An Emergence Principle for Complex Systems

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"Beware, you who seek first and final principles, for you are trampling the Garden of an angry God and He awaits you just beyond the last theorem"
Sister Miriam Godwinson

Abstract. From elementary system graph representation, systems are shown to belong to only three states: simple, complicated, and complex. First two have been studied over past centuries. Last one originates in existence of threshold above which components interaction overtakes outside interaction, leading to system self-organization which filters outer action, making it more robust with emergence of new behaviour not predictable from components study. The threshold value, expressed in terms of coupling system parameters, is verified to recovers limits found in a broad range of domains in Physics and Mathematics, giving explicit criterion for emergence in complex system. Application to man-made systems concentrates on the balance between relative system isolation when becoming complex and delegation of more "intelligence" in adequate frame between new augmented system state and supervising operator. Entering complexity state opens the possibility for the function to feedback onto the structure, ie to mimic technically the early invention of Nature.

Keywords: Complex Systems, Emergence Criterion.

1 Introduction

Accumulation of recent observations on natural phenomena with high technical development boosting the access to a wide range of new parameter indicates without doubt the existence of phenomena not following main stream laws established from patient analysis of natural phenomena over millennium long previous period. These "classical" laws are concerning phenomena "reasonably" isolated over broad range in size from galaxy to atoms when including quantum and relativistic improvements. In the mean time, technology advance and observation accuracy drove the attention on more complicated systems with always larger number of elements, for which previous laws are not sufficient to represent correctly enough their behaviour. Such systems are forming a new huge class in all scientific and technical human activities, and have reached their own status by the corner of the millennium under the name of "*complex*" systems. There is today a strong questioning about their origin and their formation [1]. This has been addressed in a very pedestrian approach [2] based on

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elementary source-sink model applied to the graph representing the aggregate of system components, showing that system structure falls into three different groups, simple, complicated and complex, with specific and explicit properties. The first two groups are usual ones approachable by the methods of scientific reductionism [3]. The third group, by its very global nature, is not reducible to the effect of its components [4], and requires some adjustment for being correctly handled, because now a key point is the way the system behaves under (or against) environment action. The system mainly self-organizes and develops a global reaction hiding details of specific component effect. A consequence is that the mechanistic notion of individual component "trajectory" pertaining to first two groups looses its meaning and should be replaced by more general "manifold" entity corresponding to accessible "invariants" under environment action. So there is emergence of new natural properties which will be discussed depending of complexity grade and which can be related to well known classes of observed phenomena. Within the pedestrian graph approach an explicit criterion for passing to complex state and to have emergence of new behaviour is given in terms of system coupling parameters and is recovering all previous expressions in Physics and in Mathematics. Advantages in application of complex structure to artificial man made systems are stressed.

2 System State Analysis

Let the graph with N nodes N_i representing a system with a finite number N of identified and separable components. There exists three types of vertices in between the N components i and outside sources e whenever an exchange exists between them.. System dynamics result from a combination of previous three different exchanges to which three characteristic fluxes can be associated for each system component the nature of which (power, information, chemical,..) unambiguously characterize system components status. First flux corresponds to ''free'' dynamics of i-th component p_{ii} along vertex V_{ii}, second one $p_{i,e}$ to transfer flux between outer source and system i-th component along vertex V_{ie}, and last one to inter components effects $p_{i,int} = \mathrm{Inf}_j\{p_{i,j}\}$, with $p_{i,j}$ the characteristic and oriented flux exchange between components i and j along vertex V_{ij}, see Fig. 1. For weak coupling $p_{ii} >> p_{i,int}$(case (**A**)), system dynamics are reducible to a set of almost independent one-component sub-systems, and system will be termed as a *simple* one. For strong outer coupling $p_{i,e} >> p_{i,int}, p_{ii}$ (case (**B**)), system dynamics can be decoupled (as other components action creates a weak coupling between them), at least locally, into a set of sub-systems controlled by as many exterior sources as there are components in the system because they can still be identified. The system can be termed as *complicated*. Finally for strong internal coupling $p_{i,int} >> p_{ie}, p_{ii}$ (case(**C**)),system dynamics are now determined by components interaction satisfying the inequality, with fundamentally different outside action compared to case (**B**). Here internal interactions dominate and shield input tracking from outer source to component. So control action can only be a ''global'' one from other system components satisfying condition for case **(B)**, so

