Bifurcation Phenomena of Opinion Dynamics in Complex Networks

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Abstract. In this paper, we study the opinion dynamics of Improved Deffuant model (IDM), where the convergence parameter μ is a function of the opposite's degree K according to the celebrity effect, in small-world network (SWN) and scale-free network (SFN). Generically, the system undergoes a phase transition from the plurality state to the polarization state and to the consensus state as the confidence parameter ϵ increasing. Furthermore, the evolution of the steady opinion s_* as a function of ϵ, and the relation between the minority steady opinion s_*^{min} and the individual connectivity k also have been analyzed. Our present work shows the crucial role of the confidence parameter and the complex system topology in the opinion dynamics of IDM.

Keywords: opinion dynamics, complex networks topology, bifurcation phenomena.

1 Introduction

Our local society, which can be well modelled as complex networks, has its own structure depending on the geography, culture and history. Recently it has also been realized that many real social networks arising in society, such as networks of sexual relationships [1], collaborations between actors [2,3] and scientists [4,5], web-based social networks [6] , peer-to-peer social network [7], and the social network of a bulletin board system in a university [8] all share some universal characteristics such as the small-world effect, high clustering coefficient property and the power-law degree distribution. Those features affect the dynamics in society systems, especially the opinion dynamics in complex networks. Many natural and man-made networks have been successfully studied as a framework of several celebrated opinion models. Nevertheless, the understanding of the opinion dynamics in complex networks remains a challenge.

Social impact theory founded by Latané [9,10], was developed as a metatheoretical framework for modelling situations where each individual is influenced by others around him/her to change his/her beliefs, attributes or behaviors. Based on social impact theory, there are two celebrated models about opinion dynamics that were proposed in recent years. One celebrated model is the binary opinion

J. Zhou (Ed.): Complex 2009, Part I, LNICST 4, pp. 1146–1153, 2009.

model that proposed by K. Sznajd-weron and J. Sznajd (S model) [11] to describe a simple mechanism of making up decisions in a closed community. In this model, the opinion of individual is a binary variable assuming the values +1 and -1 referring to two opposite opinions on a particular issue. The updating rules follow the principle of "united we stand, divided we fall". The other is the continuous model proposed by Deffuant et al (D model) [12]. In D model, the opinion s of individual can vary continuously between zero and one. Each agents selects randomly one of the other agents and checks first if an exchange of opinions makes sense. If the two opinions differ by less than ϵ $(0 < \epsilon < 1)$, each opinion moves partly in the direction of the other, by amount $\mu \Delta s$, where Δs is the opinion difference and μ the convergence parameter $(0 < \mu \leq 0.5)$; otherwise, the two refuse to discuss and no opinion is changed. The parameter ϵ is called confidence bound or confidence parameter. In our society, individual typically has a continuous opinion and always is influenced by his/her acquaintances or other external factors, such as advertisement, newspapers and broadcast, to change his/her opinion. Here, we pay most of our attention to the interaction between individuals and do not consider the influence of the external factors. On the other hand, each agent has his/her own influence of persuading others to agree with him/her, and also has his/her own ability to keep his/her opinion from changing. In our present work, we study the dynamics of continuous opinion of improved Deffuant model (IDM) with heterogeneous convergence parameters μ, which is a function of the opposite's degree k according to the celebrity effect, in complex networks.

The main goal of this paper is to study the opinion dynamics of IDM in complex networks with various confidence parameter ϵ. Generically, the system undergoes a phase transition from the plurality state to the polarization state to the consensus state as ϵ increasing, in the both small-world network (SWN) and scale-free network (SFN). Then, we focus on the evolution of the steady opinion s_* as the function of the confidence parameter ϵ in the both celebrated complex networks, and find that there exists a bifurcation diagram of the steady opinion s_* as ϵ increasing. Furthermore, we analyzed the relation between the minority steady opinion s_*^{min} and the individual connectivity k in the both SWN and SFN, and find that the process of opinion dynamics of IDM in SWN is complete different from that in SFN. Our present work reveals the dependence of the opinion dynamics of IDM on the confidence parameter ϵ between individuals and the complex system topology.

2 Improved Deffuant Model

Many real society systems can be well mapped to complex networks, which are sets of distinguishable nodes $i = 1, 2, \ldots, N$, connected by a fixed number of $l = 1, 2, \ldots, L$ indistinguishable edges. Those edges represent the different relationships among agents in society, such as friendship, collaboration, business, sexual and other interactions [13]. The network is represented by its adjacency matrix A, where $a_{ij} = 1$, if an edge connects nodes i and j and $a_{ij} = 0$, otherwise. There are no self-connections or multiple edges.

