Hypernetworks of Complex Systems

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Abstract. Hypernetworks generalise the concept of a relation between two things to relations between many things. The notion of *relational simplex* generalises the concept of network edge to relations between many elements. Relational simplices have multi-dimensional connectivity related to hypergraphs and the Galois lattice of maximally connected sets of elements. This structure acts as a kind of *backcloth* for the dynamic system *traffic* represented by numerical mappings, where the topology of the backcloth constrains the dynamics of the traffic. Simplices provide a way of defining multilevel structure. This relates to system time measured by the formation of simplices as system events. Multilevel hypernetworks are classes of sets of relational simplices that represent the system backcloth and the traffic of systems activity it supports. Hypernetworks provide a significant generalisation of network theory, enabling the integration of relational structure, logic, and topological and analytic dynamics. They provide structures that are likely to be necessary if not sufficient for a science of complex multilevel socio-technical systems.

Keywords: Complex Systems, Hypernetworks, Networks, Simplex, Simplicial Complex, Backcloth, Traffic, Multilevel Systems, Dynamics.

1 Introduction

Hypernetwork generalise the concept of a relation between two things to relations between many things. The higher dimensional analogues of the network edge are the triangle, the tetrahedron, the pentahedron, and so on (Figure 1). Thus *n*-ary relations can be represented by *polyhedra* in multidimensional space. Polyhedra provide a multidimensional generalisation of one-dimensional network edges.

An *n*-ary relation R between n elements, $x_1, x_2, .., x_n$ is defined by a proposition P_R where it is assumed that $P_R(x_1, x_2, .., x_n)$ is well formed and there is a practical procedure for deciding whether or not $P_R(x_1, x_2, .., x_n)$ is true.

Binary relations yield the usual network edge, written (a, b), where a is related to b under the relation R if and only if there is a proposition P_R with $P_R(a, b)$ = True. The graphical representation of networks uses small solid circles called *vertices* to represent the elements and *lines* between the vertices to represent relationship. These lines are called *edges* or *links*. Generally $P_R(a, b) \neq P_R(b, a)$ and $(a, b) \neq (b, a)$, and the edges of networks are said to be *oriented*. (a, b) is oriented from a to b. Oriented edges are often represented by *arrows*.

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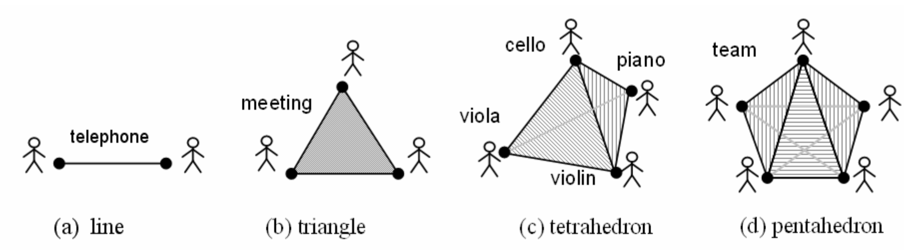


Fig. 1. Polyhedra generalise the concept of binary related to pairs to n-ary relations

Links and arrows are very powerful for representing things in complex systems. Links show that *a* and *b* are related in some way, and arrows can represent ideas such as *flow*, *transformation*, and *entailment* between the vertices *a* and *b*.

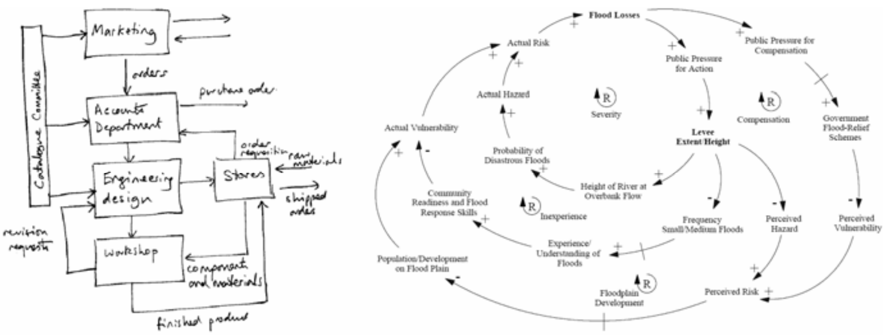
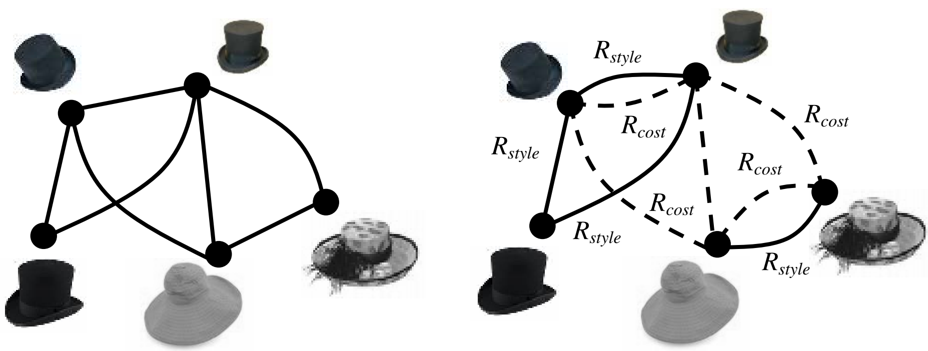


