Performance Analysis of Public Transport Systems in Nanjing Based on Network Topology

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Abstract. The urban public transport network (UPTN) in Nanjing is characterized by a complex network with topological pedestals. The empirical data indicates that it is a small-world network. Under malicious attack to the high connectivity nodes of the network, the average path-length will increase 2.5 times, the reliability and traffic capacity of the UPTN will greatly decline, and the travel expenditure will distinctively increase. The topological significance of stations and routes are redefined to help assess the small-world property of UPTNs, so as to improve city transportation. It is also found that if the urban rail transit, such as metro, is introduced to the UPTN, then the topological diameter of the network is reduced, and its structure is optimized.

Keywords: complex network, network topology, public transport system, urban rail transit.

1 Introduction

One of the common challenges that urban cities are confronted with is traffic congestion. According to the 2005 annual report on the development of urban road traffic of Nanjing, the average waiting time of citizens in Nanjing City at a bus station is 5.80 min, which is the longest time since 1999. Most of roads in Nanjing are almost saturated with traffic flow. On the other hand, the passenger volume of 2005 in the public transportation is less than that of 2003, the year when China suffered from SARS outbreaks. Facing the increasingly serious traffic problem, one needs a new approach to analyze the characteristics of urban public traffic systems (UPTS) from new perspective, so as to deal with the traffic problem in practice; a complex network approach fits to this requirement [1,2,3]. The UPTS is a complex system consisting of thousands of vehicles and passengers, and its performance is directly related to the topological structure of the

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network [4,6,5,7,8,9,10,11]. However, contrasting with the similar findings of the small-world and scale-free properties in other complex networks [1,3], the recent studies of 14 major cities of the world [12] have shown the diversity in statistical and topological properties of UPTNs due to diverging historical evolutions and other external factors such as wartime destruction and political constraints. In this paper, we analyze the characteristics of the UPTN in Nanjing from the novel perspective of topology.

2 Statistic Parameters of Network Topology

In order to quantitatively describe the characteristics of the topological structure of networks, a series of characteristic parameters have been introduced into network analyses [3], such as node degree k, degree distribution $P(k)$, node strength, s, distribution of nodes' strength, $p(s)$, characteristic path length, L, clustering coefficient, C, and betweenness centrality, BC.

2.1 Node Degree and Degree Distribution

The degree of a node, k, means that this node is connected with k edges. In the case of UPTNs, it can be described as "a bus station has k neighbor stations". Here, two stations are defined as neighbor only if one station is the successor of the other in the series serviced by a route. One of the important statistical attributes is the distribution of node degree, for not all nodes have the same number of incident edges. The distribution of nodes' degree can be depicted by density distribution function, $p(k)$, (or cumulative distribution function, $P(k) = \int_k^\infty p(\xi)d\xi$), which represents the probability that a randomly selected node in a network has k neighbor nodes, and is also equivalent to the ratio of the number of the nodes with degree k in the network to the total number of nodes of the network. In a scale-free network, both $p(k)$ and $P(k)$ take power law function forms with respect to node degree. In a UPTN, the distribution of node degree has much impact on the accessibility of the public traffic system. Carefully analyzing the degree distribution in a UPTN and figuring out which function form it satisfies can contribute to understanding the topological characteristics of the UPTN. In the case of weighted networks, another meaningful parameter of topological characteristics is the strength of a node, which is defined as the sum of the weight of all its neighbor nodes, i.e. $s_i = \sum_{j \in V_i} \omega_{ij}$. Here, ω_{ij} denotes the weight of the link from node i to its neighbor node j. In a UPTN, the strength of a node can be described as the total frequency with which different bus routes serve the bus station.

2.2 Characteristic Path Length

Characteristic path length, L, is defined as the average value of all shortest path lengths over all pairs of nodes in a network; it is a characteristic parameter describing the distance between two arbitrary nodes in a whole sense. In a UPTN, it reflects the average number of bus stations one may traverse from one place to another place using the UPTS. It is straight forward that the smaller L the more convenience one travels by bus and the better accessibility the public traffic network has.

