Global Synchronization of Generalized Complex Networks with Mixed Coupling Delays

Yang Dai[1], Yunze Cai[1], and Xiaoming Xu[1,2,3]

[1] Department of Automation, Shanghai Jiao Tong University, P.R. China
daiyang1980@gmail.com
[2] University of Shanghai For Science and Technology, P.R. China
[3] Shanghai Academy of Systems Science, P.R. China

Abstract. In this paper we propose a generalized complex networks model, which concerns asymmetric network configuration including both neutral-type coupling delay and retarded-type one. The synchronization problem of this generalized complex networks is reformulated into the asymptotical stability problem of neutral delay functional differential equations. By introducing descriptor system transformation strategy, the less conservative sufficient condition of delay-independent and independent-of-delay global synchronization criteria are derived in terms of linear matrix inequalities. A numerical example is given to support the theoretical results.

Keywords: Complex Networks, Synchronization, Retarded Delay, Neutral Delay, Linear Matrix Inequalities (LMI).

1 Introduction

During the past decade, complex networks have attracted a lot of interests in the fields of biology, physics, chemistry, engineering and human society [1,2,3,4,5,6]. With each unit regarded as a node, many nonlinear coupling nodes form the complex networks, which describe the sophisticated properties of many systems in nature. As a significant collective behavior, synchronized dynamical propensity in large networks of coupled units has been widely investigated by many researchers [7,8,9,10,11,12,13,14,15,16], since synchronization has potential applications in such places as semiconductor lasers, secure communication and electronic circuits.

Among the methods of synchronization analysis, a common technique is to linearize the dynamical nodes with respect to certain synchronized state and then some local synchronization criteria are derived based on this linearized model [8,9,10,11]. In some situations, if the synchronized state is unavailable in advance, one cannot implement linearization technique. It should be further noticeable that this approach does not solve the inherent nonlinear difficult problem of complex networks. In order to overcome this disadvantage, the global synchronization of complex networks was studied [12,13,14,15,16]. For simplicity, most of these papers considered that the network has symmetric topology structure.

J. Zhou (Ed.): Complex 2009, Part I, LNICST 4, pp. 1001–1010, 2009.

This paper focuses on a more generalized model which may be weighted network, directed network or any other configurations if only coupling matrix can be explicitly represented.

On the other hand, time delays occur commonly in the process of synchronization due to the limited transmitting capability and possible network traffic congestions. In the previous work, complex networks with coupling delay were modeled as interacting retarded delay functional differential equations [9,17,18]. However, neutral delay functional differential equations may better describe the realistic networks in the case of population ecology, distributed networks containing lossless transmission lines, heat exchangers and financial market [19,20,21]. For instance, when the complex dynamical networks are used to model a stock transaction system, each node's state is defined as an agent's behavior such as buying, selling and holding, which is driven dynamically by judging the situation at the time and the historical fluctuating rate records. Hence a complex dynamical networks model with neutral coupling delay is established [22]. In this note, we address the question of how to choose the value of coupling delay such that a given network may achieve synchronous behavior. In order to obtain the less conservative sufficient conditions, descriptor system transformation strategy is utilized during the derivative of the criteria.

The rest of paper is organized as follows. In section 2, a general complex dynamical networks with mixed coupling delays and some useful lemmas are given. In Section 3, descriptor system transformation strategy is introduced to reformulate the network model, then delay-dependent and independent-of-delay synchronization criteria are presented based on this novel model. In Section 4, a numerical example is given to illustrate the main results of this paper. Finally, conclusions are given in Section 5.

Notation: The notation used throughout the paper is fairly standard. The superscripts $'T'$ stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $P > 0$ (≥ 0) means P is real symmetric and positive definite (semi-definite). In symmetric block matrices, we use an asterisk $(*)$ to represent a term that is induced by symmetry and diag$(\ldots)$ stands for a block-diagonal matrix. The norm of a vector or matrix is denoted by $\|\cdot\|$, and $\rho(\cdot)$ denotes the spectral radius of a matrix.