Fig. 2. Network with oriented edges can represent flow and transformations



(a) The price-style hat network is ambiguous (b) real networks have heterogeneous links

Fig. 3. In complex systems the relational structure must be explicit

The notation of conventional network theory leaves R implicit in the representation of edges as (a, b). In complex systems there are generally many relations between many sets, and it is common for there to be many relations between elements. For example, two hats may be related by being the same style and costing the same. Thus $R_{style}(a, b)$ = True is not the same as $R_{cost}(a, b)$ = True. To draw the link (a, b) leaves ambiguity as to which relation holds, and the best that can be reconstructed from the notation is that $R_{style}(a, b)$ = True or $R_{cost}(a, b)$ = True.

To overcome this problem we use the *explicit relation* notation $\langle a, b; R\rangle$ to show that $R(a, b)$ = True. Then $\langle a, b; R_{style}\rangle \neq \langle a, b; R_{cost}\rangle$ is clear from the notation.

Let $X_0, X_1, \ldots, X_n$ be sets. In the usual way let the *Cartesian product* of these sets be defined as $\prod_i X_i = X_0 \times X_1 \times \ldots \times X_n = \{ \langle x_0, x_1, .., x_n\rangle \mid$ for all x_i belongs to X_i for all $i = 0, \ldots, n$ $\}$. The ordered set of elements, $\langle x_0, x_1, .., x_n\rangle$ is called an *abstract n-simplex*. Such a simplex can be represented by a polyhedron on n-dimensional space. Simplices are the natural generalisation of network edges.

A *relational simplex*, $\langle x_0, x_1, .., x_n; R\rangle$ is said to exist if the is proposition P_R such that $P_R \langle x_0, x_1, .., x_n\rangle$ is well formed and there is an operational procedure to decide whether or not $P_R \langle x_0, x_1, .., x_n\rangle$ = True. This can be extended by the definition of *temporal relational simplex* $\langle x_0, x_1, .., x_n; R; t\rangle$ where $P_R \langle x_0, x_1, .., x_n\rangle$ is observed to be true at time t. This generalises: let $\langle x_0, x_1, .., x_n; R; T\rangle$ mean that $P_R \langle x_0, x_1, .., x_n\rangle$ is observed to be true for all times t in T where T is an *interval* of time, or a set of intervals of time.

When relational propositions are defined on the same simplex, it is natural to define the *wedge operation* on two simplices, *e.g.* $\langle a, b; R_{style}\rangle \wedge \langle a, b; R_{cost}\rangle = \langle a, b; R_{style} \wedge R_{cost}\rangle$, where $R_{style} \wedge R_{cost}(a, b)$ = True if and only if $R_{style}(a, b)$ = True and $R_{cost}(a, b)$. The *vee operation* is defined for disjunction in a similar way, $R_{style} \vee R_{cost}(a, b)$ = True if and only if $R_{style}(a, b)$ = True or $R_{cost}(a, b)$.

In general we use the notation $\underline{X}$ to represent sequences of vertices such as $x_0, x_1, .., x_n$. Then the notation $\langle \underline{X}; R; T \rangle$ provides a way of combining relational structure, logic, and time, all of which are necessary if not sufficient for a science of complex systems.

2 Simplicial Complexes, Connectivity and Q-Analysis

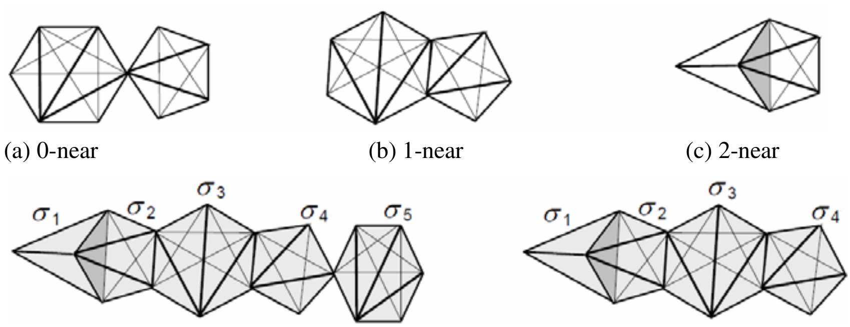
The concept of hypernetwork introduced in this paper has its origins in the work of R. H. Atkin in the nineteen seventies (Atkin 1974(a), 1977, 1981). Atkin had observed that the topological space-time structure of physics constrains the dynamics, and demonstrated that many phenomena can be summarised by the *Law of the trivial cocycle* (Atkin, 1972). As early as 1968 Atkin and his coworkers suggested the simplex as a model for relationships:

