Morphological Similarities between DBM and an Economic Geography Model of City Growth

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Abstract. An urban microeconomic model of households evolving in a 2D cellular automata allows to simulate the growth of a metropolitan area where land is devoted to housing, road network and agricultural/green areas. This system is self-organised: based on individualistic decisions of economic agents who compete on the land market, the model generates a metropolitan area with houses, roads, and agriculture. Several simulation are performed. The results show strong similarities with physical Dieletric breackdown models (DBM). In particular, phase transitions in the urban morphology occur when a control parameter reaches critical values. Population density in our model and the electric potential in DBM play similar roles, which can explain these resemblances.

1 Introduction

In this paper, we propose a model of self-generated city where households evolve in a cellular automata space. We simulate the growth of a metropolitan area where land is devoted to housing, road network and agricultural/green areas. Such as several previous papers, the results show strong similarities with the *Dieletric breackdown models* (DBM) proposed in physics. Our contribution is to show that this property holds with urban economics micro-foundations, while previous works use diffusion mechanisms close to DBM, which are far removed from household's behaviour in urban economic theory.

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1.1 The Literature

Cellular Automata (CA) have been widely used in urban geography for simulating the development of cities. Various types of modelling strategies can be distinguished. A first group favors abstract approaches for studying the interactions between groups of populations. They are linked to the concept of *self-organisation* (see, e.g., Schelling, 1971; Couclelis, 1985; Phipps, 1989). In a second trend of papers, urban growth is modelled more concretely by referring directly to physical (or biological) models to generate structures that reproduce the morphological evolution of cities (see, e.g., Batty and Longley, 1986; Batty 1991; Frankhauser, 1991; Makse et al., 1995). A third type of models introduce into the previous ones rules that describe in a heuristic way and on an aggregated level the spatial interactions between different land uses see, e.g., White and Engelen,1993; 1994). A common concern of these approaches is the absence of micro-economic fundations. Caruso et al. (2007) and Cavailhès et al. (2004) are attempts to bridge this gap, the first paper dealing with CA, the second with a fractal setting.

In the meantime, other concepts have been proposed to simulate the formation of clusters. One such example in physics is *Diffusion-limited aggregation* (DLA), proposed by Witten and Sander (1981), which is the process whereby particles following a random walk cluster to form aggregates. It was applied to simulate town growth (see, e.g., Benguigui, 1995; 1998; or, in a percolation version: Makse et al., 1998). Combining DLA with electric fields, Niemeyer et al. (1984) proposed the *Dielectric breakdown model* (DBM) to describe the patterns of dielectric breakdown of solids, liquids, and gases, and to explain the formation of the branching, self-similar Lichtenberg figures. DLA and DBM have been applied in physics (e.g., discharges in non-homogeneous material (Peruani et al.), surface thermodynamics (Bogoyavlenskiy et al., 2000)) and chemistry, such as biology (see, e.g., Chikushi and Hirota, 1998; Li et al., 1995) or urban geography (see, e.g., Batty, 1991). DLA and DBM models can only generate connected aggregates. It thus seems difficult to apply them as such at periurban zones where built areas are in patches scattered in the countryside. This lead Benguigui et al. (2001) to propose an approach where they introduce leap-frogging. Another concern of the urban applications of DLA-DBM models is the thin link to urban economics theory. Recently, authors have proposed models with economic foundations that lead to DBM-like diffusion mechanisms (see, e.g., Andersson et al., 2002).

1.2 The Self-organised Growth of a Metropolitan Area

The purpose of this paper is to investigate the development of a metropolis where economic agents living in residences scattered in a mixed residential/agricultural area commute to work to an exogenous Central Business District (CBD) by an endogenous road network. This system is self-organised (except for some elementary urban growth rules): based on individualistic decisions of agents who compete on the land market, the model generates a metropolitan area with houses, agriculture and the road network.