2 Complex Dynamical Networks Model and Preliminaries

Consider the ensemble of N identical diffusively coupled nodes, with each one being an n-dimensional dynamical system. The proposed complex dynamical networks model is described by

$$\dot{x}_i(t) = Ax_i(t)+f(x_i(t))+\sum_{j=1}^{N} G_{ij}(Bx_j(t-\tau)+C\dot{x}_j(t-\tau)), \quad i = 1,\ldots,N \quad (1)$$

where $x_i = (x_{i1},\ldots,x_{in})^T \in \mathbb{R}^n$ is the state variable of node i, $t \in \mathbb{R}$ is the continuous time variable, f is continuously differentiable map, τ is the positive constant coupling delay between nodes (it is assumed that the network has identical retarded delay and neutral delay), $B = (a_{ij}) \in \mathbb{R}^{n\times n}$ is a constant inner coupling matrix of the nodes about the retarded delay and $C = (b_{ij}) \in \mathbb{R}^{n\times n}$ regarding to the neutral one, $G = (G_{ij}) \in \mathbb{R}^{N\times N}$ is the outer-coupling matrix combining both configuration and weights of the entire networks.

When $C = 0$, the system model (1) becomes the retarded-type delay dynamical networks as [9]

$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t), t) + \sum_{j=1}^{N} G_{ij}Bx_j(t-\tau) \qquad i = 1, \dots, N \tag{2}$$

Further when $B = 0$ and $\tau = 0$, the system model (1) turns into the simple uniform dynamical networks proposed by Wang and Chen [23]

$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t), t) + \sum_{j=1}^{N} G_{ij}Bx_j(t) \qquad i = 1, \dots, N \tag{3}$$

Therefore, (1) is a general complex networks model, with (2) and (3) as the special case.

Let $s(t)$ be the synchronized state of the generalized networks satisfying $\dot{s}(t) = As(t) + f(s(t))$, which may be an equilibrium, aperiodic trajectory or a chaotic attractor of the uncoupled dynamical behavior of each node.

In the sequel, we present some definitions and useful lemmas required throughout the paper.

Definition 1. *The complex dynamical networks (1) is said to be globally asymptotically synchronized if for any initial function $\varphi_i(\theta)$ and $\dot{\varphi}_i(\theta)$, it holds:*

$$\lim_{t\to\infty} \|x_i(t) - \mathrm{s}(t)\| = 0, \qquad i = 1, \dots, N. \tag{4}$$

Lemma 1. *[24](Schur Complement) Suppose A_1, A_2, A_3 are respectively $p \times p$, $p \times q$ and $q \times q$ matrices, and A_1 is invertible, then the inequality*

$$\begin{pmatrix} A_1 & A_2 \\ A_2^T & A_3 \end{pmatrix} > 0$$

is equivalent to the following two inequalities

$$A_1 > 0, A_3 - A_2^T A_1^{-1} A_2 > 0$$

Lemma 2. *([25]) For the matrices $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{p\times q}$, the Kronecker product of A and B is a $mp \times nq$ matrix defined as*

$$A \otimes B \triangleq \begin{pmatrix} a_{11}B & a_{12}B & \dots & a_{1n}B \\ a_{21}B & a_{22}B & \dots & a_{2n}B \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1}B & a_{n2}B & \dots & a_{mn}B \end{pmatrix}$$

3 Main Results

First, the global synchronization of general complex networks is transferred into the stability of $N \times n$ dimensional neutral-type time delay systems. Then delay-dependent and independent-of-delay synchronization criteria are derived.