2.3 Clustering Coefficient

Assuming that node i has k_i neighbor nodes, there may exist at most $C_{k_i}^2$ edges among k_i nodes; in fact, there are only t edges among them, so the clustering coefficient C_i of node i , $C_i = t/C_{k_i}^2 = 2t/k_i(k_i - 1)$. The clustering coefficient of the whole network, C, is the average of the clustering coefficients over all the nodes in the network, namely, $C = \sum_{i=1}^{n} C_i/n$. The clustering coefficient of a network describes the local clustering characteristics of the network, i.e. measures the tendency that nodes in the network form cliques. In a UPTN, it reflects the local connectivity and intensive degree of public traffic routes. The larger the C, the higher the local connectivity, and the more intensive the urban public traffic routes. It ensures that there will be little impact on the accessibility between any other directly connected stations when a certain bus station is congested. Therefore it contributes to the robustness of public traffic networks.

2.4 Betweenness

Node betweenness reflects the centrality of a node in a network. It can be used to identify the hub nodes in a network. Much information and many other resource flows from a node must traverse these hub nodes to rapidest reach other nodes through shortest paths. The betweenness centrality of node i, denoted by BC_i, can be obtained by counting the sum of the fraction of shortest paths between all pairs of nodes passing through node i in the network. In a UPTN, the betweenness of a node reflects the capability that this bus station acts as transfer station. In this paper, aforementioned network topological statistic parameters will be adopted as major criteria to investigate the topological characteristics and its evolution of the UPTN.

3 Statistic and Topological Properties of the UPTN

3.1 The Construction of UPTNs

The bus station network is defined in a natural way with nodes representing bus stations, such as A and B, and if there is at less one route passing through A and B without any other stations between them, then the two stations are linked. If there are multiple links between A and B, then the number of these routes is assigned to the link (edge) between node A and node B as its weight. Obviously the modeled network is a directed weight network, which depicts the fundamental topological characteristics of UPTNs. Using its basic topological parameters such as average shortest path, shortest path distribution, degree distribution etc, one can study the topological property of the network. For the case study of Nanjing, the UPTN is modeled in this way. This network consists of 224 routes and 1542 stations [13](Fig.1).

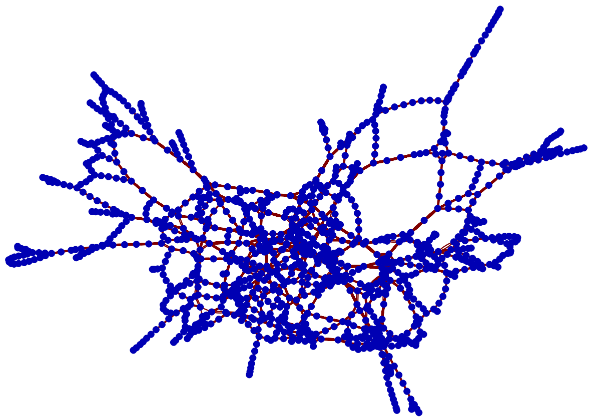

Fig. 1. The topologic structure of the UPTN in Nanjing

3.2 Small World Property Testament

To describe the small world property of the complex networks, Watts et al. [1] introduced two characteristic parameters: characteristic path length (L) and clustering coefficient (C). The small world property is mathematically characterized by the average shortest path length that depends at most logarithmically on the network size n and the clustering coefficient in such a way that

$$C \gg C_{random} \sim < k > /n \quad (1)$$

$$L \gg L_{random} \sim \ln n / \ln < k > \quad (2)$$

where, $< k >$ is the average degree over all nodes in the network, and n is the total number of nodes. The results of the average degree $< k >$, characteristic path length L, and clustering coefficient C of the UPTN in Nanjing are listed in Table 1(L_{random} and C_{random} in Table 1 are the characteristic path length and clustering coefficient of the corresponding random network with same size, respectively).

Table 1. Topological statistical parameters of the UPTN in Nanjing

n	$< k >$	L	L_{random}	C	C_{random}
1542	5.856	17.00	4.15	0.111	0.0038

From Table 1, $C/C_{random} \sim 29.25$ and $L/L_{random} \sim 4.10$ indicate the small-world property of the network. Therefore it can be concluded that the UPTN in Nanjing is a small-world network.