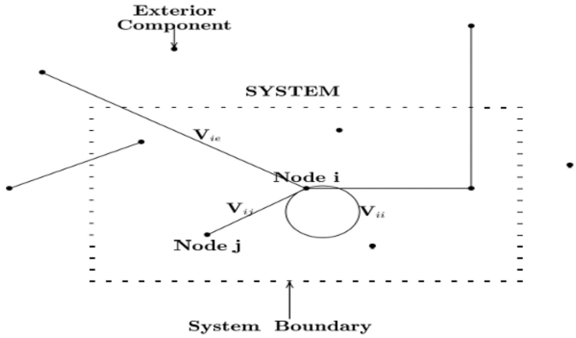

Fig. 1. Graph representation of system with its three exclusive types of vertices V_{ii}, V_{ie}, and V_{ij}

system dynamics are now also driven by internal action. For all system components passing in third state their control cannot as in case (**B**) be fixed only by outer source action because of stronger interaction effect dominating the dynamics of concerned components, and a *self-organization* takes place inside the system leading to an internal control replacing classical one from outside. So manipulating inputs with as many dof as in initial system is no longer possible because of the conflict with internal control. External system control dimension is thus *reduced*. The system will be termed as *complex* (from Latin *cum plexus* : tied up with). A very elementary test for determining if a system is passing to complex state is thus to verify that its control requires manipulation of less dof than in initial system with weaker internal interactions. So paradoxically most complex possible structure corresponds to totally autarchic system, which joins simple system definition by isolation. In fact this apparently contradictory statement is resulting from the very nature of internal interactions effect which reduces the number of invariants on which system trajectory takes place. Example is neutral gas particles for which their initial *6N* positions and velocities (the mechanical invariants of motion) are reduced to the only energy invariant (or temperature), justifying thermodynamic representation. Consequently, when increasing internal interactions, contrary to a complicated system which remains complicated from either side, a system is the less complicated seen from outside as it is more complex internally, on top of being less sensitive to outer action, a very useful property used very early by Nature and at the origin of Her evolution. In summary, exactly like there exists three states of matter (solid, liquid, gas), there are three states (simple, complicated, complex) for each system component. On a 3-d space, plotting the three values $\{p_{ii}, p_{i,e}, p_{i,int}\}$ for each system component gives a cluster of N points C_i the status of which is determined by their location with respect to boundaries of inequalities (**A**),(**B**),(**C**), see Fig. 2. Moreover, consequence of parameter variation can be analyzed, especially when crossing inequality (**C**), extremely important for control of man-made systems as this boundary is nowadays very often passed over with modern technology advance.

An immense literature exists about complexity, its definition and its properties covering an extremely wide range of domains from Philosophy to Technology [5], especially in recent years where its role has been “discovered” in many different

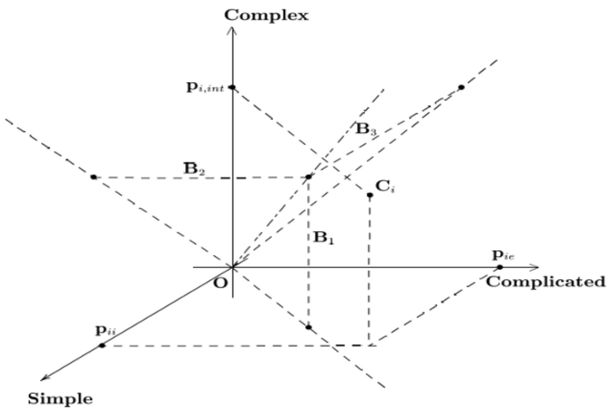

Fig. 2. System representation in [Simple-Complicated-Complex] state-space domain
- Inequality (**A**) is satisfied in tetrahedral domain (O,B_1,B_2)
- Inequality (**B**) is satisfied in tetrahedral domain (O,B_1,B_3)
- Inequality (**C**) is satisfied in tetrahedral domain (O,B_2,B_3)

fields such as networks now playing a crucial role with the ascent of Information Technologies. In the sequel emphasis is more modestly put only on more restricted complex state compared to complicated state as concerns action from outside environment (ie from control point of view) onto the system. It is intended to show from the very elementary picture above that most of important results in a broad range of domains from Mathematics to Physics can be easily recovered and that still unclear emergence phenomenon finds here a natural explanation.

3 Emergence

First the deep difference between the first two states and last complex one is in the possibility for the formers to split the system into as many independent one-component systems in a first approximation, which is impossible in the later where all interacting components have to be taken as a whole. In mathematical terms the consequence is that usual approximation methods developed for the first two states do not straightforwardly apply and have to be revised in order to handle the global aspect of the coordinated response of components in complex state, at the origin of important computer research on the problem. In specific situations, other elements also enter the description and it is interesting from description above to recover various situations observed and analytically studied for different parameters values. In general system state exhibits a mixed structure where some components are in one state and others in another one. Important examples are (weakly) inhomogeneous and continuous natural systems such as fluids with non zero gradients in a domain. Here, fluctuations are universally observed in a very large range of frequencies (roughly because the system has a very large number of components) the source of which is the free energy available in between this stationary equilibrium and complete (homogeneous) thermodynamic one, ie from the space gradients related to medium non homogeneity.