Land belongs to absentee landowners, each renting her parcel to the highest bidder, either a resident or a farmer, hence determining land occupancy. Farmers produce food stuff and a green amenity (open space, landscape, etc), which is a by-product enjoyed by neighbouring residents. Households arrive sequentially in the city and choose their location by maximizing a utility function subject to a budget constraint. Each migrant chooses freely its location, considering the commuting cost to the CBD, the rent of the residential plot determined by the competition on the land market and the surroundings. She enjoys both open space/agricultural amenity and local public goods (respectively negative and positive functions of the neighboring population density).

As households migrate into this growing city, agricultural cells are developed, while the local public authority creates a connected road network so as to provide all new households with an access to the CBD. At each step, a resident can move in another cell of the city to maximise her utility if the migrant changes her surroundings. This competition leads to an adjustment of the residential rent until all the inhabitants obtain the same utility (short run equilibrium, instantaneously reached in our framework). Thus, the city grows according to the individual choice of the economic agents, leading to a self-organization of the residential space, the road network and the open-spaces. The mean commuting cost increases with population and thus households' utility progressively decreases. Migration stops when households' utility equals the utility of the rest of the world(i.e. long run equilibrium).

Our analysis privileges the morphological properties of the emerging road network. Appropriate fractal measures are used to this end. One of the main results is the occurrence of phase transitions between linear and dendritic structures when varying the parameters. This is akin to what is observed in the literature with DBM models. Some interesting analogies between both models are elaborated.

The remainder of the paper is organised as follows. Section 2 presents the microeconomic model and the cellular automata environment where households evolve. Section 3 provides some simulations and their morphological properties. Next, Section 4 compares these findings with similar results obtained in physics and chemistry. Section 5 concludes.

2 An Economic Model for Residential Location

2.1 General Setting

We consider a closed 2D space with a set of cells i. A pointwise *central business district* (CBD) is located exogeneously at the centre of the grid, where two preexisting orthogonal roads intersect. Initially, the rest of the grid is occupied by farmers who produce stuff food under constant returns to scale and sell this output on the world market. Each cell has three possible (mutually exclusive) states: residential, road, or agricultural (or undeveloped), which we respectively denote by j, k and l. The grid is gradually filled in by residences and roads from undeveloped cells. Such conversions are irreversible.

2.2 Residential Growth

Households' preferences are represented by a Cobb-Douglas utility function $U = Z^{\delta} H^{\alpha} E^{\beta} S^{\gamma}$, whose arguments are Z, a non-residential composite good made up of every market good except housing, H, housing, E, open-space externalities (greenness), and S, local public goods externalities. Each household maximizes U under a budget constraint: $Y - \theta d = Z + SR$, where Y is the income, θ the unitary commuting cost, d the distance to the city centre and R the unitary land rent (the composite good is taken as the numeraire, hence $p_Z = 1$). The parameters $\delta \in [0,1]$, and $\alpha = 1 - \delta$ indicate respectively the preference for the composite good and for housing, whereas $\beta \geq 0$ and $\gamma \geq 0$ are respectively the preference parameters for open-space and for social externalities. The first-order conditions for a constrained optimum yield the demand functions of Z and S as well as the indirect utility function:

$$V = (Y - \theta d)R^{-\alpha}E^{\beta}S^{\gamma} \tag{1}$$

Households arrive one by one. At each time step t, the migrant picks up the cell l that maximizes its indirect utility function, considering the commuting cost θd_l^t to the city centre, the land rent R_l^t of her residential plot, the neighbourhood open-spaces E_l^t (i.e. the density of agricultural cells within a given radius around l at $t-1$), and the local public goods S_l^t (i.e. the density of residential cells in the same neighbourhood at $t-1$).

