

# Multiple Phase Transitions in the Culture Dissemination

Bing Wang<sup>1,2</sup>, Yuexing Han<sup>3</sup>, Luonan Chen<sup>4</sup>, and Kazuyuki Aihara<sup>1,2</sup>

<sup>1</sup> ERATO Aihara Complexity Modelling Project, JST, Institute of Industrial Science, The University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo, 153-8505, Japan

<sup>2</sup> Institute of Industrial Science, The University of Tokyo, Tokyo, Japan

<sup>3</sup> Graduate School of Information System, The University of Electro-Communications, 1-5-1, Chofugaoka, Chofu-Shi, Tokyo, Japan

<sup>4</sup> Department of Electrical Engineering and Electronics, Osaka Sangyo University, Daito, Osaka, 574-8530, Japan

**Abstract.** We study the coevolution process in the Axelrod's model with the consideration of agents' abilities to access to the information. With a parameter to control the ability of communication, we observe two kinds of phase transitions both for cultural domains and network fragments, respectively. With the simulation results, we find the relationship between the critical value and the controlled parameter. The results indicate that the powerful ability to access to the information benefits the dissemination of culture in the system.

**Keywords:** Self-organized systems, Complex systems, Dynamics of social systems.

## 1 Introduction

During the last few years, a great deal of efforts have been devoted to the study of social phenomena and social behaviors, such as opinion formation, rumors, disease propagation. In [1], Axelrod proposed one model to describe the dissemination of culture among interacting agents in a society system. With this model it generates a global convergence to a single culture state.

Since the Axelrod model has been proposed, The behavior of this model has been much studied in static networks [3,4], such as the effect of the network structure [5], noise [6], mass media [7,8] and cultural threshold [9,10]. Recently, the study of the relationship between the networks' topologies and the dynamical behavior on them has led to a deep understanding, and showed that the topology of the network or the interaction between agents is not static in time. Therefore, it is emergent to integrate the new framework of the coevolution of network structures and state dynamics [11,12,13,14,15,16].

In this work we investigate the influence of communication ability on the coevolution of agents' interactions and state dynamics in the Axelrod's model. Agents can interact with their neighbors who share common features with them or construct new contacts with others. The candidate agents can be chosen within

the limited spatial space due to the constraints of transportation condition, technology factor or the limited information sources. With simulations we show that the system can display two kinds of phase transitions, that is, cultural domains and network fragments, respectively. The results indicate that the more powerful the ability to access to the information, the better the ordered phase can be observed.

The rest of the paper is organized as follows. In Sec. 2, we briefly introduce the Axelrod's model and the coevolution process by taking into account the space constraints. In Sec. 3, we show the results on the effects of the spatial constraint on the dissimilation of culture. In Sec. 4, we conclude our results and give a brief discussion.

## 2 The Model

Assume a system with  $N$  agents at the sites of a square lattice  $L \times L$  without periodical condition is given, the state  $S_i$  of node  $i$  is defined as a vector of  $F$  components  $\sigma_i = (\sigma_{i1}, \dots, \sigma_{iF})$  representing cultural features such as language, music, and sports. Each component  $\sigma_{if}$  represents the preference for each culture feature. Initially for each node  $i$ ,  $\sigma_{if}$  is randomly assigned from the set  $\{0, 1, \dots, q-1\}$  with equal probability  $1/q$ . The time-discrete dynamics evolves by iterating the following steps.

(1) Select a node  $i$  and one of its neighbors  $j$  randomly. Let  $c_{ij}$  represent the number of common features shared by  $i$  and  $j$ , denoted by  $c_{ij} = \sum_k \delta_{\sigma_{ik}, \sigma_{jk}}$ .

(2) Interaction. If  $c_{ij} = 0$  or  $c_{ij} = F$ , nothing happens; otherwise if  $0 < c_{ij} < F$ ,  $i$  and  $j$  interact with probability  $c_{ij}/F$ . The interaction means that one of  $i$ 's feature  $\sigma_{if}$ , if  $\sigma_{if} \neq \sigma_{jf}$ , then set  $\sigma_{if} = \sigma_{jf}$  [1].

Step (1) and (2) are the basic process of the Axelrod's model and by Step (2) agents become more similar. With the repeat of the above process a *frozen state* can often be reached, where the node's status does not change anymore. That is, neighboring sites have an overlap equal to  $F$  or 0. The relative size of the largest component of agents sharing the same culture features  $S/N$  is a measure of the cultural diversity.

In the following we consider the coevolution process that agents search new contacts with common features to interact with access to the information.

(3) Rewiring. If  $c_{ij} = 0$ , agent  $i$  will break the link with  $j$  and look for new contacts. Agents within the geographical distance  $\alpha \times L$  can be the candidate nodes, where  $\alpha \in [1/L, \sqrt{2}]$ , and no multiple connections are allowed. Candidates sharing common features with  $i$  will be given priorities to be chosen. If there are no nodes sharing common features with  $i$ ,  $i$  will connect with a candidate randomly.

The parameter  $\alpha$  can be described as agents' abilities of communication constrained by the spatial space and technology factors.