Splitting fluctuations into two groups depending on their wavelength compared to system characteristic gradient length, large wavelength ones excited in the medium are in complicated state and, because they are sensitive to boundary conditions, can be observed and possibly acted upon as such from outside, whereas fluctuations with small wavelength are in complex state and can only be considered globally. Under their strong interactions, and because they are much less sensitive to boundary conditions, they are loosing their phase and globally excite an outflux (usually called a *transport*) expressing non equilibrium system situation counteracting input flux responsible of medium non homogeneity. Clearly emergence of this transport is a direct consequence of components self interaction when crossing condition (**C**) which now takes place on a manifold in small wavelength component state space and constraints their motion to take place on it. So transports determination is an important element qualifying system behaviour and is an active research problem studied worldwide. This feature is observed for all natural (open) systems on the *dissipative* branch [6] exchanging (particles, chemical, energy) fluxes with outside environment. Evidently the channels by which internal energy sources are related to these fluxes are playing a privileged role because they regulate the influx ultimately available for the system and finally determine its *self-organized* state[7]. Dissipative systems only exist to the expense of these fluxes, and they evolve with parameter change − such as power input − along a set of neighbouring states determined by branching due to bifurcations where internal structure changes in compatibility with boundary conditions and by following the principle of largest stability. So the picture is a transport system governing flux exchanges guided by the bifurcation system which, as a pilot, fixes the structure along which these exchanges are taking place. Finding the branching pattern thus entirely defines possible system states and determines fluctuation spectrum. Branching is found as nontrivial solutions of variation equations deduced from general system dynamics equations. Despite clear identification of phenomena physical origin, their analysis is still in progress in many situations [8,9], and cleared up for fluctuations in deformable solid bodies[10]. Moreover, modification of system dynamics is the more important as non homogeneity gradient is steeper, with extreme case of living cell systems completely encapsulating within a filtering membrane (ie a steep gradient) a space domain where very specific "memory" DNA molecules are fixing the dynamics of inside system they control, with corresponding exchange across the membrane.

However, aside dissipative pattern followed by natural systems exhibiting components with relatively elementary features (charge, mass, geometrical structure, chemical activity, wavelength, frequency..), there exists cases where complex state occurs in systems with more sophisticated components, usually called "*agents*". Examples are herds of animals, insect colonies, living cell behaviour in organs and organisms, and population activity in an economy. In all cases, when observed from outside the systems are exhibiting relatively well defined behaviours but a very important element missing in previous analysis is the influence of the *goal* the systems are seemingly aiming at. Very often the components of these systems are searching through a collective action the satisfaction of properties they cannot reach alone, and to represent this situation the specific word "*emergence*" has also been coined [11]. The point is that it is now possible to return back to previous case and in a unified picture to envision the laws of Physics themselves as emerging phenomena.

For instance for an ensemble of neutral particles with hard ball interactions, and beyond the threshold of rarefied gas, (ie when the Knudsen number $K_n = \lambda/L$ is decreasing to 0 from the value 1, with λ the particle mean free path and L a characteristic length scale), the particle system is suddenly passing from a complicated to a complex state (due to the huge value of Avogadro number). Consecutive to overtaking of collective interaction by collisions (expressed by decrease of Knudsen number below the value *1* representing the limit of constraint (**C**) for neutral particle system), it could be said that there is emergence of a pressure and a temperature, which, from a point of view outside the gas, summarizes perfectly well the representative parameters (the invariants) describing it at this global level (the thermodynamic representation). Similarly at atomic level, after baryons are assembled from primitive quark particles below some threshold energy, protons and neutrons assemble in turn themselves below another lower energy threshold into ions with only mass and charge parameters, able to combine finally with electrons to create atoms. In all cases, it can be verified that emergence of new compound system does occur when boundary condition (**C**) is crossed when applied to each component of the system and at each interaction level. Interestingly, independent of the background system and of the vocabulary, it is easily verified that all systems exhibit first emergence of self organization out of which there appears a specific behaviour. In fact it is elementarily understandable from previous source-sink model that a key point is in the accuracy of modelling the components in complex state, as long as the resulting "invariants" which will grasp all system information for interaction with environment are directly depending on this modelling. This has been at the origin of a computer "blind" search where the agents are given properties and "emerging" behaviour is obtained in a bottom-up approach, sometimes in surprising compatibility with experimental observations [12], in parallel to theoretical analysis [13]. Finally it should be observed that the logical chain:

{stimuli/parameter change} → {higher interactions between some system components} →{passage to complex state} → {system self organization} → {emergence of new behaviour}

discussed here is nothing but the sequence leading to the final step of system evolution toward more independence, and which is the feedback of "*function*" onto "*structure*", a specific property of living organisms explaining their remarkable survival capability by structure modification.