3.1 Model Transformation

Let $\mathbf{x}(t) = [x_1^T(t), \ldots, x_N^T(t)]^T$, $\mathbf{F}(\mathbf{x}(t)) = [f^T(x_1(t)), \ldots, f^T(x_N(t))]^T$, then the coupled dynamical network (1) can be rewritten as the compact form

$$\dot{\mathbf{x}}(t) = \bar{A}\mathbf{x}(t) + \mathbf{F}(\mathbf{x}(t)) + \bar{B}\mathbf{x}(t-\tau) + \bar{C}\dot{\mathbf{x}}(t-\tau), \tag{5}$$

where $\bar{A} = (I_N \otimes A)$, $\bar{B} = (G \otimes B)$, $\bar{C} = (G \otimes C)$, I_N is a $N \times N$ dimension identity matrix.

Define the synchronous error state of node i with respect to $s(t)$ as $e_i(t) = x_i(t) - s(t)$ and $\mathbf{e}(t) = [e_1^T(t), \ldots, e_N^T(t)]^T$, then the error dynamical network is denoted by:

$$\dot{\mathbf{e}}(t) = \bar{A}\mathbf{e}(t) + \mathbf{F}(\mathbf{e}(t)) + \bar{B}\mathbf{e}(t-\tau) + \bar{C}\dot{\mathbf{e}}(t-\tau), \tag{6}$$

where $\mathbf{F}(\mathbf{e}(t)) = \left[f^T(x_1) - f^T(s), \ldots, f^T(x_1) - f^T(s)\right]^T$. It may be seen that the asymptotical stability of neutral time delay system (6) is equivalent to the global synchronization of the complex networks (1) with respect to s(t).

Many efforts have been made to obtain less conservative conditions when the Lyapunov-Krasovskii theory is employed [26]. There are four basic fixed transformation methods, among which the descriptor system transformation method introduced by Fridman [27] is much better because this method may attain less conservative criteria than the others. Thus descriptor system transformation strategy is introduced in the derivation of the global synchronization criterion for generalized complex networks.

Eq. (5) may be represented in the equivalent descriptor form as follows:

$$\begin{aligned} &\dot{\mathbf{e}}(t) = \mathbf{y}(t), \\ &\mathbf{y}(t) = \bar{A}\mathbf{e}(t) + \mathbf{F}(\mathbf{e}(t)) + \bar{B}\mathbf{e}(t-\tau) + \bar{C}\dot{\mathbf{e}}(t-\tau), \end{aligned} \tag{7}$$

According to the Leibniz-Newton formula $\mathbf{e}(t) - \mathbf{e}(t-\tau) = \int_{t-\tau}^{t} \mathbf{y}(s)ds$, then we have:

$$\begin{aligned} &\dot{\mathbf{e}}(t) = \mathbf{y}(t), \\ &0 = -\mathbf{y}(t) + (\bar{A} + \bar{B})\mathbf{e}(t) + \mathbf{F}(\mathbf{e}(t)) + \bar{C}\dot{\mathbf{e}}(t-\tau) + \bar{B}\int_{t-\tau}^{t} \mathbf{y}(s)ds \end{aligned} \tag{8}$$

It is well known that for a neutral delay system to be stable, it is necessary that its neutral part must be stable. This requirement concerns the following hypothesis.

Assumption 1 (A1). The operator is defined as

$$\mathcal{D}\mathbf{e}(t) = \mathbf{e}(t) - \bar{C}\mathbf{e}(t-\tau)$$

Suppose that $\rho(\bar{C}) \leq 1$, then the operator $\mathcal{D}\mathbf{e}(t)$ is stable.

Assumption 2 (A2). Suppose that there exist a set of constant $\sigma_k \geq 0$ ($k = 1, \ldots, n$) such that the nonlinear part of uncoupled node $f(\cdot)$ satisfies local Lipschitz condition:

$$|f_k(a) - f_k(b)| \leq \sigma_k |a - b|, \quad a, b \in \mathbb{R} \tag{9}$$

Under **A2**, the existence and uniqueness of the solution to Eq.(5) is guaranteed.