A migrant evaluates the indirect utility (1) provided by each agricultural cell. Her bid rent (a reservation level of the land rent) equalises the utility she can obtain in the city with the utility of the rest of the world (open city model of urban economics). Nevertheless, several agricultural cells provide the same utility: the migrant can put the landowners in competition to obtain a decrease of the rent actually paid, until the land rent reaches Φ, the agricultural rent. Her indirect utility function is:

$$V_l^t = (Y - \theta d_l^t)\Phi^{-\alpha}(E_l^t)^{\beta}(S_l^t)^{\gamma}. \tag{2}$$

For more details about the microeconomic program, see Caruso et al. (2007). Let ρ_l be the density of residences within a given neighbourhood around each undeveloped cell l. We define the two neigbourhood externalities as $E_l^t = e^{-\mu\rho_l^{t-1}}$ and $S_l^t = e^{\nu\rho_l^{t-1}}$. (2) can be written (for simplicity, we ommit t and l indexes) $V = (Y - \theta d)\Phi^{-\alpha}e^{-\beta\mu\rho}e^{\gamma\nu\rho}$, where $\beta\mu$ and $\gamma\nu$ are undistinguishables. Thus we fix $\mu = 1$ and $\nu = 0.5$. Let

$$V^t = \max_l V_l^t. \tag{3}$$

The city continues to grow as long as the utility of an entrant as measured by (Uentrant) exceeds a given threshold $\overline{V}$, which is the utility she enjoys in the rest of the world. When $V^t = \overline{V}$, we obtain a so-called *long-run equilibrium*.

2.3 Network

Households access the CBD via a road network gradually built by the local authority, which builds new roads where necessary to provides each new household with an access to the CBD: all the residences must be linked to the existing road network. For deciding which undeveloped cells will be converted to roads, we minimize the number of new road cells to be created at each time step.

3 Morphological Properties of Patterns Generated by the S-Ghost-City Model

A Java-based software (called S-GHOST) was implemented to simulate the model described above. The parameters are the household's income Y, the share of income devoted to housing α, the agricultural rent Φ, the preference for green and social externalities β and γ, the size of a cell's neighborhood $\hat{x}$, and the transportation cost θ. Ex-aequo cells are solved randomly. One of the most striking outcome is the structural change in the urban morphology that occur when β and γ cross some thresholds. In Figure 1, we illustrate the shape of the patterns obtained for fixed β =0.25.

We observe that for γ-values lower than 0.11 residents disperse around the CBD since they look for green amenities in their vicinity and are rather indifferent about social amenities (see Figure 1 (a)). Hence, agricultural cells subsit around build-up

