Antisynchronization of Two Complex Dynamical Networks

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Abstract. A nonlinear type open-plus-closed-loop (OPCL) coupling is investigated for antisynchronization of two complex networks under unidirectional and bidirectional interactions where each node of the networks is considered as a continuous dynamical system. We present analytical results for antisynchronization in identical networks. A numerical example is given for unidirectional coupling with each node represented by a spiking-bursting type Hindmarsh-Rose neuron model. Antisynchronization for mutual interaction is allowed only to inversion symmetric dynamical systems as chosen nodes.

Keywords: Antisynchronization, dynamical networks, OPCL coupling.

1 Introduction

In recent years, studies on collective behavior of nonlinear dynamical systems has inclined more to dynamical processes in complex networks [1-7] since many real-life systems, living and nonliving, show complex network topology instead of regular links like nearest-neighbor or all-to-all global coupling. A complex network consists of a large number of nodes connected by links or edges where their connectivity, instead of being random as proposed earlier [8], shows statistical properties like small-world [3-4] or scale-free effect [2, 6, 7] in real world. In a complex dynamical network, each node is considered as a dynamical system, either continuous-time or discrete time. Understanding the process of collective behavior or synchronization in a crowd of dynamical nodes within a complex topology then becomes interesting and important [9, 10] to explain many real world phenomena in engineering networks [11] like Internet, World Wide Web, World Trade Web and in biological networks like neurons in brain, pacemaker cells in heart and genetic networks [9]. Particularly, the nodes of the complex networks are assumed as dynamical, as for example in biological systems, which evolve with time. In this context, synchronization in complex networks called as *inner synchronization* has been investigated [9, 10-13] recently to understand the interplay between dynamics of nodes and the topology of a

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complex network. In *inner synchronization*, all the nodes have a common dynamics both in amplitude and phase. Establishing conditions of synchronization and desynchronization between two or more networks is an important task of practical relevance [14]. Inducing desynchronization in networks is important from a viewpoint of overcrowding or jamming in networks like Internet, which may be avoided by breaking a state of synchrony within a network. Here, we address a process of antisynchronization in two complex dynamical networks, which may work as an alternative to desynchronization. In a state of antisynchronization in two dynamical networks, the evolution of each of the nodes of a network is locked with the corresponding node of another network in alternate time.

A complex dynamical network is described by

$$\dot{x}_i = f(x_i) + \varepsilon \sum_{j=1}^{N} a_{ij} \Gamma x_j;\ i = 1, 2, 3...., N \tag{1}$$

where $f : R^n \to R^n$ is a continuous dynamical flow that governs the local dynamics of each uncoupled node i and $x_i = (x_{i1}, x_{i2},....., x_{in})^T \in R^n$ is the state variable of a node; N is the number of nodes. The matrix A=(a_{ij})$\in R^{N\times N}$ defines the connectivity of nodes in a network whose entries follow a rule: if there is a connection between the nodes i and j ($j\neq i$), then a_{ij}=1; otherwise a_{ij}=0 ($j\neq i$); the diagonal elements of A are defined as $a_{ii} = -\sum_{j=1, j\neq i}^{N} a_{ij} = -\sum_{j=1, j\neq i}^{N} a_{ji}$, and clearly if the degree of i^{th} node is k_i then $a_{ii}=-k_i$, i=1, 2, 3,…, N. ε>0 is the coupling strength between the nodes of individual networks. $\Gamma \in R^{n\times n}$ is a constant diagonal matrix whose elements are 0 or 1 and it defines the links between the state variables of any two nodes. In a Γ matrix, if all the elements are 1, then any pair of nodes is connected by all state variables, otherwise they are partially connected if any of the elements is zero. A synchronous state of all nodes of the network is defined by $\dot{x} = f(x)$. We are concerned here with the process of synchronization, particularly, antisynchronization and how to implement them in two complex dynamical networks.

Synchronization of two complex dynamical networks was reported earlier [15] using a master-slave type unidirectional open-plus-closed-loop (OPCL) coupling [16]. In a recent Letter [17], we extended the OPCL method to establish antisynchronization and amplification or attenuation in two chaotic oscillators. Here, we extend the results further to achieve antisynchronization and/or attenuation in two dynamical networks. Once the node dynamics is known, one can design an appropriate coupling using the OPCL scheme to realize antisynchronization or to attenuate any undesired effect in one dynamical network from being transmitted to another response dynamical network.