"To examine the idea of connectivity in more detail consider, for example, a collection of people and the sociological roles which they are said to be playing. Let the role-set be denoted by Y and let it contain a finite number of roles Y1 , Y2, . . ; similarly let there be a finite number of persons X1, X2, . . . in the collection of people X. An individual person X1 plays, say, roles Y1 , Y2 ,Y3 and a second person X2 plays the roles Y2 Y3,Y4,Y5.

We now define an abstract p-simplex to be a subset of Y containing (p + 1) roles provided that there is at least one individual who plays all these roles. Thus the 2-simplex (Y1 Y2 Y3) exists since X1 plays the three roles represented therein, so also do the 3-simplex (Y2 Y3 Y4 Y5), the 0-simplex (Y5), and many others. The two simplices (Y1 Y2 Y3) and (Y2 Y3 Y4 Y5) are clearly joined by the 1-simplex (Y2 Y3) – which is referred to as a face of both the 2- and 3-simplices. The collection of all such simplices actually forms a complex K(Y) which has the property that a person is represented by one of its simplices together with all the faces of that simplex. We may note that two people who play the same roles are indistinguishable in this model.

If we use the language which has historical connections with geometry we would refer to X1 as an abstract closed triangle, whilst X2 would be an abstract closed tetrahedron: in general, with respect to this particular role-set Y, we would observe a person X as an abstract closed polyhedron. All such polyhedra are connected (if at all) by faces (which are also polyhedra) which are common – the whole structure, or complex, therefore exhibits a connectivity which possesses a natural classification in terms of the dimensionality of the various polyhedra and their common faces. This connectivity seems to be a natural expression of the possibility of communication among the persons in the structure.

Thus our persons X1 and X2 can communicate with each other because they have a connection via their common simplices (Y2 Y3), (Y2) and (Y3). This connection exhibits the fact that X1 "sees" X2 via the common faces of their separate polyhedra. On the other hand X2 has many faces (15 in all, if we include the whole tetrahedron) any one of which might serve as a connecting face between himself and someone else." (Atkin *et al*, 1968).



(a) 0-near (b) 1-near (c) 2-near

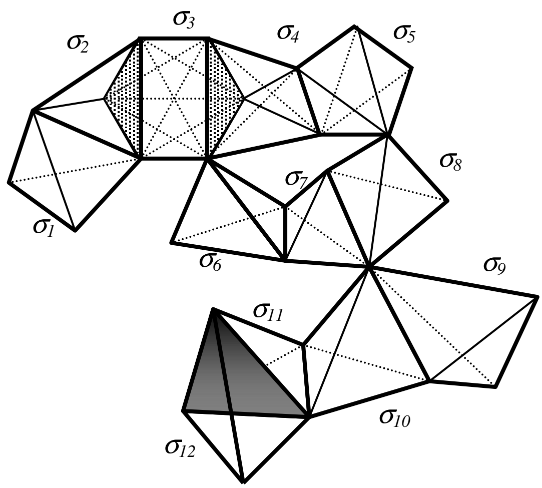
(d) σ_1 is 0-connected to σ_5 (e) σ_1 is 1-connected to σ_4

Fig. 4. q-nearness and q-connectivity

Let $\sigma_p = \langle x_0, x_1, .., x_p \rangle$ be an abstract p-simplex with vertices $x_0, x_1, .., x_p$. The simplex $\sigma_q = \langle x'_0, x'_1, .., x'_q \rangle$ is a *q-dimensional face*, or *q*-face, of σ_p if and only if every vertex of σ_q is also a vertex of σ_p, *i.e.* σ_q is a face of σ_p if and only if $\{x_0, x_1, .., x_q\} \subseteq \{x_0, x_1, .., x_p\}$. Let $\sigma_p = \langle x_0, x_1, .., x_p \rangle$ and $\sigma_{p'} = \langle x'_0, x'_1, .., x'_{p'} \rangle$ be two abstract simplices. Their *shared face* is defined as $\sigma_p \cap \sigma_{p'} = \sigma_{p''} = \langle x''_0, x''_1, .., x''_{p''} \rangle$ where $\{ x''_0, x''_1, .., x''_{p''}\} = \{ x_0, x_1, .., x_p \} \cap \{ x'_0, x'_1, .., x'_{p'}\}$.