3.3 Scale-Free Property Testament

Many empirical studies show that most of the real networks display a power law shaped degree distribution, and the power law function curve decreases relatively slow, which results in existence of nodes with large degree. Networks whose degree distribution obeys a power-law form are called as scale-free networks. Barabási and Albert [2] attributed the self-organization of real systems into the scale-free structure to two major factors: growth and preferential linking. Scale-free networks often have the small-world property as well. However, according to

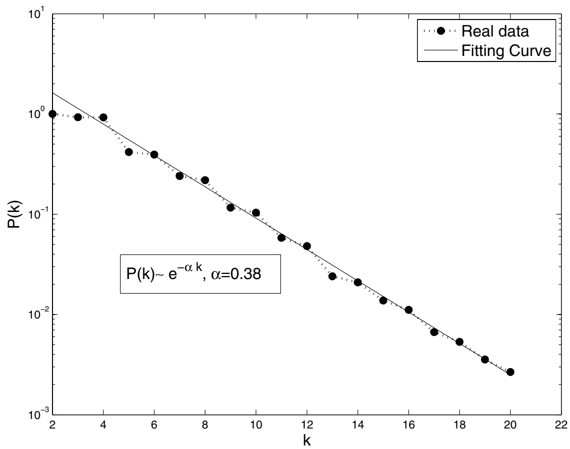

Fig. 2. The degree cumulative distribution of the UPTN in Nanjing

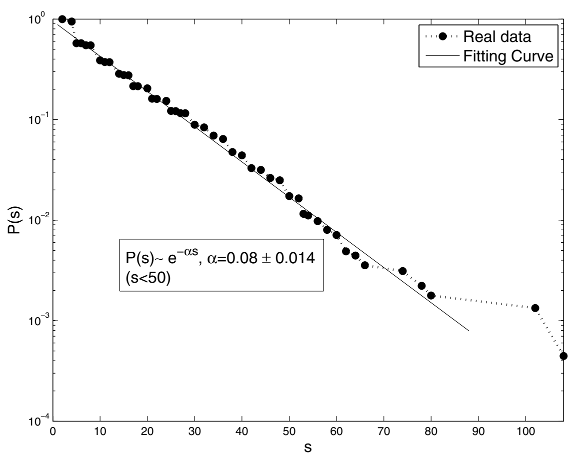

Fig. 3. Semi-logarithmic plot of the strength cumulative distribution of the UPTN in Nanjing

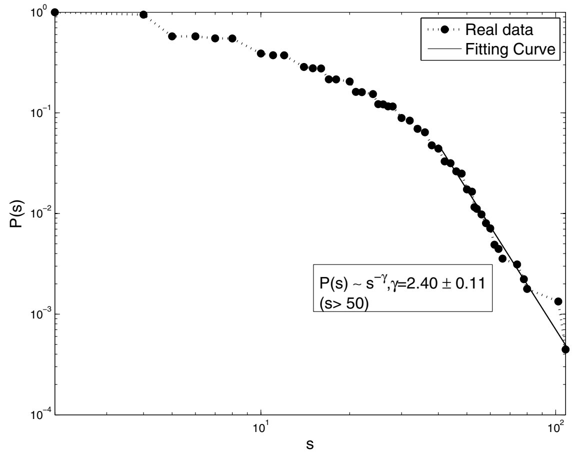

Fig. 4. Log-log scale plot of the strength cumulative distribution of the UPTN in Nanjing

our empirical data of the UPTN in Nanjing, not all the degree distribution and the strength distribution have a typical power law form. The degree distribution (Fig. 2) more likely takes an exponential form, while the strength distribution takes a certain form in between a power law form and an exponential one. The nodes whose strengths are less than 50 seem to take an exponential distribution (Fig. 3), while the nodes with a larger strength appear to take a power law distribution (Fig. 4); which is consistent with some research reports by other scholars [12,14], who have also found that the degree distributions of UPTNs in some cities such as Berlin, Hamburg, Moscow, Hong Kong, Beijing do not take a power law form. Obviously, the finite-size and spatial constraint of the UPTN are two major factors which hinder the formation of the scale-free property. On the other hand, one can see from Fig. 3. and Fig. 4. that the fat-tail distribution results in the coexistence of a few hub nodes and a larger number of poor connected nodes.