The relevance of synchronization between two dynamical networks was explained in [15] by citing an interesting example of two economic worlds: one developing and another developed. It explained how a developed economy influences the developing world economy and considered unidirectional influence while studying synchronization in two such networks. Although this unidirectional effect is strongly felt in a recent economic crisis in the United States that is followed by immediate crash in the share market network of many countries, the reality is more complex. It is natural that both the economies (developed or developing) influence each other to evolve a new world economic order. It is obviously more realistic to consider mutual interactions between the networks either economic networks or social networks to derive a true picture. Accordingly, in addition to the unidirectional effect, we address the mutual OPCL coupling issue to realize antisynchronization in two complex dynamical networks.

The paper is structured as follows. In the next section, antisynchronization using unidirectional OPCL coupling in two oscillators is described. The theory is then extended to complex dynamical network in section 3. Mutual interaction in two dynamical networks is described in section 4. Results are summarized in section 5.

2 Antisynchronization in Two Oscillators

We briefly introduce the general scheme [16, 17] of unidirectional OPCL coupling in two chaotic oscillators: a chaotic driver is defined by $\dot{y} = f(y), \quad y \in R^n$. The model of the chaotic oscillator with parameters is assumed known *a priori*. It drives another chaotic oscillator $\dot{x} = f(x), x \in R^n$ to achieve a goal dynamics $g(t)=\alpha y(t)$ as a desired response, where α is a constant. The response system after coupling is given by

$$\dot{x} = f(x) + D(x, g). \tag{2}$$

where the coupling term $D(x, g)$ is defined by

$$D(x, g) = \dot{g} - f(g) + \left(H - \frac{\partial f(g)}{\partial g} \right)(x - g). \tag{3}$$

$\frac{\partial f(g)}{\partial g}$ is the *Jacobian* and H is an arbitrary constant Hurwitz matrix (nxn), whose eigenvalues have all negative real parts. The error signal of the coupled system is defined by $e = x - g$ when $f(x)$ can be written, using the Taylor series expansion,

$$f(x) = f(g) + \frac{\partial f(g)}{\partial g}(x - g) + ... \tag{4}$$

Keeping the first order terms in (4) and substituting in (3), the error dynamics is obtained as $\dot{e} = He$ from (2) and this ensures that $e \to 0$ as $t \to \infty$ and the synchronization is asymptotically stable. The Hurwitz matrix can be easily constructed from the *Jacobian* of the known model of the interacting oscillators. The elements of the Hurwitz matrix, H_{ij} , are then chosen such that $\left(H - \frac{\partial f(g)}{\partial g} \right)_{ij}$ is zero when $\left(\frac{\partial f(g)}{\partial g}\right)_{ij}$ is a constant in (3). If $\left(\frac{\partial f(g)}{\partial g}\right)_{ij}$ involves a state variable, we define $H_{ij} = p_{ij}$ where p_{ij} is a constant. The parameter values, p_{ij}, are so selected as to satisfy the Routh-Hurwitz (RH) criterion. For a 3D dynamical system, the characteristic equation of the H matrix is

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0 \tag{5}$$

where a_i (i=1, 2, 3) are coefficients.

The corresponding RH criterion [16] is given by

$$a_1>0, \quad a_1a_2>a_3, \quad a_3>0 \tag{6}$$

The selection of the parameters p_{ij} is so appropriately made that the RH criterion is fulfilled and thereby ensures synchronization that is asymptotically stable even in presence of any parameter mismatch [18]. The multiplying constant α in the goal dynamics can be used as a control parameter to realize CS (α=1), AS (α=-1), attenuation ($|\alpha| < 1$) or amplification ($|\alpha| > 1$).