Let $\Sigma = \text{diag}\{\sigma_k, \ldots, \sigma_n\}$ and $L = I_N \otimes \Sigma$. For a non-negative definite matrix S, there exists a decomposition $S = U^T \Lambda U$ where $U = [\mathbf{u}_1, \ldots, \mathbf{u}_N]^T$, $\mathbf{u}_i = [u_{i1}, \ldots, u_{in}]^T$ and $\Lambda = \text{diag}\{\lambda_1, \ldots, \lambda_{N\times n}\}$ with $\lambda_i \geq 0$ ($i = 1, \ldots, N$). According to Eq.(9), it may be readily deduced that:

$$\begin{aligned} M(t) &= \mathbf{e}^T(t) LSL\mathbf{e}(t) - F^T(\mathbf{e}) SF(\mathbf{e}) \\ &= \mathbf{e}^T(t) LU^T \Lambda UL\mathbf{e}(t) - F^T(\mathbf{e}) U^T \Lambda UF(\mathbf{e}) \\ &= \sum_{i=1}^{N} \sum_{j=1}^{n} \lambda_{i\times j} \left[e_{ij}^2 \sigma_j^2 - f_j^2(e_{ij}) \right] \\ &< 0 \end{aligned} \tag{10}$$

3.2 Synchronization Criteria

For the given dynamical network, we can establish the following delay-dependent synchronization criterion in terms of linear matrix inequality.

Proposition 1. *Under A1 and A2, the states of complex networks (1) is globally asymptotically synchronized for a given scalar $\tau > 0$, if there exist positive definite matrices $P_1 = P_1^T > 0$, $Q = Q^T > 0$, $R = R^T > 0$, non-negative definite matrix $S \geq 0$, and any matrices P_2, P_3 such that the following LMI is feasible:*

$$\begin{pmatrix} (\bar{A}+\bar{B})^T P_2 + P_2^T(\bar{A}+\bar{B}) + LSL & * & * & * & * \\ P_1 - P_2 + P_3^T(\bar{A}+\bar{B}) & -P_3 - P_3^T + Q + \tau R & * & * & * \\ \tau P_2^T \bar{B} & \tau P_3^T \bar{B} & -\tau R & * & * \\ P_2^T \bar{C} & P_3^T \bar{C} & 0 & -Q & * \\ 0 & 0 & 0 & 0 & -S \end{pmatrix} < 0 \tag{11}$$

Proof. Choose the Lyapunov-Krasovskii functional as follows:

$$V(\mathbf{e}_t, t) = V_1 + V_2 + V_3 \tag{12}$$

where

$$V_1 = \left[\mathbf{e}^T(t) \; \mathbf{y}^T(t) \right] EP \begin{bmatrix} \mathbf{e}(t) \\ \mathbf{y}(t) \end{bmatrix} = \mathbf{e}^T(t) P_1 \mathbf{e}(t) \tag{13}$$

in which $E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$, $P = \begin{pmatrix} P_1 & 0 \\ P_2 & P_3 \end{pmatrix}$, $P_1 = P_1^T > 0$

$$V_2 = \int_{t-\tau}^{t} \mathbf{y}^T(s) Q\mathbf{y}(s) ds, \qquad Q > 0 \tag{14}$$

$$V_3 = \int_{-\tau}^{0}\int_{t+\theta}^{t} \mathbf{y}^T(s)R\mathbf{y}(s)ds, \qquad R > 0 \tag{15}$$

The time derivative of $V(\mathbf{e}_t, t)$ along the trajectories of (8) is given by

$$\dot{V}(\mathbf{e}_t, t) = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \tag{16}$$