4 Mathematical Representation of Complex Systems Emergence

The few basic previous examples from common sense observation illustrate the generality of the elements described above providing a unified base for complex system paradigm. From atomic nucleus to galaxy natural systems are seen to be constituted by aggregates of identifiable components (which, as already stressed, can be themselves, at each observation level, aggregates of smaller components) with well defined properties. These aggregates have been said to exhibit a complex behaviour when interaction between the components –or some of them– is overtaking their interactions with exterior environment. Similarly living beings are exhibiting the same behaviour, as observed with gregarious species, and in artificial man-made systems the same phenomenon is occurring when the overtaking conditions are satisfied. This is the case for high enough performance level systems because the components are then tightly packed, as for high torque compact electrical actuators. Despite an extremely large variety of possible situations there are few basic interactive processes leading to complex self-organization. First exist weak gradients natural systems discussed in previous paragraph and entering the more general class of *reducible systems* mainly because it can be shown that their complete dynamics including generated fluctuations can be reduced by projection onto {initial state plus large wavelength components} dynamics without small wavelength components dynamics (in complex state), now globally represented by transport coefficients modifying initial dynamics, see Fig. 3. Their mathematical analysis is still in progress in non Gaussian case, due to difficulty for specific small parameter ordering to analytically express transport coefficients despite their source is well identified [8,9,14].

More generally, a system may be in complete complex state, examples of which are atomic nucleus, herd of animals, and galaxies. Despite their very different space sizes, the systems exhibit always the same characteristic feature to finally depend on an extremely restricted number of parameters as compared to the aggregate of their initial components. Searching the way to extract directly the remaining ''control'' parameters of such systems from their dynamics is a fundamental issue which today motivates a huge research effort worldwide, especially in relation with information networking. Extensive analytical and numerical study has been developed for differential systems of generic form

$$\frac{dX}{dt} = A(t, X, v) + \lambda.F(t, X, u) + \mu.S(t, X) \tag{1}$$

where $X = \mathrm{col}(X_1, X_2, .. X_n)$ is system state space, λ,μ are n-vector coupling parameters, and $A(.,.,.) : \mathbf{R}_1^+ \times \mathbf{R}_n \times \mathbf{R}_p \to \mathbf{R}_n$, $F(.,.,.) : \mathbf{R}_1^+ \times \mathbf{R}_n \times \mathbf{R}_q \times \mathbf{U} \to \mathbf{R}_n$, $S(.,.) : \mathbf{R}_1^+ \times \mathbf{R}_n \to \mathbf{R}_n$ are three specific $q_1 \times q_2$−matrix terms ($q_1, q_2 \leq n$) corresponding to isolated free flight,

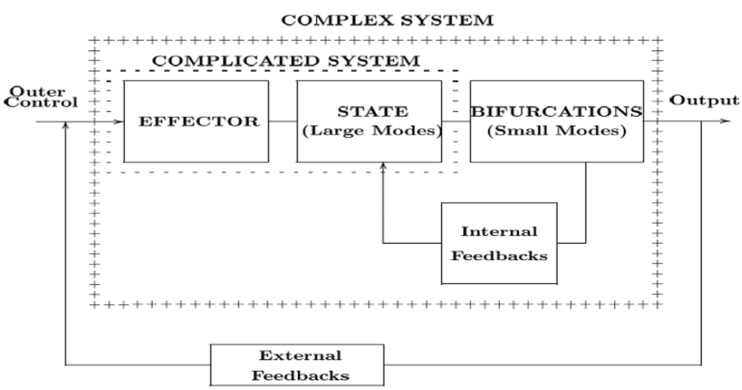

Fig. 3. Schematic block representation of (controlled) complex system

nonlinear internal interactions and source terms respectively (the linear and source terms in the right hand side of eq.(**1**) are here split apart to indicate their respective role). The other variables $v=v(X,t) \in \mathbf{R}_p$ and $u(X,t) \in \mathbf{U}$ in A and F function account more generally for possible feedback evolution of $X(t)$ onto their own dynamics as it often occurs in systems when splitting parameters into given and manipulated control ones. For fixed $u(.,.)$, $v(.,.)$, depending on the values of λ,μ components the system will be in simple, complicated or complex state described in §1. Increase in λ components moves the system into complex state, and eq.(**1**) transforms for very large λ into a singular system with small parameter in front of derivative amenable to asymptotic analysis [15]. For instance, 2–d Van der Pol system exhibits individual mode oscillations in state (**A**), driven oscillations in state (**B**). Under strong coupling between components when condition (**C**) is crossed relaxation oscillations are produced on a restricted (closed) 1–d curve representing the manifold on which complex state motion takes place. So even with two dof, complex state can occur, illustrating the fact that complexity is completely independent of complication with which it is very often confused.