4 Effectiveness Analysis of the UPTN Based on Topological Statistics

4.1 Improvement of City Transportation by Enhancing the Small-World Property of UPTN

A few of shortcuts should be built between critical nodes, which may shorten the average path length and improve the reliability of the whole network. To do this, one shall first identify the critical nodes in the network. One possible way is to identify these critical nodes according to the role that nodes play in topological structure. It is known that node degree reflects the total number of a given station's connections, while *BC* represents the capability that the station acts as a transfer station. The two parameters are both the important characteristic parameters measuring the importance a node has in topological structure. To integrate the description of the two properties, the geometric mean of the two parameters denoted as SI, is applied to describe the topological importance of a node, where, $SI = \sqrt{K \cdot BC}$. According to this evaluation index, the sequencing analysis of the topological significance of 1542 stations in the UPTN in Nanjing is performed, with the first 10 critical nodes listed in Table 2.

Table 2. First 10 topologic significance stations in Nanjing UPTN

Station	Chalukou	Xinzhuang	Zhongyang men	Nanjing railway station	Huamugongsi
SI value	1.26×10^{-2}	1.21×10^{-2}	1.19×10^{-2}	0.91×10^{-2}	0.87×10^{-2}
Station	Hedingqiao	Yuhuatai	Gongjiaozong gongsi	Changlelu	Xinjiekou
SI value	0.79×10^{-2}	0.76×10^{-2}	0.75×10^{-2}	0.74×10^{-2}	0.69×10^{-2}

Aiming to improve the traffic capability between the main city and new towns of Nanjing, the Xincheng bus company offers a No.101 bus service, which starts from Dongshanzongzhan, passes through Jingfashichang, Dajiedongzhan, Xinyilu, Chengzhong, Zhushanlu, Fuqianlu, Gongxiaoshangxia, Wuyihuayuan, Hedingqiao, Shijiali, Chalukou, Yanhuihongcun, and ends at Zhonghuamennei. According to the company's estimation, bus route No.101 will run efficiently. In our analysis (Table 2), Chalukou, Hedingqiao, and Zhonghuamennei rank the first, sixth, fifteenth place in the significance index of 1542 stations, respectively, which provides the theoretic basis for running this route. In the light of station significance index, one can define the significance index of a route, LI, by the average of SI over all stations of the route. Assuming that a route has m stations and the significance index of node i is SI_i , then the significance index of the route is $LI = \sum_{i=1}^{m} SI_i/m$.

4.2 Robustness and Vulnerability Analysis of UPTNs

Small-world networks have a common feature that they are robust against random attacks, yet vulnerable to malicious attacks. The robustness (vulnerability) of a network can be measured to find out whether the network will still connected after some of nodes have been deleted. Owing to the inequality of the role that different nodes in a network play, a few of critical nodes play a key role in the running of the whole network; which makes the network to be highly vulnerable when these nodes are attacked deliberately. That is to say, if only those small amount of nodes (not more than 5%) with largest connectivity are halted or become congested, which may cause the entire network fail to work. So the small-world scale-free property brings about the advantage of effectiveness and fast communication of the network, but might result in the quick spread of congestion as well. For this reason, we should attach much attention to the construction of the key stations from the following aspects: protecting key nodes and attaching much importance to "long-range link". Protecting the transport capability of key stations is able to enhance the transfer capability of the whole network as well as to prevent traffic jam from spreading quickly. As learned from the study on small-world networks research, a few shortcuts will shorten the average distance L significantly, which improves the transit capability of the network. This has been demonstrated by the example of Nanjing Xincheng bus company's bus route No.101 operation. To assess the robustness of Nanjing UPTN, all the stations have been sequenced by their node degree and betweenness (BC), respectively. After the removal of the 1% stations with the largest degree and betweenness, and the 1% randomly selected stations separately, the average degree $< k >$, characteristic path length L, clustering coefficient C, and other topological characteristic parameters are recalculated with the results from both cases listed in Table 3.