From (13) to (15), we have

$$\begin{aligned}\dot{V}_1 &= 2\mathbf{e}(t)P_1\dot{\mathbf{e}}(t) = 2\left[\mathbf{e}^T(t)\ \mathbf{y}^T(t)\right]P^T\begin{bmatrix}\mathbf{e}(t)\\0\end{bmatrix}\\ &= 2\left[\mathbf{e}^T(t)\ \mathbf{y}^T(t)\right]P^T\\ &\quad\times\begin{bmatrix}\mathbf{y}(t)\\ -\mathbf{y}(t)+(\bar{A}+\bar{B})\mathbf{e}(t)+F(\mathbf{e})+\bar{C}\mathbf{y}(t-\tau)-\bar{B}\int_{t-\tau}^{t}\mathbf{y}(s)ds\end{bmatrix}\end{aligned} \tag{17}$$

$$\dot{V}_2 = \mathbf{y}^T(t)Q\mathbf{y}(t) - \mathbf{y}^T(t-\tau)Q\mathbf{y}(t-\tau) \tag{18}$$

$$\dot{V}_3 = \tau\mathbf{y}^T(t)R\mathbf{y}(t) - \int_{t-\tau}^{t}\mathbf{y}^T(s)R\mathbf{y}(s)ds \tag{19}$$

Let $\xi(t) = [\mathbf{e}^T(t), \mathbf{y}^T(t), \mathbf{y}^T(t-\tau), F^T(\mathbf{e})]^T$. Consider the fact that $M(t) \geq 0$, then $\dot{V}(\mathbf{e}_t, t)$ satisfies that

$$\dot{V}(\mathbf{e}_t, t) \leq \dot{V}(\mathbf{e}_t, t) + M(t) = \xi^T(t)\Psi\xi(t) + \nu - \int_{t-\tau}^{t}\mathbf{y}^T(s)R\mathbf{y}(s)ds \tag{20}$$

where

$$\Psi = \begin{bmatrix}\Psi_{11} & P^T\begin{bmatrix}0\\ \bar{C}\end{bmatrix} & 0\\ \left[0\ \bar{C}\right]P & -Q & 0\\ 0 & 0 & -S\end{bmatrix} \tag{21}$$

$$\Psi_{11} = P^T\begin{bmatrix}0 & I\\ \bar{A}+\bar{B} & -I\end{bmatrix} + \begin{bmatrix}0 & \bar{A}^T+\bar{B}^T\\ I & -I\end{bmatrix}P + \begin{bmatrix}0 & 0\\ 0 & \tau R+Q\end{bmatrix} + \begin{bmatrix}LSL & 0\\ 0 & 0\end{bmatrix} \tag{22}$$

$$\nu = -2\int_{t-\tau}^{t}\left[\mathbf{e}^T(t)\ \mathbf{y}^T(t)\right]P^T\begin{bmatrix}0\\ \bar{B}\end{bmatrix}\mathbf{y}(s)ds \tag{23}$$

For any $Nn \times Nn$ dimension positive definite matrix $R > 0$, it holds

$$\nu \leq \tau\left[\mathbf{e}^T\ \mathbf{y}^T\right]P^T\begin{bmatrix}0\\ \bar{B}\end{bmatrix}R^{-1}\left[0\ \bar{B}^T\right]P\begin{bmatrix}\mathbf{e}^T\\ \mathbf{y}^T\end{bmatrix} + \int_{t-\tau}^{t}\mathbf{y}^T(s)R\mathbf{y}(s)ds \tag{24}$$

Combining (20) and (24), then apply Schur complements to the first term of Ψ_{11}. It yields that $\dot{V}(\mathbf{e}_t, t) < 0$ if the LMI shown in (25) are feasible.

$$\begin{bmatrix}\Psi & \tau P^T\begin{bmatrix}0\\ \bar{B}\end{bmatrix} & P^T\begin{bmatrix}0\\ \bar{C}\end{bmatrix} & 0\\ \tau\left[0\ \bar{B}^T\right]P & -\tau R & 0 & 0\\ \left[0\ C^T\right]P & 0 & -Q & 0\\ 0 & 0 & 0 & -S\end{bmatrix} < 0 \tag{25}$$

It can been easily seen that (25) is equivalent to (11). According to the Lyapunov theory, the synchronous error states of the network are asymptotically stable, which implies Proposition 1. This completes the proof. □

Remark 1. For a certain network, all the parameters are given explicitly. Applying Proposition 1, the upper bound of coupling delay τ_{max} can be obtained by solving (11) with the help of MATLAB LMI toolbox [24]. It is worth noting that $\tau \leq \tau_{max}$ is only sufficient condition to guarantee the global synchronization for the complex networks. That is, there may exist a scalar $\tau' > \tau_{max}$ such that the network can be synchronized with this coupling delay. It is an intrinsically limitation of Lyapunov theory, but there is no better substitution analysis tool for this sophisticated problem.