More generally, it has been repeatedly observed, especially on systems close to Hamiltonian ones[16], that system representative point in n–dimensional state space follows a more and more chaotic trajectory when crossing bifurcation values and at the end fills up a complete domain[17]. Of course sensitivity is largest when the system exhibits resonances, ie is close to conservative, and adapted mathematical expansion methods have to be worked out[18]. In this case, it is easily verified that resonance overlapping condition[19] is nothing but application of condition (**C**). Because systems are basically non integrable[20], this is a direct evidence of increasing effect of internal interactions which reduce system dynamics to stay on attractor manifold of degree $p < n$, so that system dynamics are now layered on this manifold. This also expresses the fact that trajectories on the remaining n–p dimension space are becoming totally indistinguishable (from outside) when taken care of by internal interactions of n–p components going to complex state. So system trajectories reorganize here in equivalence classes which cannot be further split, a dual way to express the fact that there exists an invariant manifold on which system trajectories are lying. Continuing to control these components by regular previous control [21] worked out for complicated state and specially designed for tracking a prescribed trajectory, is no longer possible and a new approach is required which carefully respects internal system action due to complex state self-organization. More global methods of functional analysis [22] related to function space embedding in adapted function spaces [23] by fixed point theorem[24] are now in order as shown for reducible case [25], because they are providing the correct framework to grasp the new structure of system trajectory which cannot be fully tracked as before. Basically the method is again to counteract impreciseness in an element by robustness to its variation, a method very largely followed by living organisms. More generally, another very influential parameter is the range of inter-component interaction, because this determines completely the build up of system clustering when becoming complex. Obviously long range interactions are leading to more intricate response with more difficult analysis. Examples are stars in a galaxy, electromagnetic interactions between ions and electrons in a plasma, animals in a herd and social behaviour of human population in economic trading such as stock market with internet link. In all cases a new element is coming from the size of the neighbouring domain each system component is sensitive to, and implies a time extension to past neighbouring components trajectories. In this case, the resulting complex behaviour is more generally determined by interactive component effects over a past time interval and weighted according to their importance [2]. Finally one can summarize previous analysis by the universal.

Emergence Principle for Complex Systems: When interaction between some system components takes over by satisfying condition (**C**), they cluster into a subsystem the dynamics of which are only guided by the invariants of the generated manifold, and which are the only control parameters left by this (internal) reorganization.

This principle provides the way to express explicitly in mathematical terms the boundaries of domains in parameter space where complexity state is reached once a model of the system is given, and to fix its dynamical behaviour in new state by determination of its invariants.

5 Discussion and Conclusion

Systems exhibiting behaviours which do not fit with main stream scientific laws established from patient observations of Nature over past centuries have been repeatedly observed over last decades with the ascent of modern technology, where new natural and man made systems with very intricate structure implying a large number of heterogeneous components in strong interaction have been observed and developed. Application of usual laws is often unable to describe their dynamics, because they stay outside the domain of complicated multi component systems only covered by use of reductionism method. The main reason is in the overtaking of component interaction strength which dominates enough over other effects to force the system to close on itself and to manifest an internal self-organization responsible of its new behaviour. Differently said in elementary terms, the new paradigm is that "increasing interactions between components lead to their isolation" as easily verifiable. Such systems are termed as "complex", and their main feature is that components in complex state are internally ruled through this self-organization so that they are less sensitive to environment action. So natural complex systems are structurally more robust than complicated ones as evidenced by observation of living organisms, the most complex known ones. Analysis of complex systems dynamics shows the possibility of "emergence" of a new behaviour not included into the set of initial components behaviour as a direct consequence of self-organization opening the possibility for the "function" to feedback onto the "structure", as illustrated by development of living systems on Earth. The criterion for crossing "complexity" barrier has been explicitly expressed in terms of system coupling parameters by condition (**C**), which is verified to cover all known formulae in broad range of applications in Physics and Mathematics, thus providing a general emergence condition for a complex system.