In algebraic topology a set of simplices with all its faces is called a *simplicial complex*. Atkin defined two simplices to be *q-near* if they shared a *q*-dimensional face, and he defined two simplices to be *q-connected* if there was a chain of pairwise *q*-near simplices between them.

Technically, *q*-connectivity is the transitive closure of the *q*-nearness relations, and is an equivalence relation on a set of simplices with dimension *q* or greater. As such it partitions those simplices into equivalence classes of *q-connected components*. Atkin defined a listing of those components and related statistics to be a *Q-analysis*.



$q = 5$
$\{\sigma_3\}$

$q=4$
$\{\sigma_3\}\ \{\sigma_4\}\ \{\sigma_5\}$

$q = 3$
$\{\sigma_3\}\ \{\sigma_4\}\{\sigma_5\}\ \{\sigma_1\}\ \{\sigma_2\}\ \{\sigma_6\}$
$\{\sigma_7\}\ \{\sigma_8\}\ \{\sigma_9\}\ \{\sigma_{10}\}\ \{\sigma_{11}\}\ \{\sigma_{12}\}$

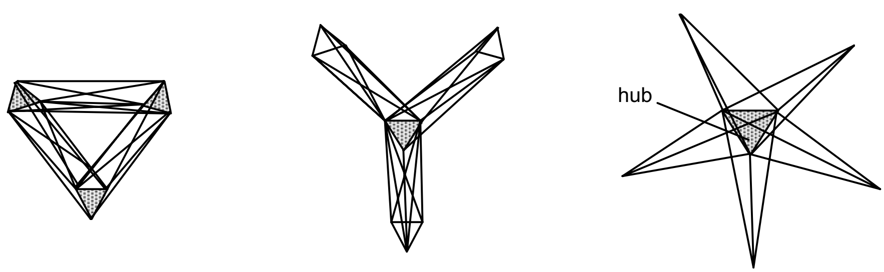
$q = 2$
$\{\sigma_2, \sigma_3, \sigma_4\}\ \{\sigma_5\}\ \{\sigma_1\}\ \{\sigma_2\}\ \{\sigma_6\}$
$\{\sigma_7\}\ \{\sigma_8\}\ \{\sigma_9\}\ \{\sigma_{10}\}\ \{\sigma_{11}, \sigma_{12}\}$

$q = 1$
$\{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5\}\ \{\sigma_6, \sigma_7, \sigma_8\}$
$\{\sigma_9, \sigma_{10}, \sigma_{11}, \sigma_{12}\}$

$q = 0$
$\{\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6, \sigma_7, \sigma_8, \sigma_9,$
$\sigma_{10}, \sigma_{11}, \sigma_{12}\}$

Fig. 5. A Q-analysis

Again using ideas from algebraic topology, Atkin defined a discrete analogue of homotopy that he called pseudo-homotopy, or shomotpty. Intuitively, two closed loops on a surface are homotopic if they can be continuously defined into each other. Loops cannot be continuously deformed across holes, *e.g.* the torus has different homotopy to the sphere, and the homotopy properties of a simplicial complex relate to its topological structure.



(a) all simplices are 2-near (b) all simplices are 2-near (c) a star-hub configuration

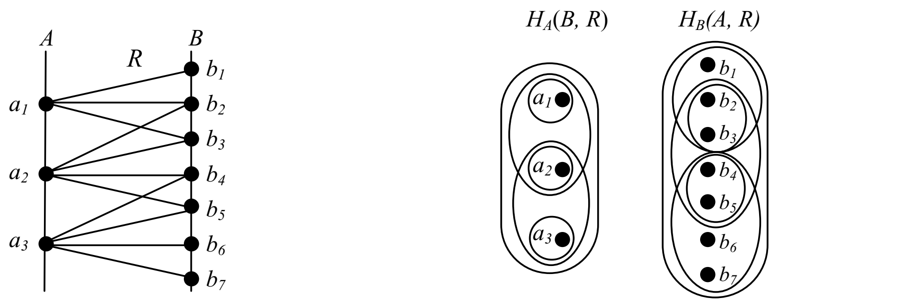
Fig. 6. Star-hub configurations

This work highlighted the need to discriminate between the configurations shown in Figures 6 (a) and 6(b), where all the simplices are q-near to each other, but one configuration has a hole while the other does not. This leads to the definition of the star-hub configuration shown in Figure 6(c) (Johnson (1983).

3 Hypergraphs, Galois Connections, Maximal Rectangles, Star-Hubs

Most binary relations hold between different sets, A and B, giving rise to *bipartite networks* such as that shown in Figure 7(a) with the elements of A arranged in a line, the elements of B arranged in a line, and lines drawn between a and b when a is R-related to B.