Remark 2. As it is seen above, the delay-dependent criterion of Proposition 1 is so powerful that the independent-of-delay case is also involved. Let $\tau = 0$, then the independent-of-delay synchronization criterion may be readily derived.

Corollary 1. *Under A1 and A2, the states of complex networks (1) is globally asymptotically synchronized for all* $0 \leq \tau < \infty$*, if there exist positive definite matrices* $P_1 = P_1^T > 0$*,* $Q = Q^T > 0$*,* $R = R^T > 0$*, non-negative definite matrix* $S \geq 0$*, and any matrices* P_2*,* P_3 *such that the following LMI is feasible:*

$$\begin{pmatrix} (\bar{A}+\bar{B})^T P_2 + P_2^T(\bar{A}+\bar{B}) + LSL & * & * & * \\ P_1 - P_2 + P_3^T(\bar{A}+\bar{B}) & -P_3 - P_3^T + Q & * & * \\ P_2^T \bar{C} & P_3^T \bar{C} & -Q & * \\ 0 & 0 & 0 & -S \end{pmatrix} < 0 \qquad (26)$$

4 Numerical Examples

In this section, a numerical example is presented to illustrate the usefulness of developed theoretical results.

Consider a 2-order nonlinear system as the dynamical node of the complex networks (1) which is described by

$$\dot{x_i} = Ax_i + f(x_i, t) \qquad (27)$$

where $A = \begin{pmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{pmatrix}$, $f(x_i,t) = [f_1(x_i), f_2(x_i)]^T$, $f_1(x_i) = 0.4(|x_{i1}+1| - |x_{i1}-1|)$, $f_2(x_i) = 0.1sin(x_{i2})$. It may be easily calculated that the Lipschiz constant matrix of function $f(\cdot)$ is $\Sigma = \mathrm{diag}\{0.8, 0.1\}$.

The dynamical networks consist of 8 identical nodes, which interconnect in the nearest-neighbor topology structure. The coupling configuration are given as follows: the inner-coupling matrices are

$$B = \begin{pmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{pmatrix}, \qquad C = \begin{pmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{pmatrix},$$

and the outer-coupling matrix is

$$G = 0.5 \times \begin{bmatrix} 2&1&0&0&0&0&0&1 \\ 1&2&1&0&0&0&0&0 \\ 0&1&2&1&0&0&0&0 \\ 0&0&1&2&1&0&0&0 \\ 0&0&1&2&1&0&0&0 \\ 0&0&0&1&2&1&0&0 \\ 0&0&0&0&0&1&2&1 \\ 1&0&0&0&0&0&1&2 \end{bmatrix}$$

The spectral radius of neutral part matrix is $\rho(G \otimes C) = 0.4 < 1$, which satisfies the precondition of Proposition 1. In sequence, the delay-dependent synchronization criterion of the network is validated. Applying Proposition 1, we can obtain the upper bound of delay $\tau_{max} = 0.22$. It guarantees that the complex networks is globally synchronized for $\tau \leq \tau_{max}$.

Define the k-th element of the state errors between node 1 and node i as $e_{ki} = x_{k1} - x_{ki}$ ($k = 1, 2$ and $i = 2, \ldots, 8$). Take the initial values of the network as random number in $[0, 1]$, then the curves of synchronous state errors for the case of $\tau = 0.1$ and $\tau = 0.8$ are shown in Fig. 1 and 2, respectively.