Previous properties are of up-most importance when applied to man made industrial systems now appearing in open and global economy with an always increasing number of heterogeneous components to be operated all together for production of higher value objects. Previous centralized control structure cannot be maintained for keeping complete system mastery of larger number of elements going to complex state due to higher value of coupling parameters. In parallel, reduction of input control parameters by transforming the system into a (partially) complex one by clustering some components into bigger parts reduces system fragility. The industrial challenge civilisation is facing today justifies if any the needs to study and to create these complex systems [26]. On Fig. 2, this would mean to vary adequate parameters to move the representative points along complex axis in order to fix exactly new system status. In any case, internal non controlled dynamics are taken care of by system self organization resulting from passing to complex state, implying that precise trajectory control is now delegated to system. The challenging difficulty is that to comply with new structure, some "intelligence" has also to be delegated to the system, leading for the operator to a more supervisory position [27]. In present case, this is contribution to trajectory management by shifting usual (imposed) trajectory control to more elaborated task control [28], a way followed by all living creatures in their daily life to guarantee strong robustness while still keeping accuracy and preciseness. This illustrates the limited possibility of behaviours from laws of Physics because they are tightly linking *information flux* related to the described action to *power flux* implied in them. This opens on searching an adequate merging of information flux mastery from recent Information Technology development with power flux mastery resulting from classical long term mechanical development [29]. Though apparently loosing some hand on such systems, it has been surprisingly possible along this line to find explicit conditions in terms of system parameters expressing somewhat contradictory high preciseness (by asymptotic stability condition) and strong robustness (against unknown system and environment parts)[30]. In this way system dynamics are finally controlled and asymptotic stability can be demonstrated, but in general to the price of a not necessarily decreasing exponential asymptotic type.

References

1. Pattee, H.H.: Hierarchy Theory, the Challenge of Complex Systems. Braziller, New York (1973); Galbraith, J.: Designing Complex Organisations. Addison-Wesley, Reading (1973); Hayek, F.A.: The Theory of Complex Phenomena. In: Bunge, M. (ed.) The Critical Approach to Science and Philosophy, pp. 332–349. Collier McMillan, London (1964); Trappler (ed.): Power, Autonomy, Utopia: New Approaches towards Complex Systems. Plenum Press, New York (1986); Dyke, C.: The Evolutionary Dynamics of Complex Systems. OUP, New York (1988); Funtowicz, S., Ravetz, J.: Emergent Complex Systems. Futures 26, 568–582 (1994); Haken, H.: Information and Self–Organization: a Macroscopic Approach to Complex Systems. Springer, Heidelberg 1988)
2. Cotsaftis, M.: What Makes a System Complex: an Approach to Self-Organization and Emergence, Survey talk ECCS, Dresden, Germany, October 6-8, 2007 (2007), `http://www.arxiv/nl/07060440`
3. Laplace, P.S.: OEuvres Complètes, Part 6, Gauthiers−Villard, Paris (1878-1912); Lagrange J.L.: Mécanique Analytique, vol. I-II. Albert Blanchard, Paris (1788)

4. Aritotle: Physics, part I, chap. 9; Anderson P.W.: More is Different. Science 177, 393–396 (1972)
5. see reference [7] in ref [2] above
6. Prigogine, I., Nicolis, G.: Self−Organization in Non−Equilibrium Systems: From Dissipative Structures to Order through Fluctuations. J. Wiley and Sons, New−York (1977); Klimontovitch, Y.L.: Theory of Open Systems. Kluwer, Dordrecht (1995); Kleidon, A., Lorenz, R.D. (eds.): Non-equilibrium Thermodynamics and the Production of Entropy: Life, Earth, and Beyond. Springer, New−York (2005)
7. Kadomtsev, B.B.: Self−Organization and Transport in Tokamak Plasma, Plasma Phys. and Nucl. Fus., 34(13), 1931–1938 (1992); Biebricher, C.K., Nicolis, G., Schuster, P.: Self−Organization in the Physico−Chemical and Life Sciences, vol.16546, EU Report (1995); Schweitzer, F.: Multi−Agent Approach to the Self−Organization of Networks. In: Reed−Tsochas, F., Johnson, N.F., Efstathiou, J. (eds.) Understanding and Managing Complex Agent−Based Dynamical Networks. World Scientific, Singapore (2007); Ebeling, W., Schweitzer, F.: Self-Organization, Active Brownian Dynamics, and Biological Applications. Nova Acta Leopoldina NF 88(332), 169–188 (2003)
8. Strand, P.I.: Predictive Simulations of Transport in Tokamaks, PhD Thesis, Dept. of Electromagnetics, Chalmers Univ. of Technology (1999)
9. Zakharov, V.E., L'vov, V.S., Falkovich, G.: Kolmogorov Spectra of Turbulence. Springer, New−York (1992); Klimontovich, Y.L.: Turbulent Motion and Structure of Chaos. Kluwer, Dordrecht (1991); Frisch, U.: Turbulence. Cambridge Univ. Press, Mass (1995)
10. Cotsaftis, M.: Lectures on Advanced Dynamics, Taiwan Univ., Taipeh, R.O.C (1993)
11. Fromm, J.: The Emergence of Complexity. Kassel Univ. Press (2004); Johnson, S.: Emergence. Penguin Books, New−York (2001)
12. Camazine, S., Deneubourg, J.L., Franks, N.R., Sneyd, J., Theraulaz, G., Bonabeau, E.: Self−Organization in Biological Systems. Princeton Univ. Press, Princeton (2002); Shen, W.-M., et al.: Hormone−Inspired Self−Organization and Distributed Control of Robotic Swarms. Autonomous Robots 17, 93–105 (2004); Bonabeau, E., Dorigo, M., Theraulaz, G.: Swarm Intelligence: from Natural to Artificial Systems. Oxford Univ. Press, New−York (1999)
13. Yates, F.E.(ed.) : Self−Organizing Systems: The Emergence of Order. Plenum Press (1987); Lewin, R.: Complexity−Life at the Edge of Chaos. Macmillan, Basingstoke (1993); Lerner, A.Y.(ed.) : Principles of Self−Organization. Mir, Moscow (1966); Nolfi, S., Floreano, D.: Evolutionary Robotics : the Biology, Intelligence and Technology of Self–Organizing Machines. The MIT Press, Cambridge (2000); Serra, R., Andretta, M., Compiani, M., Zanarini, G.: Introduction to the Physics of Complex Systems (the Mesoscopic Approach to Fluctuations, Nonlinearity and Self–Organization). Pergamon Press (1986)
14. Chandrasekhar, S.: Stochastic Problems in Physics and Astronomy. Rev. Mod. Phys. 15(1), 1–89 (1943); Risken, H.: The Fokker-Planck Equation, 2nd edn. Springer, Berlin (1996)
15. Grasman, J.: Asymptotic Methods for Relaxation Oscillations and Applications. Applied Math Sciences, vol. 63. Springer, New−York (1987)
16. Salamon, D.: The Kolmogorov–Arnold–Moser Theorem, ETH-Zürich (preprint) (1986); Arnold, V.I.: Mathematical Methods of Classical Mechanics, 2nd edn. Springer, Heidelberg (1989); Moser, J., Zehnder, E.: Lectures on Hamiltonian Systems. Lecture Notes, ITCP (1991)

