Queueing Transition of Directed Polymer in Random Media with a Defect

Jae Hwan Lee and Jin Min Kim

Department of Physics, Soongsil University, Seoul 156-743, Korea
jmkim@ssu.ac.kr

Abstract. We study a queueing transition of directed polymer in random media with an attractive defect at the center of the one dimensional substrate. The end to end distance Δx of the polymer follows $\Delta x \sim t^{1/z}$ with $z = 3/2$, for weak defect strength ϵ where t is the polymer length. If $\epsilon \geq \epsilon_c$ then the polymer is localized with finite Δx in long t limit. The transition is related to the queueing phenomena of the asymmetric simple exclusion process.

Keywords: directed polymer in random media, queueing transition.

1 Introduction

A traffic jam occuring in a bottleneck, or near a road under construction is a typical example of queueing phenomena. The asymmetric simple exclusion process (ASEP) [1,2,3,4] have been introduced to explain the relation between the queueing phenomena and nonequilibrium driven dynamic process. In the ASEP, most particles jump into a vacant neighboring site in one direction with hopping probability one, but a selected site of hopping rate r with $r < 1$ plays a role of the bottleneck. Many studies have suggested that the critical hopping rate is one, $r_c = 1$, i.e., the nonzero hopping rate always gives rise to queueing phenomena [5,6,7,8,9]. However, some recent studies on the ASEP with a slow bond insist that a queueing transition of a jamming state at $r_c < 1$. The overall flux passing through the defect site is rarely influenced if the defect strength is not strong enough [10,11,12,13,14].

The ASEP can be interpreted as surface roughening problem of crystal growth in the body-centered solid-on-solid interface model [15,16,17] where the height difference between nearest neighbors is restricted by ± 1. An increase (decrease) of surface height is equivalent to the presence (absence) of a particle in the ASEP. Both models belongs to the Kardar-Parisi-Zhang (KPZ) [18] universality class of two dimensional problems.

The directed polymer problem in random media (DPRM) [19,20,21,22] is well described by the KPZ equation and can be mapped into the ASEP. An attractive line defect in two-dimensional DPRM is related to the slow bond in the ASEP. Some arguments on the DPRM have proposed $r_c = 1$ [5,6,7,8,9], for the queueing transition. If a queueing transition exists at $r_c < 1$ in the ASEP problem, it would

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be interesting to find the critical point and phase transtion in the DPRM. Here, we study the DPRM with an attractive defect and measure various physical quantities as a function of in order to observe whether there exist the transition or not.

The Hamiltonian of DPRM with a defect at $\mathbf{x} = 0$ is

$$\mathcal{H} = \int dt \left[\gamma \left(\frac{d\mathbf{x}}{dt} \right)^2 + \mu(\mathbf{x}, t) - \epsilon\delta(\mathbf{x}) \right], \tag{1}$$

where $\mathbf{x}$ is the $d-1$ dimensional transverse vector, t is the polymer length perpendicular to the substrate, and $-\epsilon\delta(\mathbf{x})$ means a time-independent defect at $\mathbf{x} = 0$. There are three competing terms: a bending energy γ forcing the polymer straight against a transverse bending, the impurity $\mu(\mathbf{x}, t)$ assigned to each point $(\mathbf{x}, t)$ preferring the polymer to be deformed through the minimum impurities, and the attractive defect potential at $\mathbf{x} = 0$ making the polymer return to the origin. The random potential $\mu(\mathbf{x}, t)$ is a white noise satisfying

$$\langle \mu(\mathbf{x}, t)\mu(\mathbf{x}', t') \rangle = 2D\delta(t - t')\delta^{d-1}(\mathbf{x} - \mathbf{x}'). \tag{2}$$

The partition function $Z(\mathbf{x}, t)$ for the polymer, starting from $(0, 0)$, and ending at $(\mathbf{x}, t)$, can be written as the path integral

$$\begin{aligned} Z(\mathbf{x}, t) = & \int_{(0,0)}^{(\mathbf{x},t)} \mathcal{D}\mathbf{x}'(t') \\ & \times \exp\left\{ -\frac{1}{T} \int_0^t dt' \left[\gamma \left(\frac{d\mathbf{x}'}{dt'} \right)^2 + \mu(\mathbf{x}', t') - \epsilon\delta(\mathbf{x}') \right] \right\}, \end{aligned} \tag{3}$$

where T is temperature.

2 Discrete Model and Numerical Results

At zero temperature, entropy is ignored and the problem in Eq. (1) becomes much simplified by finding the optimal path and its energy $E(x, t)$ among all the pathes arriving at (x, t). The initial energy $E(x, 0) = 0$ is given at $t = 0$. A continuous random number between 0 and 1 with uniform distribution is assigned for the randomness $\mu(x, t)$ on a discrete structure. In addition, the attractive defect potential $-\epsilon$ is given at the center site $x = 0$.