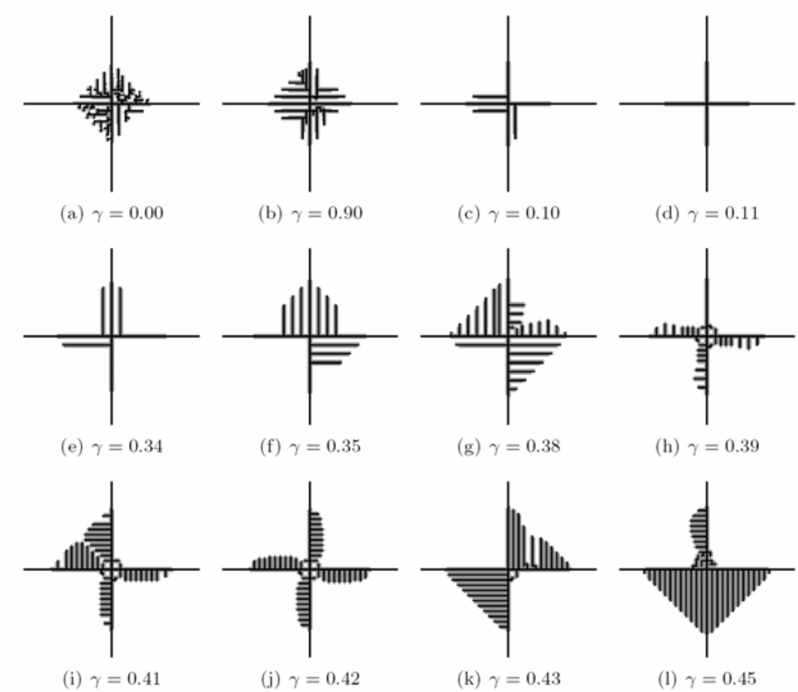

Fig. 1. Long-run equilibria for β= 0.25 and varying γ

cells. At the threshold $\gamma = 0.11$ and above, preferences for social and environmental amenities compensate. The residents line up on the sides of the two preexisting orthogonal roads in order to minimize the commuting cost, because moving to a lateral location would increase this cost without providing enough social or open-space compensation (Figure 1 (d)). The compensation effect is evident when comparing Figure 1 (d) to Figure 2 (a) where $\beta = 0.0$ and $\gamma = 0.0$, i.e. where residents are indifferent to their neighbourhood.

In Figure 1, beyond a second critical value of $\gamma = 0.34$ the city expands again along lateral streets (surfacic pattern). Migrants accept the higher commuting cost of a lateral cell because this cell provides more local public goods than in the cross-like city, due to a higher density. As γ increases further, residents cluster together more densely as they taste for social contact increases. However, green cells separating the built-up cells still exist up to γ around 0.40. Beyond that value, the preference for social contacts dominates residential choice: compact clusters emerge. Moreover, singular phenomena may occur as shows the example of Figure 1 (i), (j) and (l). Here, the access to green areas is blocked by culs-de-sacs so that migrants have to go further to have still access to the CBD.

We now come to the second series considered, which refers to $\gamma = 0$ and a set of different values of β (Figure 2).

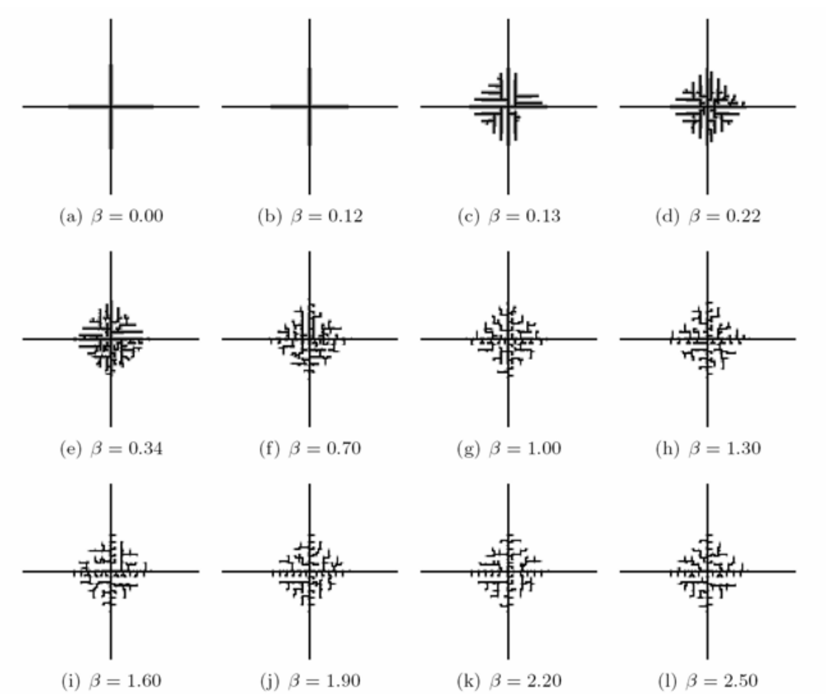

Fig. 2. Long-run equilibria for $\gamma = 0$ and varying β

Hence residents do not have any taste for local public goods, but they are more or less interested by green amenities. For β-values less than 0.12, both preferences are very small, and a cross-like city is again obtained. Beyond this threshold the increasing preference for green amenities dominates and dendritic patterns occur where houses and roads are separated by undeveloped cells (Figures 2 (c) to (l)). This pattern remains stable all over the range considered that runs up to $\beta = 3.0$.