A *hypergraph* is a set A with a class of its subsets. Let $\sigma_R(a) = \{\ b \mid a$ is R-related to $b\}$. For any $A' \subseteq A$ let $\sigma_R(A') = \{\ b \mid a$ is R-related to b for all a belong to $A'\}$. Then let $H_A(B, R) = \{\sigma_R\ (A') \mid$ for all $A' \subseteq A\}$ and $H_B(A, R) = \{\sigma_R\ (B') \mid$ for all $B' \subseteq B\}$. These are hypergraphs, as illustrated in Figure 7(b).

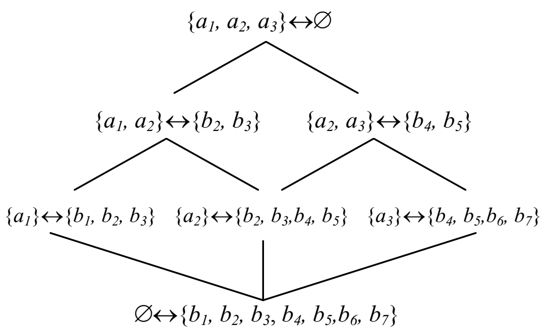


(a) the bipartite network of R

(b) the hypergraphs of R

R	b_1	b_2	b_3	b_4	b_5	b_6	b_7
a_1	1	1	1	0	0	0	0
a_2	0	1	1	1	1	0	0
a_3	0	0	0	1	1	1	1

(c) maximal rectangles



(d) the Galois lattice of R

Fig. 7. Bipartite network, hypergraphs and Galois lattice for a relation R between sets A and B

It can be shown that Then $H_A(B, R)$ and $H_B(A, R)$ are in one-to-one correspondence. Intuitively the subsets of A and B are paired as $A' \leftrightarrow B'$ so that every member of A' is R-related to every member of B'. A' and B' are *maximal* in the sense that no element outside A' is related to all the elements of B', and no element outside B' is related to all the elements of A'. If the relation R is represented by an incidence matrix with entry $m_{ij} = 1$ if $a_i \, R \, b_j$ and equals zero otherwise, then the rows and columns can be arranged to show the $A' \leftrightarrow B'$ pairs as blocks of ones in so-called *maximal rectangles*. For example, Figure 7(c) shows the two maximal rectangles corresponding to the pairs$\{a_1, a_2\} \leftrightarrow \{b_2, b_3\}$ and $\{a_2, a_3\} \leftrightarrow \{b_4, b_5\}$. This one-to-one correspondence is called a *Galois connection* and the pairs sets can be arranged as a *Galois lattice* (Barbut and Monjardet, 1970) as shown in Figure 7(d).

4 Relational Simplices and Multilevel Hypernetworks

The ideas sketched in the previous section are essentially set-theoretic. Hypernetworks enrich this set-theoretic approach by making a distinction between sets and structured sets, and this enables a powerful approach to representing the dynamics of complex multilevel systems.

The main idea is that imposing an n-ary relation on a set of elements creates an object at a higher level in the representation. This is illustrated below by the three blocks a, b, and c being assembled by the relation R into a structure, R: $\{a, b, c\} \rightarrow \langle a, b, c; R \rangle$, that is given the *name* arch. If the elements of the structure exist at, say, *Level N* then the structured object can be said to exist at a higher level, say *Level N+1*. In this case the higher level structure has an *emergent property* not possessed by its elements, namely there is a gap between the assembled blocks.

As another example, consider assembling sets of road segments to form paths between origins and destinations, as illustrated in Figure 9. For the origin-destination pair A and A' the set of roads r_1, r_2 and r_3 can be assembled into a *path*, $\langle r_1, r_2, r_3; R_{AA'} \rangle$,

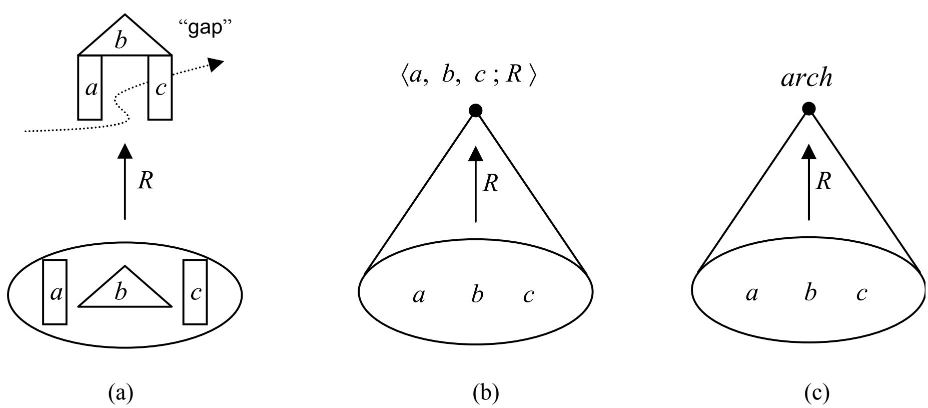
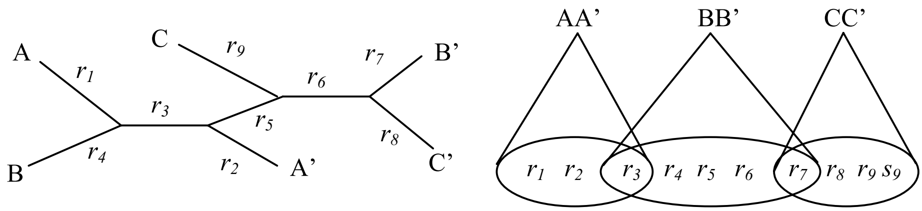