To realize our model simulation, we employ the celebrated small-world network (SWN) proposed by Watts and Strogatz [2] and scale-free network (SFN) proposed by Barabási and Albert [3] to study the opinion dynamics. The SWN is defined on a lattice consisting of N nodes arranged in a ring. Initially each node is connected to all of its neighbors up to some fixed range K to make the network have average coordination number $z = 2K$, and randomness is then introduced by rewiring edges between two randomly chosen nodes with rewiring probability ϕ. The random rewiring process introduces ϕNK long ranges which connect nodes that otherwise would be part of different neighborhoods. By varying ϕ one can closely monitor the transition between order ($\phi = 0$) and randomness ($\phi = 1$) [14]. And the scale-free network is built following the principle of growth and preferential attachment. The SFN of size N is generated starting from a randomly connected core of m_0 nodes and a set $U(0)$of $(N-m_0)$unconnected nodes. At each time step, a new node is chosen from $U(0)$ and linked to $m(m < m_0)$ other nodes with the probability of Π that a new node will be connected to node i depending on the degree k_i of node i, i.e, $\Pi(k_i) = k_i/\sum_j k_j$. Numerical simulations indicated that this network evolves into a scale-invariant state with the probability that a node has k edges following a power law with an exponent $\gamma = 3$ [14].

Many previous works about D model have considered the situation where the convergence parameters μ between pairwise agents are uniform [12,15,16]. However, in our society, we often change our opinion as the one of individual who is a famous expert about the particular issue according to the celebrity effect. In our present work, we assume that the larger the agent's connectivity is, the more famous expert the agent will be. Hence, the convergence parameters μ between pairwise agents are different, which is a function of the opposite's connectivity k.

We choose a pairwise agents i and j randomly at each time step. If the two opinions differ more than a fixed threshold parameter $\epsilon(0 < \epsilon < 1)$, called the confidence parameter, both opinions refuse to discuss and no opinion is changed. If, instead, $|s_i(t) - s_j(t)| < \epsilon$, then each opinion moves partly in the direction of the other as:

$$\begin{cases} s_i(t+1) = s_i(t) + \mu_j[s_j(t) - s_i(t)], \text{with} \quad \text{prob.} p_i; \\ s_j(t+1) = s_j(t) + \mu_i[s_i(t) - s_j(t)], \text{with} \quad \text{prob.} p_j. \end{cases} \tag{1}$$

where, $\mu_j = k_j/(2 * k_{max})$ is the convergence parameter $(0 < \mu \leq 1/2)$ that agent j interacts other agents and k_{max} is the largest connectivity degree in social complex system. The probability $p_j (= 1 - \mu_j)$ is the probability that agent j is persuaded to change his/her opinion, since each agent has the ability to keep his/her opinion from changing. The famous expert changes his/her opinion with smaller probability.

3 Results

We simulate the opinion dynamics of IDM, Eq. (1) in SWN and SFN of size $N = 1000$. The initial opinion of agent varies continuously from zero to one with a uniform distribution. All the results have been averaged over at least 100 realizations, with each running lasting for at least 2×10^4 updating steps. We choose about 400 different pairwise agents randomly at each updating step.

Generally, the system of opinion dynamics reaches a steady state from the plurality state to polarization state or to the consensus state as time elapses, which has also been found in many previous works about D model. Hence, the pictures of the evolution of opinions as a function of time steps t are not shown in our present work. Here, we focus on the dependence of the opinion dynamics of IDM on the confidence parameter ϵ and the complex system structure topology.

In Fig. 1 we represent the evolution of the steady opinion s_* as a function of confidence parameter ϵ both in SWN ($\times$) and SFN ($+$). We find that there exists bifurcation phenomena of the steady opinion s_* as the confidence parameter ϵ decreases from one to zero. Namely, the system undergoes a phase transition from the consensus state to the polarization state and then to the plurality state with ϵ decreasing. Here, the polarization state is defined as that the individuals can be divided into two or more camps according to their opinions. Each camp has its opinion that different from others obviously. The consensus state is defined as that all the individuals share the same opinion. On the other hand, we find that the steady opinion s_* of consensus state is around 0.5 in SWN and SFN. However, the fluctuation of steady opinion s_* in SFN is larger than that in SWN, probably caused by the topology structure of complex networks, which will be explained below. A detailed finite-size scaling analysis performed for both complex networks shows that the critical value of polarization and the critical value of consensus, (ϵ_p, ϵ_c), corresponds in SFN to (0.21(4), 0.48(2)), and in SWN to (0.15(5), 0.40(3)), accordingly, as shown in Fig. 1. From the bifurcation diagram of steady opinion s_* as the function of confidence parameter ϵ, the ability of polarization and consensus of SWN is much stronger than that of SFN. Namely, the ability of polarization and consensus depends on the heterogeneous property of complex networks, the more heterogeneous the complex network is, the weaker the ability of polarization and consensus of complex network will be.