3 Complex Dynamical Network: Unidirectional Coupling

We extend the unidirectional coupling scheme to complex dynamical networks to realize antisynchronization and attenuation. As described earlier [15], an analytical approach is possible to establish synchronization using OPCL coupling between two dynamical networks for identical connectivity matrix and it is found unchanged for the proposed generalization here. The driving network may be expressed by (1) and the response network is defined by

$$\dot{y}_i = f(y_i) + \alpha\dot{x}_i - f(\alpha x_i) + \left(H - \frac{\partial f(\alpha x_i)}{\partial(\alpha x_i)}\right)[y_i(t) - \alpha x_i(t)] + \varepsilon\sum_{j=1}^{N} b_{ij}\Gamma y_j \tag{7}$$

$y_i = [y_{i1}, y_{i2}, y_{i3}]^T$ is the state variable of the i^th^ node of the response network; other notations have similar meaning as above, and A=(a_{ij}) $\in R^{n\times n}$, B=(b_{ij}) $\in R^{n\times n}$ are symmetric or asymmetric matrices; each row sum of A and B equal to zero. Networks (1) and (7) achieve synchronization if

$$\lim_{t\to+\infty} \|y_i(t) - \alpha x_i(t)\| = 0, \; i = 1, 2, 3..., \mathrm{N} \tag{8}$$

For simplification, we assume two networks having identical topology (A=B). Then linearizing the error system, $e_i(t) = y_i(t) - \alpha x_i(t)$, around x_i , we obtain

$$\dot{e}_i = He_i + \varepsilon\sum_{j=1}^{N} a_{ij}\Gamma e_j; \; i = 1, 2, 3...., N \tag{9}$$

which can be simplified as

$$\dot{e} = He + \varepsilon \Gamma e A^T \tag{10}$$

T stands for transpose and $e = [e_1, e_2, ..., e_N]$ denotes $n \times N$ matrix. The coupling matrix may be decomposed by taking $A^T = SJS^{-1}$ where J is a Jordan canonical form with complex eigenvalues $\lambda \in C$ and S contains the corresponding eigenvectors. If we define, $\eta = eS$, using eq.(10), we can easily derive

$$\dot{\eta} = H\eta + \varepsilon \Gamma \eta J \tag{11}$$

where $J = [J_1, J_2, ..., J_h]^T$ is a block diagonal matrix and J_k is a block corresponding to the m_k multiple eigenvalues λ_k of A.

$$J_k = \begin{bmatrix} \lambda_k & 1 & 0 & ... & 0 \\ 0 & \lambda_k & 1 & ... & 0 \\ . & . & . & . & . \\ 0 & 0 & . & \lambda_k & 1 \\ 0 & 0 & .... & 0 & \lambda_k \end{bmatrix} \tag{12}$$

Assuming $\eta = [\eta_1, \eta_2, .., \eta_h]^T$, $\eta_k = [\eta_{k,1}, \eta_{k,2}, .., \eta_{h,m_k}]^T$ and, since the sum of every row of the matrix A is zero and J_1 is a 1×1 matrix, we can assume λ_1=0. Now if λ_1=0, it satisfies $\dot{\eta}_1 = H\eta_1$ and hence the zero solution of $\dot{\eta}_1 = H\eta_1$ is asymptotically stable if H is a Hurwitz matrix. In this way one can easily establish asymptotic stability of all zero solutions for k>1; details may be found in [15] that confirms synchronization of the dynamical networks (1) and (7) once A=B and H is a Hurwitz matrix. The analysis presented in ref.15 remains unaffected by the introduction of the parameter α, where we set a goal dynamics at each node of the networks as $y(t)=\alpha x(t)$. Hence we can realize synchronization (α=1), antisynchronization (α=−1) or attenuation ($|\alpha < 1|$) simply by a choice of the α-value.

We present a numerical example where each i^{th} node of both the networks is described by spiking-bursting type Hindmarsh-Rose neuron model [19],

$$\begin{aligned} &\dot{x}_{i1} = x_{i2} - a x_{i1}^3 + b x_{i1}^2 - x_{i3} + I, \quad \dot{x}_{i2} = c - d x_{i1}^2 - x_{i2}, \\ &\dot{x}_{i3} = r\{s(x_{i1} + 1.6) - x_{i3}\}. \end{aligned} \tag{13}$$

where a=1, b=3, c=1.0, d=5.0 and s=5.0. The state variables x_{i1} and x_{i2} correspond to fast oscillation and x_{i3} represents the slow dynamics as decided by a choice of r=0.003. The bias current I=4.1 sets the oscillatory mode in a chaotic regime. The H matrix of the model (13) is given by