We use fractal analysis to characterize the shape of the road network (see e.g. Benguigui, 1995 or Lu and Tang, 2004). The method used here is described in Frankhauser (19..) and the results are produced in Figure 3 where the fractal dimension (Y-axis) depends on β ($\gamma = 0.0$) (Display A) or γ ($\beta = 0.25$) (Display B). In Display (A), after the linear city ($\beta < 0.12$) the fractal dimension is $1.7 - 1.8$ or so when β is comprised between 0.3 and 2.5. In Display (B), the fractal dimension decreases from 2 or so when $\gamma = 0$ to 1 in the cross-like city ($\gamma = 0.11$ to $\gamma = 0.34$) and then increases in an uneven pattern, most of the values being comprised between 1.5 and 2.

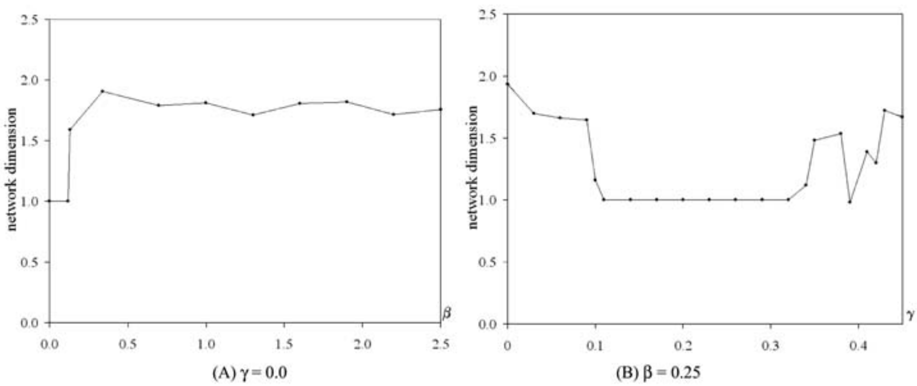

Fig. 3. Fractal dimension for β varying (γ = 0.0) and for γ varying (β = 0.25)

Note that similar simulations can be performed with constant values for β and γ but changing the size of the neighbourhood. Results are presented in figure 4 that shows that the length of the neighbourhood is also an important parameter affecting the form of the urban development. We simulate a case where households strongly taste social amenities ($\gamma = 0.5$), without any taste for green externalities ($\beta = 0$). When the length of the neighbourhood is small ($\hat{x} = 4$), agents enjoy a dense environment far from the CBD and they do not care what happen at the city fringe, which is beyond their horizon. When the length increases ($\hat{x} = 7$), agricultural sites enter in the viewshed of peripherical residents who refuse fareway locations: the city shrinks. A phase transition occurs when $\hat{x} = 8$: the pattern becomes regular at this threshold value. Finally, when $\hat{x} = 12$ the pattern collapses again into a cross-like city where few households accept to live in.