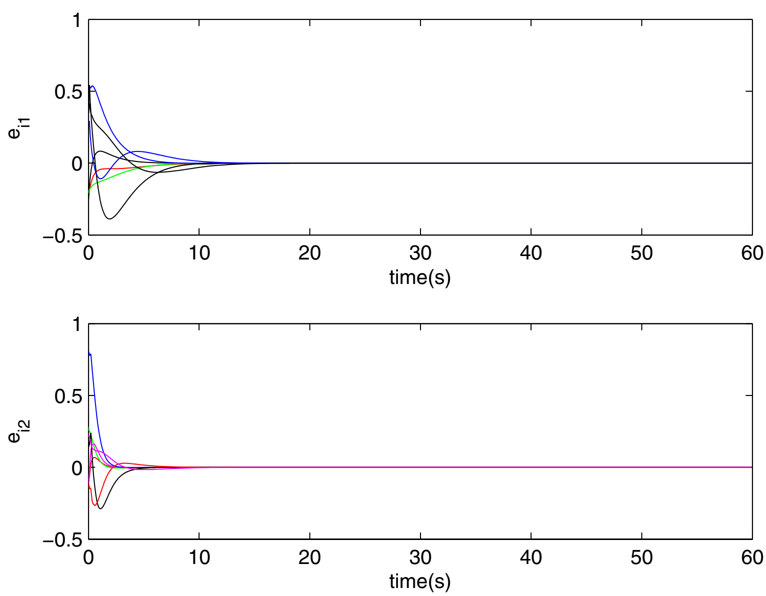

Fig. 1. Synchronous state errors of complex networks (27) for the case of $\tau = 0.1$

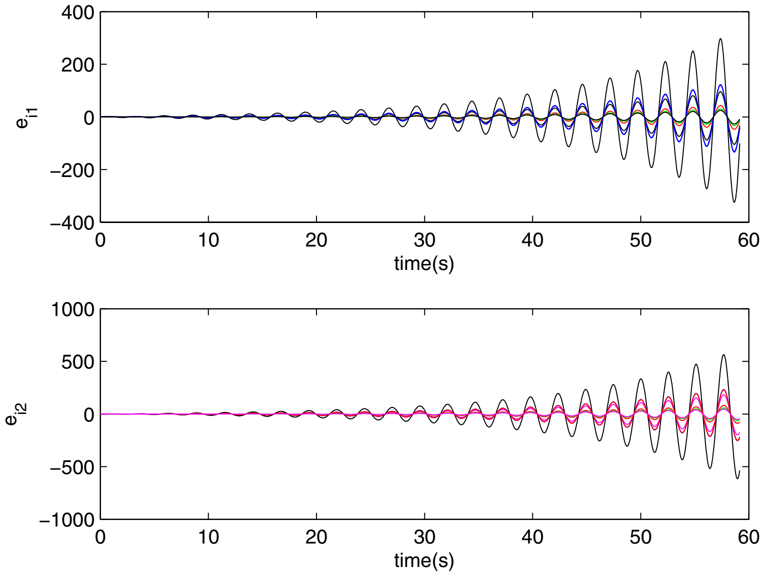

Fig. 2. Synchronous state error of complex networks (27) for the case of $\tau = 0.8$

5 Conclusions

In this paper, we discuss the global asymptotical synchronization problem for the generalized complex dynamical networks with mixed coupling delay. Compared with the previous work, the proposed model provides additionally mathematical description including the derivative of past states in the present system equation. Under some assumptions, the synchronization problem is reformulated into the asymptotically stability of nonlinear neutral delay functional differential equation. Furthermore the delay-dependent and delay-independent synchronization criterion are derived. Both of them can be readily verified by MATLAB LMI toolbox.

Acknowledgments

This work was supported by the National Key Fundamental Research Program (Grant No.2002CB312201-03) and the National Natural Science Foundation of China (Grant No.60575036).