$$H = \begin{bmatrix} p_1 & 1 & -1 \\ p_2 & -1 & 0 \\ rs & 0 & -r \end{bmatrix} \tag{14}$$

where p_1 and p_2 are parameters. It can be analytically established [16, 17] that if p_2=0 and p_1<1+r, H is a Hurwitz matrix with eigenvalues all with negative real parts. We set the Γ=diag(1, 0, 0) to establish a scalar coupling between the nodes within the individual networks. In [15], the authors assumed that all state variables of each node of the individual networks were coupled. It is found, in numerical simulations, that scalar coupling or fewer coupling as set by Γ=diag(1, 0, 0) suffices to realize synchronization or antisynchronization. We choose two undirected networks each having N=10 nodes, where A is given by

$$A=\begin{bmatrix} -4 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & -5 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & -5 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & -4 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & -5 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & -5 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 1 & -6 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & -5 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & -4 & 1 \\ 1 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & -5 \end{bmatrix} \tag{15}$$

Since the coupling matrix A is symmetric, its first eigenvalue is zero and the rest are negative. Once the parameter p_1<1+r (p_2=0) is ensured, eq.(10) confirms that the real parts of the eignenvalues of $He+\varepsilon\lambda_k\Gamma$ (λ_k is the set of eigenvalue of A) are negative for arbitrary value of p_1<1+r. Networks (1) and (7) will develop synchronization for α=1 when each node of the network (1) develops an identical dynamics with each corresponding node of the network (7). This result is already reported earlier [15], however, we introduced a general framework here to choose any desired value of α. As a result, antisynchronization can also be established by a choice of α=-1, when corresponding nodes of the networks develop identical dynamics but in opposite phase as shown in Fig.1. Attenuating the amplitude of the dynamics in a driver network is also possible at a response network by simply choosing ($|\alpha|<1$), details of which are redundant. For numerical simulations, the initial conditions are randomly chosen and the synchronization error is measured by

$$\begin{aligned} \|e(t)\| = \max\{ & \max_{1\le i\le 10}|x_{i1}(t)\mp y_{i1}(t)|, \\ & \max_{1\le i\le 10}|x_{i2}(t)\mp y_{i2}(t)|, \\ & \max_{1\le i\le 10}|x_{i3}(t)\mp y_{i3}(t)|\}, \text{ for } t\in[0,+\infty). \end{aligned} \tag{16}$$

Minus (-) sign denotes synchronization and plus (+) sign for antisynchronization.

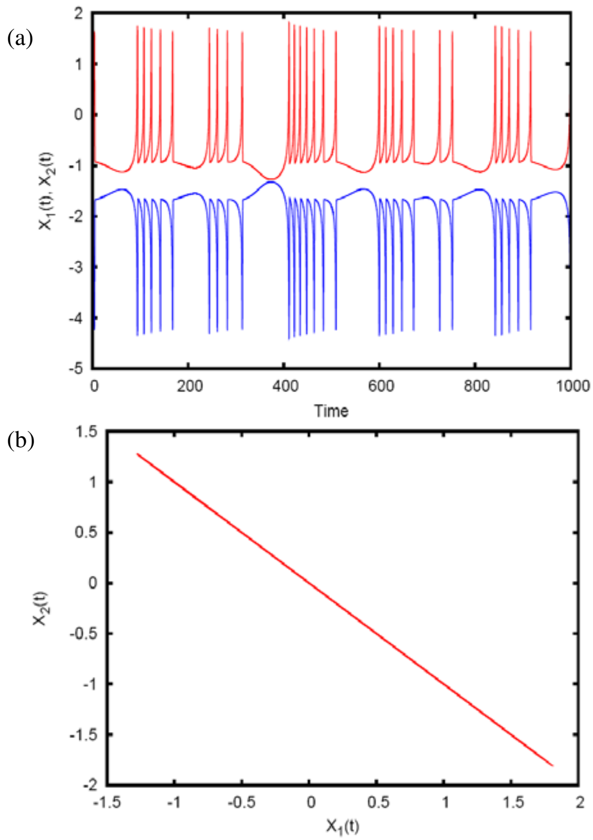

Fig. 1. Antisynchronization in complex networks, (a) time series [x_{i1}(t) in red and y_{i1}(t) in blue] in any two corresponding nodes of the driver and response networks, (b) state variable x_{i1}(t) of a node of driver network is plotted against state variable y_{i1}(t) of a corresponding node in the response. p_1=-1.5, ε=10^{-6}.