17. Alliwood, K.T., Sauer, T.D., Yorke, J.A.: Chaos. An Introduction to Dynamical Systems. Springer, New–York (1997); Wiggins, S.: Chaotic Transport in Dynamical Systems. Springer, New−York (1991); Strogatz, S.H.: Nonlinear Dynamics and Chaos with Applications to Physics, Biology, Chemistry, and Engineering. Addison−Wesley, Reading (1994); Magnitskii, N.A., Vasilevich, S.V.: New Methods for Chaotic Dynamics. World Scientific Series on Nonlinear Science, Ser.A, Singapore (2006); Sagdeev, R.Z., Usikov, D.A., Zaslavsky, G.M.: Nonlinear Physics; from Pendulum to Turbulence and Chaos. Harwood Academic, New−York (1988)
18. Poincaré, H.: Les Méthodes Nouvelles de la Mécanique Céleste, vol. 3. Gauthier−Villars, Paris (1892–1899); Arnold, V.I., Koslov, V.V., Neishtadt, A.I.: Dynamic Systems III : Mathematical Aspects of Classical and Celestial Mechanics. Springer, Berlin (1993); Broer, H.W., Huitema, G.B., Sevryuk, M.B.: Quasi−Periodicity in Families of Dynamical Systems: Order amidst Chaos, LNM, vol. 1645. Springer, New−York (1996); Katok, A., Hasselblatt, B.: Introduction to the Modern Theory of Dynamical Systems, Cambridge Univ. Press. Mass. (1996); Bogoliubov, N.N., Mitropolskii, Yu.A., Samoilenko, A.M.: Methods of Accelerated Convergence in Nonlinear Mechanics. Springer, Berlin (1976)
19. Chirikov, B.V.: Statistical Properties of a Nonlinear String. Sov. Phys. Dokl. 1, 30 (1966); A Universal Instability of Many Dimensional Oscillator System. Phys. Rept. 52, 263–279 (1979); Lichtenberg, A.J., Lieberman, M.A.: Regular and Chaotic Dynamics, 2nd edn. Springer, New−York (1992)
20. Hirsch, M., Pugh, C., Shub, M.: Invariant Manifolds. Lecture Notes in Math. vol. 583. Springer, Berlin (1977); Tabor, M.: Chaos and Integrability in Nonlinear Dynamics, an Introduction. Wiley and Sons, New−York (1989); Goriely, A.: Integrability and non Integrability of Dynamical Systems. World Sci. Publ., Singapore (2001)
21. Lefschetz, S.: Stability of Nonlinear Control Systems. Academic Press, New−York (1965); Dullerud, G.E., Paganini, F.: A Course in Robust Control Theory : a Convex Approach. Springer, New−York (2000); Leonov, A.A., Ponomarenko, I.V., Smirnova, V.B.: Frequency Domain Methods for Nonlinear Analysis : Theory and Applications. World Scientific Publ., Singapore (1996); Astrom, K.J.: Control of Complex Systems. Springer, Berlin (2000); Grigorenko, I.: Optimal Control and Forecasting of Complex Dynamical Systems. World Scientific, Singapore (2006); Ng, G.W.: Application of Neural Networks to Adaptive Control of Nonlinear Systems. Research Studies Press, London (1997); Emelyanov S.V., Burovoi, A., Levada, F.Y.: Control of Indefinite Nonlinear Dynamic Systems: Induced Internal Feedback. Springer, London (1998); Fradkov, A.L., Miroshnik, I.V., Nikiforov, V.O.: Nonlinear and Adaptive Control of Complex Systems. Kluwer Acad. Publ., Dordrecht (1999); Krstic, M., Kanellakopoulos, I., Kokotovic, P.V.: Nonlinear and Adaptive Control Design. J. Wiley and Sons, New-York (1995); Francis, B.A., Tanenbaum, A.R. (eds.) : Feedback Control, Nonlinear Systems and Complexity. Springer, New-York (1995); Solodovnikov, V.V., Tumarkin, V.I.: Complexity Theory and the Design of Control Systems, Theory and Methods of Systems Analysis, vol. 28. Nauka, Moscow (1990); Wang, L.X.: Adaptive Fuzzy Systems and Control. Prentice Hall, Englewwod Cliffs (1994)
22. Deimling, K.: Nonlinear Functional Analysis. Springer, Berlin (1985); Krasnoselskii, M.A.: Asymptotics of Nonlinearities and Operator Equations. Birkhauser, Boston (1995); Vainberg, M.M.: Variational Methods for the Study of Nonlinear Operators. Holden−Day, San Francisco (1964); Zeidler, E.: Nonlinear Functional Analysis and its Applications, vol. I-III-IV. Springer, New−York (1985-1986-1988); Bendat, J.S.: Nonlinear Systems Techniques and Applications. J. Wiley and Sons, New−York (1998); Bobylev, N.A., Burman, Y.M., Korovin, S.K.: Nonlinear Analysis and Applications. Walter de Gruyter, Berlin (1994); Bogaesvski, V.N., Povzner, A.: Algebraic Methods in Nonlinear Perturbation Theory. Springer, New−York (1991); Hale, J.K., Verduyn Lunel, S.M.: Theory of Functional Differential Equations. Springer, New-York (1993)

