municipalities according to the following procedure. First, we randomly choose two sites to merge among all the sites. When both of them are not cities, we merge them to produce a new city if the sum of those populations becomes larger than 0.95, while we merge them to produce a town if the sum is less than 0.95. On the other hand, when at least one site is a city, we merge those two sites with a probability $\beta = 0.5$ to become a new city. The probability β is introduced owing to the fact that the frequency of the merger of towns was much larger than that of cities. We iterate this merging process until the number of cities increases by 77 on average rather than when the first migration stage is finished. In our model, the increase in the number of cities affects the power exponent after the merger. In general, the power exponent increases with an increase in the number of cities generated by the merger.

4 Simulation Result

Figure 3(a) shows the time evolution of the power exponent of rank-size distributions of cities in 10^5, 10^6, and 10^7 simulation steps without mergers. Error bars that are almost invisible on a few data marks are standard deviation obtained by least-squares linear regression. In this simulation, α is fixed at $\alpha = 0.3$. This figure shows that the power exponent converges to the stationary value around $b = 1.094 \pm 0.001$. Thus, our model can reproduce the power-law distribution of cities that converges to Zipf's law although the number of cities keeps increasing after the power exponent becomes $b = 1$, which slightly increases the power exponent.

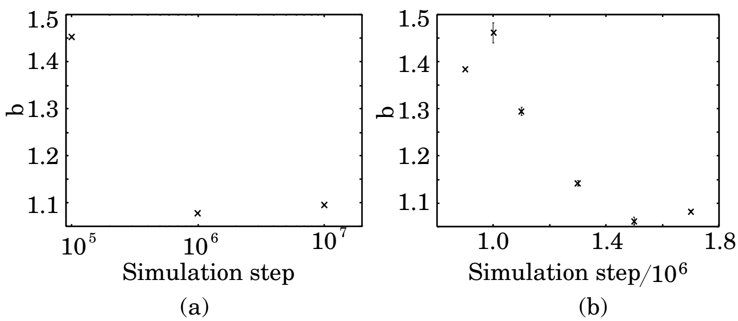

Fig. 3. Time evolution of power exponents of rank-size distribution of cities both (a) without and (b) with mergers

Next, we investigate how the great merger affects the rank-size distributions of cities through the time evolution of the power exponent b. We carry out the first population migration of 10^6 simulation steps. After that, we merge some of those sites, followed by the second population migration of 7×10^5 simulation steps.

Figure 3(b) shows the relation between the power exponent b and the simulation step. Error bars that are almost invisible on a few data marks are standard deviation obtained by least-squares linear regression. Data point at 10^6 steps shows the power exponent b after the merger has finished. We find that b converges to unity after the increase in b due to the merger. Thus, our model can reproduce the time evolution of b qualitatively.

5 Concluding Remarks

We have investigated the time evolution of the rank-size distribution of cities to show how the power exponent might change if there were no merger. The rank-size distribution shows that the power-law behavior and the time evolution of the power exponent markedly change when the great merger of municipalities occurs.

Our future task is, by the use of our model, to reproduce the time evolution of the rank-size distribution of cities without both mergers and the promotion to cities. In addition, we need to investigate the rank-size distribution of all the municipalities.

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