Synchronization between the networks is independent of *inner synchronization* of individual networks. It is obtained even for very low value of ε=10^{-6} when there is no *inner synchronization.* The synchronization between the two networks is fastest when there is no *inner synchronization*. The speed of synchronization is shown in Fig.2 for different ε -values. For larger coupling ε>0.4, the speed is not much changing with increase in coupling (ε). Similarly, antisynchronization can also be achieved for nonsymmetric A, i.e., when the inner connectivity of the network is directed. We obtained antisynchronization for A≠B in similar vain as described in [15], however, no analytical treatment is possible; numerical results is only done, for which details are redundant since it is almost a repetition of the results in ref.15. We rather prefer to extend the results to mutual interactions in two complex dynamical networks. Note that the results are checked with larger number of nodes in the networks (N=100).

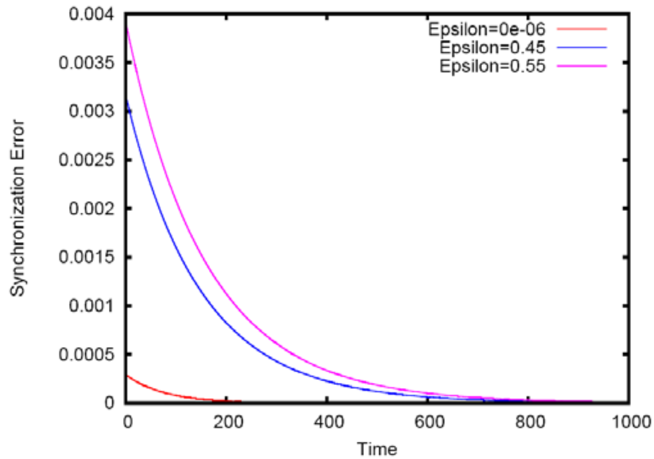

Fig. 2. Dependence of synchronization with coupling. $\varepsilon=10^{-6}$, 0.45 and 0.55, p_1=-1.5, p_2=0

4 Complex Networks: Mutual Coupling

Synchronization for mutual or bidirectional interactions in two complex networks was not investigated in the previous study [15]. We develop the theory of synchronization in dynamical networks for mutual interaction using the OPCL coupling scheme and then extend it to antisynchronization. Note that the mutual OPCL coupling in two chaotic oscillators was reported earlier [20, 21] for synchronization, but antisynchronization was never investigated. A modification in the theory is needed to realize mutual antisynchronization in two chaotic oscillators, however, it is found limited to inversion symmetric dynamical systems only. Details of antisynchronization using mutual OPCL coupling are reported elsewhere [22]. Two oscillators under mutual OPCL coupling are given by

$$\dot{x} = f(x) + D_x(x, y)\ ; \quad x \in R^n\ , \tag{17}$$

$$\dot{y} = f(y) + D_y(x, y)\ ; \quad y \in R^n\ ,$$

where $$D_x(x, y) = \left(H - \frac{df}{dx}\bigg|_{x = s_+} \right)\left(\frac{x-y}{2}\right)\ , \tag{18}$$

and $$D_y(x, y) = \left(H - \frac{df}{dy}\bigg|_{y = s_+} \right)\left(\frac{y-x}{2}\right)\ , \tag{19}$$

$$s_+(t) = \left(\frac{x(t) + y(t)}{2} \right)$$ is the synchronization manifold.

It can be easily established [21] that the error dynamics e=(x-y) is now governed by $\dot{e} = He$ and its zero error solution or the synchronization is asymptotically stable once

H is a Hurwitz matrix by an appropriate choice of the parameters. For realizing antisynchronization, we modify the coupling terms in (18) and (19) by

$$D_x(x,y) = \left(H - \frac{df}{dx}\bigg|_{x=s_-} \right)\left(\frac{x+y}{2}\right), \tag{20}$$

$$D_y(x,y) = \left(H - \frac{df}{dy}\bigg|_{x=s_-} \right)\left(\frac{x+y}{2}\right), \tag{21}$$

and antisynchronization manifold is $s_-(t) = \left(\frac{x(t)-y(t)}{2}\right)$.