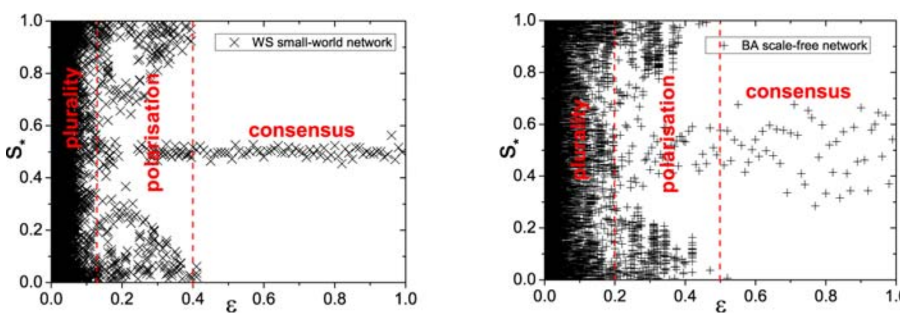

Fig. 1. (color online) Bifurcation diagram for the steady opinion s_* as a function of the confidence parameter ϵ in small-world network ($\times$ SWN) and in scale-free network ($+$ SFN) for one evolution case. The parameters of the two complex networks are: $N = 1000$, $\phi = 0.05$, $z = 18$, $m_0 = 10$, $m = 6$.

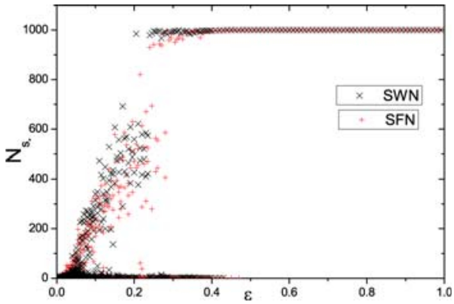

Fig. 2. (color online)Plots for the evolution of the N_{s_*} as a function of the confidence parameter ϵ in SFN (+) and in SWN (×). The network parameters are as in Fig. 1.

Further light can be shed on the dependence of the opinion dynamics on the complex network topology. In order to do this, we define the physical quantity N_{s_*} as the number of agents with the same steady opinion s_* and study the evolution of N_{s_*} as a function of ϵ in the both SWN and SFN.

In Fig. 2 we represent the evolution of the N_{s_*} as a function of the confidence parameter ϵ in SFN and SWN respectively. In the polarization region in the both complex networks, all individuals can be divided into more than one camps according to their opinions. Each camp has its opinion and size that is different from others. Here, we call the steady opinion s_* that a few individuals share, i.e., $N_{s_*} < 10$, the minority opinion s_*^{min}. Of course, there also exists the second-largest and the third-largest camps in the both complex networks, see the middle part of the picture in Fig. 2. The evolution of the largest and the second-largest camps as a function of confidence parameter ϵ of D model on adaptive networks has been analyzed by Balazs Kozma and Alain Barrat [17]. Surprisingly, we cannot find any signals to identify the different roles between the topology of SWN and that of SFN in opinion dynamics of IDM. To see this, we focus on the minority steady opinion s_*^{min} and study the relationship between the s_*^{min} and the connectivity degree k of the SWN and SFN, see the dots lie on the horizontal axis in Fig. 2.

As well known, the obvious difference of SWN and SFN is the connectivity degree distribution, the bell-form distribution to SWN and the power-law distribution to SFN accordingly. In order to analyze the relation between s_*^{min} and the connectivity degree k, we define the relative connectivity degree λ as follows,

$$\lambda = \frac{k}{k_{max}} \tag{2}$$

where, k_{max} is the largest connectivity degree in complex network. The larger the relative connectivity degree λ of one agent is, the more famous the agent is, who plays the more important role in affecting others to change their opinions.