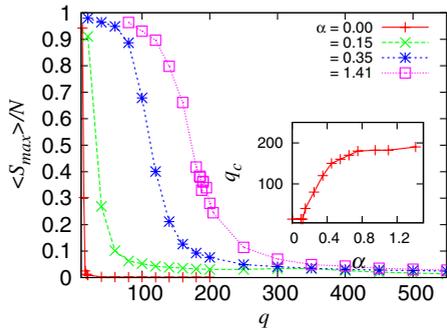
### 3 Simulation Results

We will measure the ordered parameter, the size of the giant component with the same cultural features  $S_{max}$  divided by  $N$  for the described model and the Axelrod’s model, respectively.

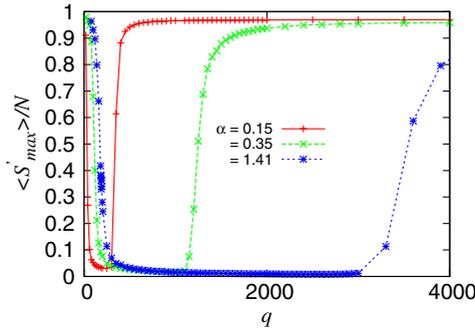
In Fig. 1, we first display  $S_{max}/N$  versus  $q$  for different values of  $\alpha$ . The curve for  $\alpha = 0$  corresponds to the results in the Axelrod’s model. It is shown that there exists a phase transition for each curve and the critical value  $q_c$  increases with  $\alpha$ , which means that the powerful ability to obtain the information favors the homocultural state. The critical value  $q_c$  can be evaluated by verifying the point at which  $S_{max}/N$  achieves the maximal variance. In the inset of Fig. 1, we show the critical value of  $q_c$  versus  $\alpha$  in a more careful way. It clearly shows that with the increase of  $\alpha$ , more agents can be ideal candidates to interact and it becomes possible to find compatible ones to interact, which leads to the homocultural state, and a higher  $q_c$  can be achieved.

In the rewiring process, agents will find new contacts and rewire with them, which will induce the broken of the network into several parts and make the size of the giant component for the network structure denoted by  $S'_{max}$  also reduce, see Fig. 2. However, if  $q$  is very large, since most of the agents do not share any common cultural features and they cannot interact with each other to become more similar. In this case, the rewiring process governs the dynamics and the network reconnects again, which leads to a higher value of  $S'_{max}$  again, see Fig. 2.

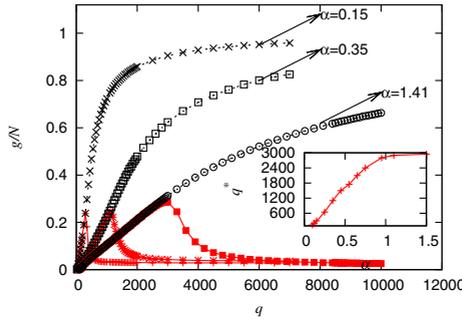
We can see that for each  $\alpha$ ,  $S'_{max}/N$  first decreases to the lowest value and then sharply increases to approach to the state where almost all the nodes are connected. In order to discover the exact value of  $q^*$ , we analyze the number of components for cultural domains and network fragments, denoted by  $g$  with black and red curves in Fig. 3. For  $q < q^*$ , the number of groups for cultural domains and network fragments coincide with each other and agents in the same network component possess the same cultural traits. However, when  $q > q^*$ , cultural



**Fig. 1.** (Color online) The ordered parameter  $S_{max}/N$  for cultural domains versus  $q$  for  $\alpha = 0.0, 0.15, 0.35, 1.41$  (from left to right).  $\alpha = 0$  corresponds to the case of the Axelrod’s model. The inset shows the critical value  $q_c$  versus  $\alpha$ .



**Fig. 2.** (Color online) The ordered parameter  $\langle S'_{max} \rangle / N$  for network structure versus  $q$  for  $\alpha = 0.15, 0.35$ , and  $1.41$



**Fig. 3.** (Color online) The ordered parameter  $g/N$  for network fragments (red) and cultural domains (black) versus  $q$  for  $\alpha = 0.15, 0.35$  and  $1.41$ . The inset shows the critical value  $q^*$ , at which the network fragments and cultural domains do not coincide, versus  $\alpha$ .

traits and network components do not coincide anymore and the number of groups for cultural domains increases while the number of groups for network fragments decreases. In other words, each component possesses several cultural states. The inset in Fig. 3 shows the exact value of  $q^*$  versus  $\alpha$ . We find that  $q^*$  grows with  $\alpha$  until most of compatible pairs have been found.

### 4 Conclusion and Discussion

In this work, we studied the coevolution of agents' interactions and the update of status in the Axelrod's model by tuning agents' abilities to access to the information. With the increase of the parameter, the model shows two kinds of phase transitions both for the cultural domains and the network components. The numerical results indicate that the critical value  $q_c$  for cultural domains increases with the controlled parameter  $\alpha$  soon and then does not change much

anymore. This phenomena can also be observed for  $q^*$  for the network components. It indicates that the powerful ability to access to the information benefits the dissemination of culture in the system.

The study of coevolution of network structure and nodes' states is still at the beginning. In this work, for simplicity, we assume that all agents' abilities to access to information is the same, which is not related with the network topology or nodes' positions. Other extensions to consider different abilities to access to the information, such as depending on agent's connectivity may bring more interesting results.

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