To realize antisynchronization, an additional condition $f(y) = -f(-y)$ is necessary to be satisfied, which actually defines inversion symmetry of any dynamical flow. The asymptotically stable antisynchronization is then ensured once H is a Hurwitz matrix. The error dynamics is again governed by $\dot{e} = He$ where the error state is $e = (x+y)$ for antisynchronization. The antisynchronization in two dynamical networks is thus restricted by the inversion symmetry property of a dynamical node as also reported earlier [23] for two chaotic oscillators. We define two mutually coupled complex dynamical networks by

$$\dot{x}_i = f(x_i) + D_x(x_i, y_i) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j;\ \mathrm{i} = 1, 2, \ldots, \mathrm{N} \tag{22}$$

$$\dot{y}_i = f(y_i) + D_y(x_i, y_i) + c\sum_{j=1}^{N} b_{ij}\Gamma y_j;\ \mathrm{j} = 1, 2, \ldots, \mathrm{N} \tag{23}$$

where

$$D_x(x_i,y_i) = \left(H - \frac{df}{dx_i}\bigg|_{x_i=s_i} \right)\left(\frac{x_i \mp y_i}{2}\right), \tag{24}$$

$$D_y(x_i,y_i) = \left(H - \frac{df}{dy_i}\bigg|_{y_i=s_i} \right)\left(\frac{y_i \mp x_i}{2}\right), \tag{25}$$

and

$$s_i = \left(\frac{x_i \mp y_i}{2}\right)$$

The mathematical structure of the error dynamics remains similar to (10) and (11). All notations and their meanings remain same as earlier. Once the connectivity matrix is again assumed symmetric (A=B), the analytical approach in (10)-(11) for unidirectional coupling remains same for implementing synchronization and antisynchronization in complex networks under mutual coupling and hence we do not repeat them here. However, in numerical examples, we take a model system [24, 25] that is inversion symmetric. The model of the dynamics of ith node of a network is

$$\dot{x}_{i1} = 0.49x_{i2} - x_{i3},\ \dot{x}_{i2} = x_{i1} - x_{i2},\ \dot{x}_{i3} = x_{i1}^3 - x_{i2}. \tag{26}$$

We confirm antisynchronization in two mutually coupled complex dynamical networks in Fig.3 using each node as represented by the model (26). Synchronization in two mutually interacting complex networks using the Hindmarsh-Rose model representing unit node dynamics is achieved in numerical simulations but details are not presented here.

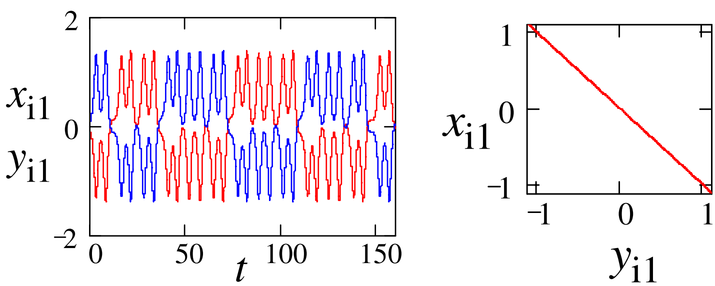

Fig. 3. Antisynchronization of complex dynamical networks for mutual coupling. x_{i1} and y_{i1} are the similar state variables of corresponding *i*th nodes of the driver and response networks, (a) time series of x_{i1} and y_{i1} in red and in blue, (b) x_{i1}(t) is plotted against y_{i1}(t).

5 Summary

We focused on antisynchronization in two complex dynamical networks for unidirectional as well as bidirectional interactions using OPCL coupling. We mainly extended the previous results [15] on synchronization in two complex dynamical networks under unidirectional interaction. However, we encounter one limitation in realizing antisynchronization in mutually coupled networks. The dynamical flow at each node of the networks must have the inversion symmetry property. While the unidirectional OPCL coupling has no such restriction in inducing anytisynchronization, but its bidirectional version fails to overcome this restriction. The synchronization between the networks is independent of inner synchrony of the individual network. It is interesting to note that the speed of synchrony is faster when there is no *inner synchronization* in the individual networks.

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