Fig. 8. n-ary relations map objects to higher levels of representation



(a) AA', BB' and CC' define paths in the networks (b) the paths as structured sets of roads

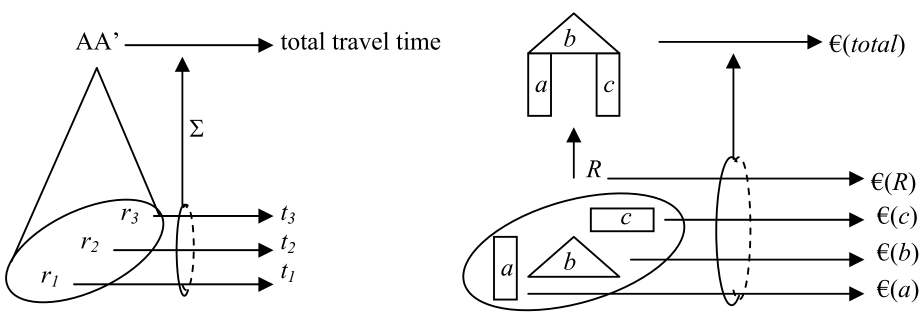
Fig. 9. Links assembled to routes in a road network

enabling vehicles to travel from *A* to *A'*. The fact that this path is a structure and not a set can be seen from the necessity for the assembly relation to order the roads as r_1 followed by r_3 (not r_2) followed by r_2. The simplices $\langle r_1, r_2, r_3; R_{AA'}\rangle$ and $\langle r_3, r_4, r_5, r_6, r_7; R_{BB'}\rangle$ are connected through the vertex $\langle r_3\rangle$ and this is where their traffic interacts, with AA' traffic delaying BB' traffic and *vice-versa*.

5 Backcloth and Traffic

Networks allow a distinction to be made between relatively static infrastructure such as a road or computer network and relatively dynamics flows such as vehicles or information on that infrastructure. Generally the infrastructure is *relational* while the flows are *numerical*. Atkin suggested the metaphor of a structural *backcloth* supporting a traffic of flows measured by numbers. For example, the flow of vehicles through a road network is traffic on the relatively fixed infrastructure of roads. The term can be generalised to mappings related to the flows, such as the travel time on a road network.

Traffic can exist at many levels across a system, and aggregates over the relational structure. For example, the travel time on the path chosen between the origin and destination, $\langle r_1, r_2, r_3; \mathrm{R}_{\mathrm{AA'}}\rangle$, is the sum of the travel times on the individual roads as vertices, as shown in Figure 10(a). Sometimes the relations themselves carry numerical traffic, *e.g.*.in Figure 10(b) the cost of the arch is the sum of the cost of its components



(a) travel time traffic (b) cost traffic for assembling the arch

Fig. 10. Mappings as traffic on multidimensional structure

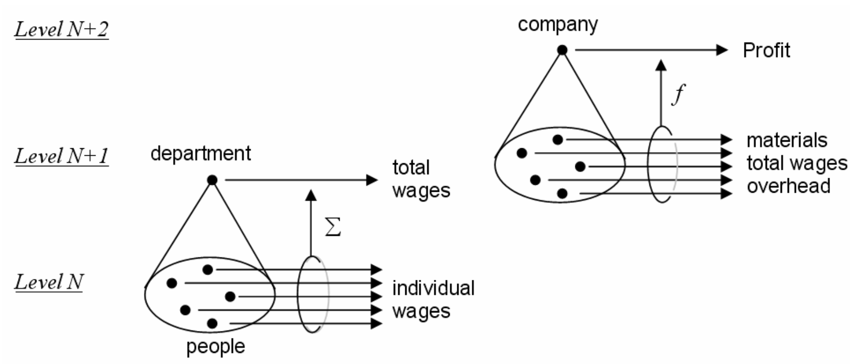


Fig. 11. Aggregating traffic over the backcloth in multilevel systems

plus the assembly cost. It could include other costs such as tax and profit, with a non-linear relationship between lower and higher level traffic.