Table 3. Changes in characteristic parameters of Nanjing UPTN after the removal of 1% stations

Type of removal	1% largest degree	1% largest BC	1% random selected	No removal
$< k >$	5.452	5.633	5.774	5.856
L	42.8959	40.3965	26.5748	17.0056
L_{random}	4.32	4.24	4.18	4.15
C	0.0973	0.1053	0.1078	0.111
C_{random}	0.00357	0.00278	0.00378	0.00380

It can be learned from the Table 3 that the small-world property of the network do not change in whatever way one delete the 1% stations. However, under the deliberate attacks the average path length increases 2.5 times relative to the original one, which suggests that the reliability and transit capacity of the network are heavily declined and the travel cost of citizens will increase remarkably. In comparison, change in the average path length due to the removal of 1% randomly selected vertexes is much less than that caused by malicious attacks. This is accordant with the robustness and vulnerability of small-world networks.

5 Synergetic Relation between Urban Rail Transit and UPTN

Urban rapid rail transit systems enjoy the advantage of large volume, high speed, less pollution and energy consumption. Vigorously development of urban rail transit has a great significance in mitigating the congestion of urban transportation and in improving urban atmosphere environment. Meanwhile, rail transit promotes the optimization of the spatial structure, and quickens the communication between the city center and sub-centers of a city as well.

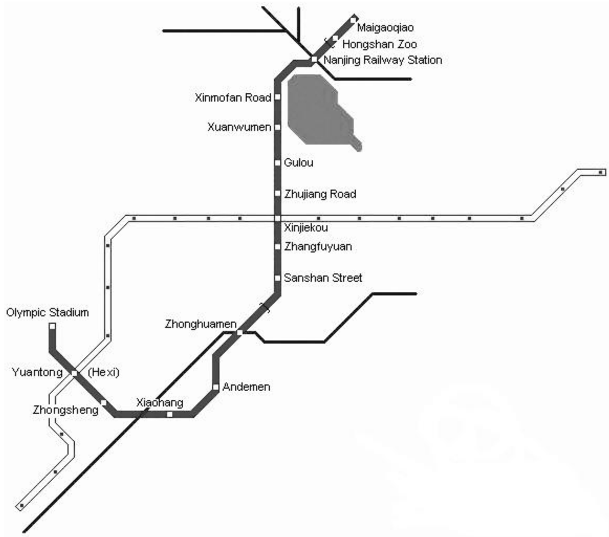

Fig. 5. Map of Nanjing Metro Line 1 and 2 (source:www.urbanrail.net)

Table 4. Changes in characteristic parameters of Nanjing UPTN before and after the joint of Metro Line 1 and 2 (R is the diameter of the network, i.e. the longest one of all shortest paths in the network)

Traffic mode	Normal bus	Bus and Metro Line 1	Bus and Metro Line1,2
$< k >$	5.856	5.869	5.885
L	17.00	16.59	16.59
R^*	56	53	53
L_{random}	4.15	4.14	4.14
C	0.111	0.111	0.111
C_{random}	0.0038	0.00278	0.00378

In the case of Nanjing rail transit system, Nanjing Metro Line 1 runs across main city from north to south, while Line 2 traverses main city from east to west, which effectively shortens the spatiotemporal distance between main city and three new towns (Xianlin, Hexi, Jiangning). This conclusion can be verified by the analysis of the diameter of Nanjing UPTN. Table 4 demonstrates the variation of the topological characteristics of the UPTN before and after the joint of the rail transits.

It can be seen from Table 4 that despite of the long length of the two Metro Lines the small-world property of the network is strengthened after their joint. Not only the average path length L but also the network diameter R reduce as well. The latter is more suggestive of that the spatiotemporal distances between the center and sub-centers of the city have been optimized effectively.

6 Conclusion

Topological structure of the UPTN determines the connectivity and accessibility of public transportation service in such a kind of complex networks. Therefore, it is meaningful to study the topological feature such as the small-world and scale-free properties of the networks. The case study of the UPTN in Nanjing shows that it does have such properties. Meanwhile, it is demonstrated theoretically and practically that the “long-range link” of a network system takes a very important role in enhancing the accessibility of the public transportation. It should be mentioned that the development of urban rail transit shortens the topological diameter of the whole network, which contributes to optimization of city’s spatial structure and communication between the center and sub-centers of the city. The complex network model can be used to analyze dynamic behaviors of urban public transport systems, and furthermore to forecast and control the systems, therefore it may possess engineering meaning and potential application value.

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