In Fig. 3 we represent the evolution of the relative connectivity degree λ as a function of the ϵ and the s_*^{min} in the polarization region in the both SWN and SFN. Along with the results in Fig. 1, we find that the steady opinion s_*

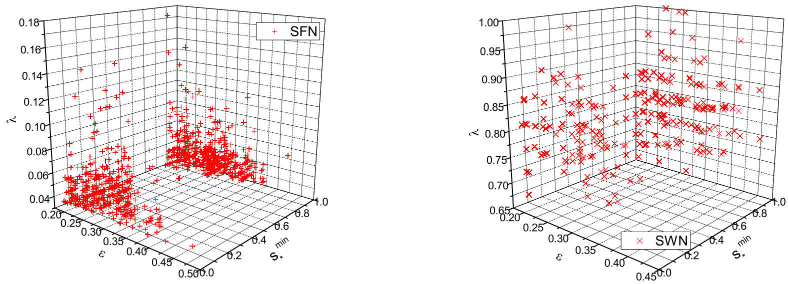

Fig. 3. (color online)Plots for the evolution of the relative degree λ as a function of confidence parameter ϵ and the minority steady opinion s_*^{min} in SFN (+) and SWN(×) when $N_{s_*} < 10$. The network parameters are as in Fig. 1.

is away from the middle opinion 0.5 in polarization region and is about 0.5 in the consensus state, which show the role of the compromise factor in our model algorithm. In the polarization region, there exists the steady opinions that smaller than 0.5 and more than 0.5 simultaneity in the system. The larger the confidence parameter ϵ, the farther the steady opinion will be away from 0.5.

Surprisingly, we also find that there exists a huge difference of the λ of agents with the minority steady opinion in the both celebrated complex networks. The relative connectivity degrees λ with the minority steady opinion s_*^{min} are smaller than 0.18 in SFN and larger than 0.65 in SWN respectively, which indicates that the process of the opinion formation in SFN is much different from that in SWN. As well known, the SFN following the algorithm of growth and preference attachment ia a disassortative complex network, i.e., the agents with higher connectivity, which plays the same role as hubs in the Internet, always are connected with those with smaller connectivity degree. Since the more difference between those agents' connectivity, agents with smaller connectivity are always persuaded to move their opinions enough in the direction of their nearest neighbor who has larger connectivity. Namely, the common people always change their opinions following the famous experts according to the celebrity effect. Note that the process of the polarization and consensus starts from the agents with highest connectivity and then to his nearest neighbors and last to all the agents in SFN, the agents with smaller connectivity away from all the agents who have higher connectivity will be separated as the minority with larger probability. On the other hand, the difference between agents' connectivity is much smaller due to the bell-form connectivity distribution in SWN. Hence, each pairwise agents with almost the same connectivity reaches the consensus opinion easily. Although agents change their opinions according to the celebrity effect in our present work, the agents who have higher connectivity degrees will be separated as the minority with larger probability. As time elapses and the interaction between individuals in SWN, the system will be divided into more than one camps and each camps share the same opinion firstly, then, larger camps can be made of those smaller camps. The difference process of opinion formation in the both

SFN and SWN, along with the larger fluctuation in SFN in Fig. 1, is caused by the heterogeneous property of complex network.

4 Conclusion

We propose an Improved Deffuant model (IDM) of opinion dynamics in SWN and in SFN, where the convergence parameter μ is the function of the opposite's connectivity degree k according to the celebrity effect. Generically, the opinion dynamics reaches the steady state (polarization state or consensus state, which is related to the confidence parameter ϵ) in the both celebrated complex networks. We find that the steady opinion s_* undergoes a bifurcation phenomenon as the confidence parameter ϵ increases, from the plurality state to the polarization state and to the consensus state. In order to show the effect of complex network topology, we pay most of our attention to the property of the agents with the same minority steady opinion s_*^{min} in the polarization region. We find that there also exists a bifurcation phenomena of the s_*^{min} as ϵ increases in SFN and in SWN, which is caused by the compromise factor in our model. Further, we find that a few agents who have smaller connectivity degree are persuaded easily as the minority with larger probability in SFN; otherwise, a few agents who have higher connectivity degree are persuaded easily as the minority with larger probability in SWN. All those results indicate that the process of the polarization and consensus of opinion dynamics in SFN is different from that in SWN, along with the fluctuation in SFN in Fig. 1, probably caused by the heterogeneous property of complex networks. Our present work opens new paths to understand the bifurcation phenomena of opinion formation in complex networks.

Acknowledgments

L.Guo thanks Prof. W. Li for helpful suggestions and comments. This work was supported by the National Natural Science Foundation of China under Grant No.s 70571027, 10635020 and by the Ministry of Education in China under Grant No. 306022.

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