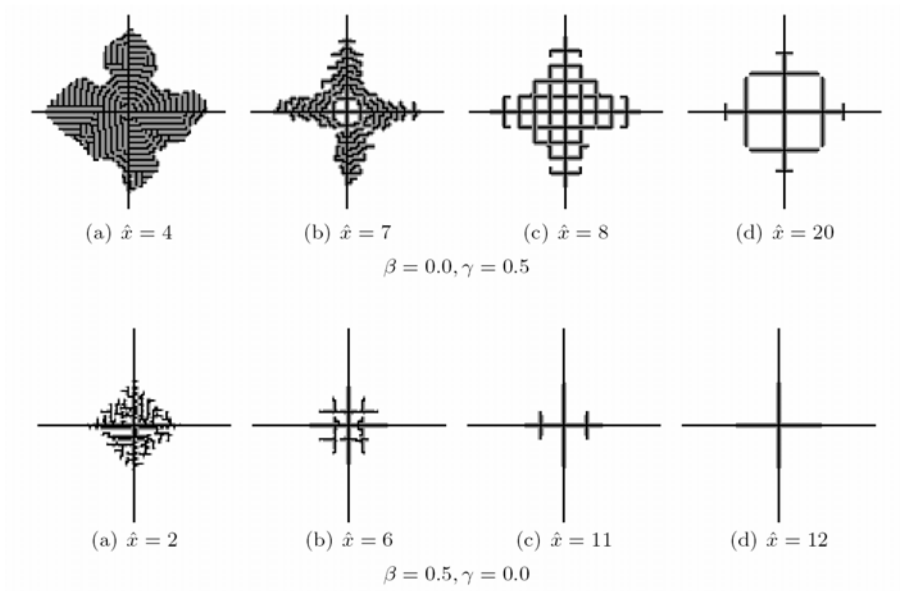

Fig. 4. Long-run equilibria for 2 series of β and γ and varying $\hat{x}$

4 Comparing Our Model to DBM

Figure 5 shows an obvious similarity between some of our simulations and the patterns obtained by Bogoyavlenskiy et al. (2000) when using an enlarged DBM-model for studying the relationship between surface thermodynamics and crystal morphology. Phase transitions are observed in both cases, which are further investigated in this section.

The basic DLA model mimics the diffusion of particles which may stick on a pre-existing seed and added subsequently. They generate a fractal cluster with a fractal dimension of about $D = 1.7$. Niemeyer et al. (1984) introduced the *Dielectric breackdown model* (DBM) to simulate electric discharge patterns. As shown already by Pietronero and Wissman (1984), a formal link can then be established between the DLA-model and the DBM-model. In DBM, the electrodynamic Laplace equation which describes the spatial variation of the electric potential Φ is transcribed into a discrete equation to compute the electric potential for each cell at a given simulation step. Then a site i' located in the immediate neighbourhood of the already generated discharge is chosen randomly, where the probability to choose i' depends on the potential:

$$p_{i \to i'} = \frac{(\Phi_{i'})^{\eta}}{\sum_{i'} (\Phi_{i'})^{\eta}} \tag{4}$$

After having selected the site i', a link is created between i' and the discharge, and the potential in i' falls to zero. Parameter η plays an important role for the

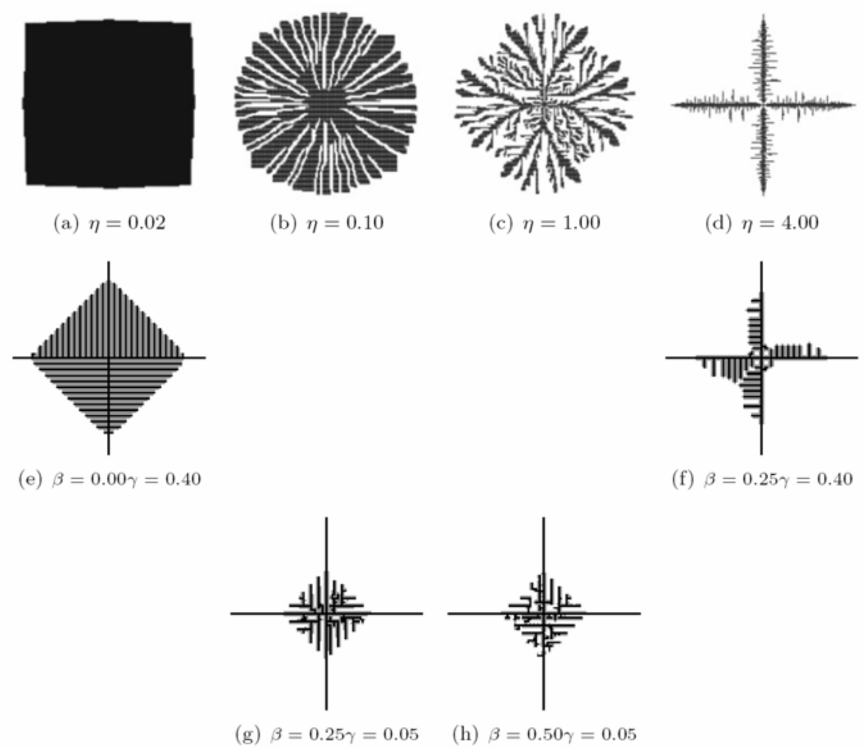

Fig. 5. A complex diffusive growth model ((a) to (d)) and our results ((e) to (h))