23. Amerio, L., Prouse, G.: Almost−Periodic Functions and Functional Equations. Van Nostrand-Reinhold, New−York (1971); Mazja, V.G.: Sobolev Spaces. Springer, New−York (1985); Rao, M.M., Ren, Z.D.: Theory of Orlicz Spaces. Marcel Dekker, New−York (1991)
24. Jiang, T.H.: Fixed−Point Theory. Springer, New−York (1985); Zeidler, E.: Nonlinear Functional Analysis and its Applications, vol. I. Springer, New−York (1986); Burton T.A.: Stability by Fixed Point Theory for Functional Differential Equations. Dover Publ., New−York (2006)
25. Cotsaftis, M.: Recent Advances in Control of Complex Systems, Survey Lecture. In: Proc. ESDA 1996, Montpellier, France, ASME, vol. I, p. 1 (1996); Cotsaftis, M.: Comportement et Contrôle des Systèmes Complexes, Diderot, Paris (1997); Cotsaftis, M.: Popov Criterion Revisited for Other Nonlinear Systems. In: Proc. ISIC 2003 (International Symposium on Intelligent Control), October 5-8, Houston, Texas (2003)
26. Cotsaftis, M.: A Passage to Complex Systems. In: Proc. CoSSoM 2006, Toulouse, France, September 24-27 (2006); to be published in Springer Series “Understanding Complex Systems”
27. Cotsaftis, M.: Beyond Mechatronics, Toward Global Machine Intelligence. In: Proc. ICMT 2005, Kuala–Lumpur, December 6-8 (2005); Kosko, B.: Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence. Prentice−Hall, Englewood Cliffs (1991)
28. Cotsaftis, M.: From Trajectory Control to Task Control – Emergence of Self Organization in Complex Systems. In: Aziz-Alaoui, M.A., Bertelle, C. (eds.) Emergent Properties in Natural and Artificial Dynamical Systems, pp. 3–22. Springer, Heidelberg (2006); also: On the Definition of Task Oriented Intelligent Control. In: Proc. ISIC 2002 Conf., Vancouver, October 27-30 (2002)
29. Cotsaftis, M.: Merging Information Technologies with Mechatronics – The Autonomous Intelligence Challenge. In: Proc. IEECON 2006, Paris, November 6-10 (2006)
30. Cotsaftis, M.: Robust Asymptotically Stable Control for Intelligent Unknown Mechatronic Systems. In: Proc. ICARCV 2006, Singapore, December 5-8 (2006)