First we consider a triangular structure. The polymer starts from $x = 0$ and its path is restricted by $|x(t) - x(t + 1)| = 0$ or 1. There is a bending energy γ against a transverse jump $|x(t) - x(t + 1)| = 1$. The minimum energy $E(x, t)$ for the polymer ending at (x, t) can be obtained recursively [20,21,22]: in $d = 1 + 1$,

$$\begin{aligned} E(x, t + 1) = \min\{ & E(x, t) + \mu(x, t) - \epsilon\delta_{x,0}, \\ & E(x - 1, t) + \mu(x - 1, t) + \gamma - \epsilon\delta_{x-1,0}, \\ & E(x + 1, t) + \mu(x + 1, t) + \gamma - \epsilon\delta_{x+1,0} \}, \end{aligned} \tag{4}$$

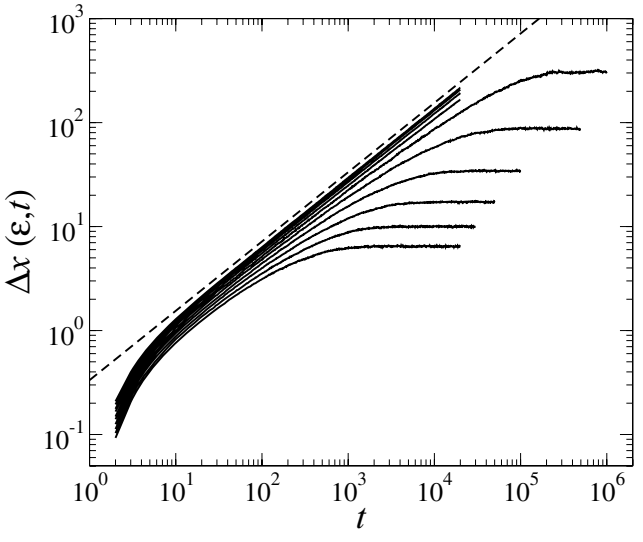

Fig. 1. $\Delta x(\epsilon, t)$ as a function of t on a discrete triangular structure with $\epsilon = 0.00$, 0.01, 0.02, 0.03, 0.04, 0.05, $\cdots$, 0.09 from top to bottom. The guideline has the slope of 2/3.

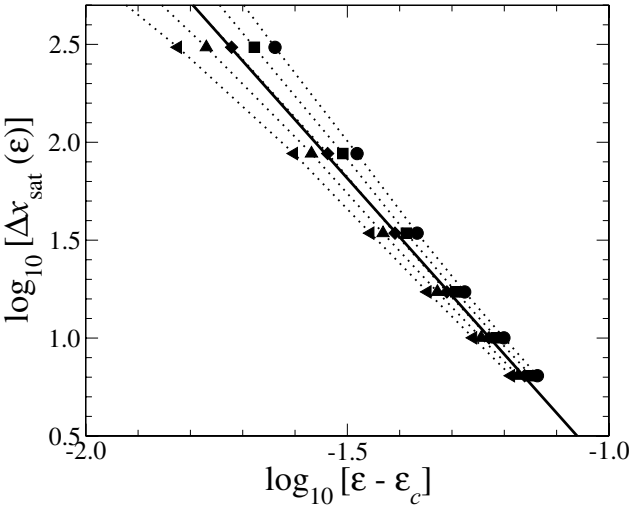

Fig. 2. $\Delta x_{sat}(\epsilon)$ as a function of $\epsilon - \epsilon_c$ for arbitrary critical values $\epsilon_c = 0.011$, 0.016, 0.021, 0.026, and 0.031 from right to left. The most straight line is obtained at $\epsilon_c = 0.021$, where $\Delta x_{sat} \sim (\epsilon - \epsilon_c)^{-\delta}$ with $\delta = 3.00$.

where $\min\{A, B, C\}$ takes the minimum value among A, B, and C. We shall write $d = 1 + 1$ to indicate that there is one transverse and one longitudinal direction. Following Eq. (4) the polymer energy at each site is updated in parallel.

We monitor the end to end distance Δx of the polymer as a function of the polymer length t. In general it increases with t. Without defect, Δx follows

$$\Delta x \sim t^{1/z}, \tag{5}$$

with $z = 3/2$ [23]. The optimal path is affected by the strength of the defect ϵ. Its contribution to Δx is negligible as long as ϵ is smaller than ϵ_c as shown in Fig. 1 where Δx still shows the power law behavior with $z = 3/2$. For $\epsilon > \epsilon_c$, Δx increases with t and then approaches a finite value Δx_{sat} in long time limit. Actually Δx_{sat} depends on ϵ. We assume that Δx_{sat} diverges as ϵ approaches ϵ_c following a scaling law

$$\Delta x_{sat} \sim (\epsilon - \epsilon_c)^{-\delta}. \tag{6}$$

The log-log plot of Δx_{sat} against $(\epsilon - \epsilon_c)$ for various values of ϵ_c is given in Fig. 2, where we estimate $\epsilon_c \approx 0.021$ with $\delta \approx 3.0$.

3 Summary

A directed polymer in random media with an attractive defect in the middle of one dimensional substrate is studied. The end to end distance Δx of the polymer is monitored as a function of the polymer length t and ϵ. We find that there is a phase transition at $\epsilon_c \approx 0.021$. We measure Δx in a triangular structure to avoid the finite system size effect. Due to the triangular structure, the simulation is limited up to $t_{max} \leq 10^6$ depending on ϵ. For $\epsilon < \epsilon_c$, the contribution of the defect to Δx is negligible. For $\epsilon > \epsilon_c$, Δx becomes finite. This behavior supports the queueing transition at finite ϵ_c [13]. Analytic works and larger simulations are required to get more accurate values of the critical exponents.

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