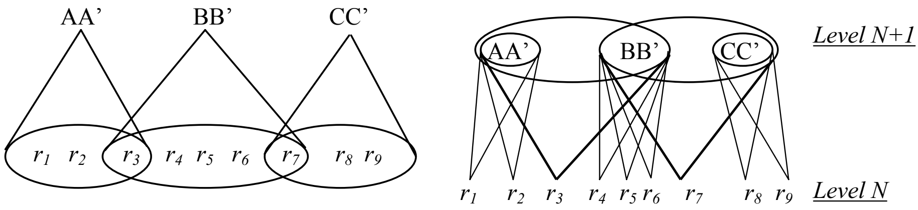
Figure 11 illustrates the bottom-up aggregation of traffic in a multilevel system, with micro-level traffic aggregating into meso-level traffic, and meso-level traffic aggregating into macro-level traffic. Generally there is both bottom-up and top-down interaction between traffic at all levels. For example, an overall price could be set for a project at a high level of representation, and this price would have a top-down influence on the lower level traffic.

A *multilevel hypernetwork* is any class of sets of relational simplices with sets of mapping on the simplices and their faces. In other words multilevel hypernetworks provide a means for representing the backcloth and the traffic of multilevel systems.

6 Hierarchical Cones in Heterarchical Systems

Given an *n*-ary relation such as $R\colon\{x_0, x_1, x_2\} \to \langle x_0, x_1, x_2 \,;\, R \rangle$, the imposition of the relation creates a new object $\langle x_0, x_1, x_2 \,;\, R \rangle$. When modelling complex systems it is common to give this new structure a *name*, say *y*. For example, the blocks were assembled into an *arch*. We extend the notation for representing simplices to say that $\langle x_0, x_1, x_2 \,;\, y;\, R \rangle$ is a *hierarchical cone*. The set of components $\{x_0, x_1, x_2\}$ is called the *base* of the cone, and the name *y* is called its *apex*. The we say that the components and the name exist at different levels, $Level(x_i) < Level(y)$ for all x_i. This gives an absolute criterion for multilevel discrimination in complex systems. The whole assembly cannot be a component of a component. For example, the carburettor may be part of the car, but the car is not part of the carburettor. Similarly a brick may be part of the house but the house is not part of the brick.

Suppose a set of components *X* is assembled into many named things collected together in the set *Y*. Then there is a relation between the higher level elements in *Y* and the lower level elements in *X*. For example, Figure 12 shows the top down relationship between the roads and the paths that pass through them. In this case we have the 'interesting' cones $\langle r_3$; AA', BB'; $R_{down}\rangle$ and $\langle r_6$; BB', CC'; $R_{down}\rangle$, where the meaning of R_{down} is discussed below.



(a) bottom-up cones (c) top-down cones

Fig. 12. Bottom-up and top-down hierarchical cones

The relation $R_{AA'}$ assembles roads into a path between A and A', and the relation $R_{BB'}$ assembled roads into a path between B and B'. What does it mean to write

$$\langle r_1, r_2, r_3; AA'; R_{AA'}\rangle \cap \langle r_3, r_4, r_5, r_6, r_7; BB'; R_{BB'}\rangle = \langle r_3; AA', BB', R_{down}\rangle$$

or more generally

$$\langle X; y; R\rangle \cap \langle X'; y'; R'\rangle \cap \langle X''; y''; R''\rangle \overset{\text{def}}{=} \langle X \cap X' \cap X''; y, y', y''; R \oplus R' \oplus R''\rangle$$

as the combination of cones? The expression $R \oplus R' \oplus R''$ concerns the combination of bottom-up n-ary relations to form new top down relations whose meaning depends on context. It can be noted that $X \cap X' \cap X'' \leftrightarrow \{y, y', y''\}$ is a Galois pair in which all the Xs are related to all the ys. The answer to the questions of what it means to write $\langle r_3; AA', BB', R_{down}\rangle$ is that the paths between AA' and BB' share the vertex write $\langle r_3\rangle$.

7 Q-Transmission

In almost all systems the topological structure of the backcloth constrains the traffic dynamics. This is obviously the case in electrical networks where components are connected together in ways decided by the designer to achieve specified electrical flows. Change the topology, as with a short circuit, and dramatically different behaviour can emerge.