shape of the generated patterns. As show by Sanchez et al. (1993), for $\eta = 0$ compact clusters appear the fractal dimension of which is two. On the contrary, ribbon-like structures are observed when $\eta > 4$ For higher values the fractal dimension drops even beyond one. Batty (1991) interprets η in the context of urban growth as a parameter describing different types of planning policies.

As saw in Section 1, since his introduction the DBM has been used in many physical domains as well as in biology and urban geography.

Let us remind that in our model an agricultural cell can be converted either into an urbanized or a transportation cell, with a possibility of leap-frogging, but with network cells that remain connected. This allows a comparison with DBM, which is made here on the morphology of the road network.

Let us consider series of pattern where β is fixed (e.g., $\beta = 0.25$; see Figure 1 and Figure 3(B)). For low γ -values ($\gamma < 0.11$) we obtain ramified patterns with a fractal dimension 1.7 or so. When γ exceeds the critical value $\gamma = 0.11$, the road network becomes cross-linear. Even if the morphological changes are here more threshold-like, the resemblance with DBM is clear. Indeed both models tackle with the diffusive growth of a network in space. It is possible to find a formal link between the two models. Remind that the choice of the site for a new house depends on the indirect utility function (eq:fullindirectutility), the variables E_l and S_l depending on the density ρ_l. This reminds the electric potential, which value depends on the neighbourhood via the Laplace-operator. Moreover, morphological changes occur in our model when varying β and/or γ. This situation reminds the role of η in the DBM. In both cases the structural changes depend on an exponent that weights the local potential or density. In our model, a first transition occurs for $\beta = 0.25$ when γ reaches the value 0.11 (Figure 1). However, a second transition is observed: for higher γ-values ($\gamma = 0.34$) a ramified street network appears again. Moreover the features of the ramified networks vary since there exist networks consisting of rather parallel streets whereas in other cases, e.g. when $\gamma = 0$, the networks are highly ramified. Such differences with respect to DBM are not surprising. As pointed out, the relationship linking the density to E_l and S_l is non-linear and hence the relationship used to determine the potentials E_l and S_l is more complex than in DBM. Moreover the model includes other parameters who may be at the origin of the additional phase transitions.

5 Conclusion

In this paper, we have presented a model simulating the joined expansion of residential areas, road network, and green areas in a metropolitan area. Our purpose was to build a simulation model on sound microeconomic foundations. We use a standard urban economics model: a household maximises a utility function, which includes tastes for the residential surroundings (local public goods and green/open space amenity), under a budget constraint including a commuting to work cost and a residential rent. As usual in standard urban economics, equilibrium is reached on the land market. These economic agents arrive sequentially in a 2D cellular automata gird and freely choose their location, considering the cost of each site (commuting, rent) and the enjoyed surroundings. As the city grows, new roads are built by a local authority to provide the migrants with an access to a pre-existing Central Business District (CBD), by a connected road network. Several simulations are made by using a software implemented in Java.

An analogy is observed with results obtained with physical DLA and DBM models. In particular, phase transitions in the urban morphology occur when a control parameter reaches critical values. Such analogy is also obtained by previous works; yet, it occurs here with different theoretical foundations. On the one hand, population density in our model and the electric potential in DBM play similar roles. On the other hand, the spatial model is different: diffusion in DBM (the potential of all the cells of the grill play a part in determining the potential of a cell), and a window vicinity here (only the neighbouring cells play a role in determining the density). Therefore, the resemblance is more surprising. Needles to say, a substantial amount of effort has to be made to decipher the analogy, and to pass from an abstract model to actual urban mophologies (calibration with realistic parameter values, etc.).

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