References

1. Watts, D.J., Strogatz, S.H.: Collective dynamics of small-world networks. Nature 393, 440–442 (1998)
2. Watts, D.J.: Small-Worlds: The dynamics of networks between order and randomness. Princeton University Press, Princeton (1999)

3. Barabási, A.L., Albert, R.: Emergence of scaling in random networks. Science 286, 509–512 (1999)
4. Barabási, A.L., Albert, R., Jeong, H.: Mean-field theory for scale-free random networks. Physcia A 272, 173–187 (1999)
5. Wang, X.: Complex networks: topology, dynamics and synchronization. Int. J. Bifurc. Chaos 12, 885–916 (2002)
6. Boccaletti, S., Latora, V., Moreno, Y., Chavez, M., Hwang, D.U.: Complex networks: Structure and dynamics. Physics Reports 424, 175–308 (2006)
7. Wu, C.: Synchronization in coupled chaotic circuits and systems. In: Nonlinear science. World Scientific, Singapore (2002)
8. Pecora, L.M., Carroll, T.L.: Master stability functions for synchronized coupled systems. Phys. Rev. Lett. 80, 2109 (1998)
9. Li, C., Chen, G.: Synchronization in general complex dynamical networks with coupling delays. Physica A 343, 263–278 (2004)
10. Zhou, J., Chen, T.: Synchronization in general complex delayed dynamical networks. IEEE Trans. Circuit Syst. I 53, 733–744 (2006)
11. Zhou, C., Motter, A.E., Kurths, J.: Universality in the synchronization of weighted random networks. Phys. Rev. Lett. 96, 034101 (2006)
12. Li, Z., Chen, G.: Global synchronization and asymptotic stability of complex dynamical networks. IEEE Trans. Circuit Syst. II 53, 28–33 (2006)
13. Zhou, J., Xiang, L., Liu, Z.: Global synchronization in general complex delayed dynamical networks and its applications. Physica A 385, 729–742 (2007)
14. Liu, B., Teoc, K.L., Liu, X.: Global synchronization of dynamical networks with coupling time delays. Physics Letters A 368, 53–63 (2007)
15. Liu, X., Wang, J., Huang, L.: Global synchronization for a class of dynamical complex networks. Physica A 386, 543–556 (2007)
16. Hung, Y.: Paths to globally generalized synchronization in scale-free networks. Physical Review E 77, 016202 (2008)
17. Lu, W., Chen, T., Chen, G.: Synchronization analysis of linearly coupled systems described by differential equations with a coupling delay. Physica D 221, 118–134 (2006)
18. Wang, L., Dai, H., Sun, Y.: Synchronization criteria for a generalized complex delayed dynamical network model. Physica A 383, 703–713 (2007)
19. Hale, J.K., Verduyn Lunel, S.M.: Introduction to functional differential equations. Springer, New York (1993)
20. Kuang, Y.: Delay differential equations with applications in population dynamics. Academic Press, Boston (1993)
21. Niculescu, S.I.: Delay effects on stability: a robust control approach. Lecture Notes in Control and Information Sciences, vol. 269. Springer, London (2001)
22. Dai, Y., Cai, Y., Xu, X.: Synchronization criteria for complex dynamical networks with neutral-type coupling delay. Physica A 387, 4673–4682 (2008)
23. Wang, X., Chen, G.: Synchronization in scale-free dynamical networks: robustness and fragility. IEEE Trans. Circuit Syst. I 49, 54–62 (2002)
24. Boyd, S., Ghaoui, L., Feron, E., Balakrishnan, V.: Linear matrix inequalities in system and control theory. SIAM, Philadelphia (1994)
25. Horn, P.A., Johnson, C.R.: Matrix analysis. Cambridge University Press, New York (1985)
26. Kolmanovskii, V.B., Myshkis, A.D.: Introduction to the theory and applications of functional differential equations. Kluwer Academic Publishers, Dordrecht (1999)
27. Fridman, E.: New Lyapunov-Krasovskii functionals for stability of linear retarded and neutral type systems. Syst. Control Lett. 43, 309–319 (2001)