As a simple example consider the road network in Figure 9. This can be represented as three connected simplices as shown in Figure 13. Suppose that there is a large increase in travel demand between A and A' resulting in higher traffic flows along the path $\langle r_1, r_2, r_3; R_{AA'}\rangle$. Then the traffic on $\langle r_3\rangle$ will be heavier than usual, acting a barrier to the traffic on BB'. After $\langle r_3\rangle$ the BB' traffic will be less. This lighter traffic on BB' means there will be lighter traffic than usual on $\langle r_7\rangle$ resulting in freer flow traffic on $\langle r_7, r_8, r_9; R_{CC'}\rangle$ and reduced CC' travel times. In this case the dynamic behaviour on $\langle r_1, r_2, r_3; R_{AA'}\rangle$ is *transmitted* to $\langle r_7, r_8, r_9; R_{CC'}\rangle$, even though the two path simplices share no road links. In other words, for the dynamics of one part of the system to affect the dynamics of another part of the system it is sufficient that they are connected by a chain of connected.

In this case just sharing a single vertex was sufficient for transmission to occur. In general the more highly connected the simplices, the greater is the magnitude of transmission. It can be hypothesised that some processes need two simplices to be

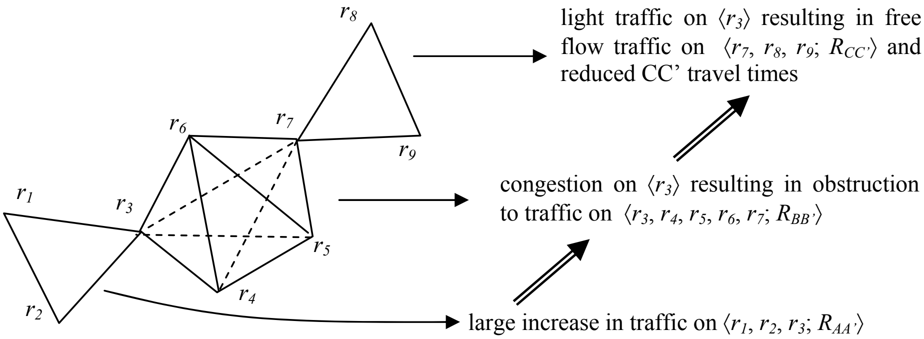


Fig. 13. Transmission of dynamics between connected simplices

q-near for one to affect the other. In this case the *q*-connectivity of the backcloth constrains what are called the *q-transmission* dynamics of the system. In many cases simply being connected is sufficient for transmission to occur.

8 Time and Structural Events

Apart from giving a way to represent multilevel structure, simplices give a way of measuring time as *system events*. Consider the arch in Figure 8 where there is a transition from a state in which the arch is not built to a state where the arch is built, R: $\{a, b, c\} \rightarrow \langle a, b, c; R \rangle$. The moment that the simplex $\langle a, b, c; R \rangle$ comes into being marks what Atkin (1981) called an *event*. Events in physical space-time are usually measured by physical systems such as pendulums or oscillating crystals. These may or may not be synchronised with events in system time. For example, it may be planned to open the bridge to traffic on 1st June, but if it is not complete it will not be opened. The defining event for opening the bridge is when it is finished, *.i.e.* after the many polyhedral events that define the bridge existing in a safe form.

Matching system time to clock time lies at the heart of much design and management. Human beings consume resource in clock time – we eat periodically in clock time, are paid in clock time, pay rent in clock time, and so on. This has to be resolved against system time when human beings are involved creating system events. In large complex systems there are events at every level on different time scales.

Much mathematical modelling has focussed on formulae relating numerical properties of systems. More recently it has become widely acknowledged that the underlying network topology plays a large role in the dynamics of systems. But it is necessary to go beyond this to consider the evolution of the networks and how the relations change. Put simply, when does a link appear and disappear in a network? What causes links to form or to break?

This relates to the possibility of making predictions in complex systems science. Predictions can be classified as

<u>Simple Type-I predictions: changes in mappings</u>
Type-I-1, Single level
Type-I-2, Multiple level

<u>Simple Type-II predictions: changes in relational backcloth</u>
Type II-1, Single Level
Type II-2, Multiple Level

The combination of types I-2 and II-2 is the norm for complex socio-technical systems, and this presents a big challenge in that hypernetworks may help to meet.

9 Conclusions

This paper has briefly introduced the notion of *relational simplex* which generalises the concept of network edge to relations between many elements. Relational simplices have higher dimensional connectivity related to hypergraphs and the Galois lattice of maximally connected sets of elements. This structure acts as a kind of *backcloth* for the dynamic system *traffic* represented by numerical mappings, where the topology of the backcloth constrains the dynamics of the traffic. Simplices provide a way of defining multilevel structure, where this relates to system time measured by the formation of simplices as system events. Multilevel hypernetworks are classes of sets of relational simplices that represent the system backcloth and support the traffic of systems activity. Hypernetworks provide a significant generalisation of network theory and provide structures that are likely to be necessary if not sufficient for a science of complex multilevel socio-technical systems.

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