Hitting Time Analysis for Stochastic Communication

Paul Bogdan and Radu Marculescu

Carnegie Mellon University, Pittsburgh, PA, USA

Abstract. This paper investigates the benefits of a recently proposed communication approach, namely *on-chip stochastic communication*, and proposes an analytical model for computing its mean hitting time. Towards this end, we model the stochastic communication as a branching process taking place on a finite mesh and estimate the mean number of communication rounds.

Keywords: Network-on-Chip, reliable communication, hitting time.

1 Introduction

Shrinking geometries, scaling down supply voltages, and increasing clock frequencies have a negative impact on System-on-Chip (SoC) reliability [8]. Thus, there is a great need for scalable and reliable communication protocols among the SoC components. The traditional acknowledgement/request protocols are not adequate in such an error prone environment. To mitigate the impact of unpredictable faults in the deep submicron domain, a biologically-inspired communication was proposed in [4]. In that paper, the authors quantify the stochastic communication node coverage, but do not evaluate concrete performance metrics (*e.g.*, mean hitting time). However, evaluating the hitting time for a source-destination pair of nodes is important as it can serve as an input parameter for task mapping and scheduling problems in SoCs.

Starting from these ideas, this paper provides a theoretical framework for computing the mean hitting time between any two nodes in a mesh under stochastic communication protocol. We model the stochastic communication as a branching process in which each node that receives a copy of the disseminated packet, duplicates and sends it probabilistically to a subset of its neighboring nodes. In contrast to a *single* random walk, where a message is sent randomly only to a neighbor at any given time, the stochastic communication consists of *multiple* random walks that start at each node and provide a higher dissemination speed and robustness to the communication protocol. A major issue raised by the hitting time analysis for stochastic communication is that the diffusion process takes place on finite cyclic graphs. Also due to link or node failures, some of these random walks end prematurely. We model this behavior as an annihilation process. Thus, not only the topology, but also the protocol features consisting of branching and annihilation phases, make the hitting time analysis difficult. By estimating the hitting time, we gain insight into the consequences of link/node failures on node-to-node communication delay; this can help us adjust the forwarding probability such that the destination node receives at least one copy.

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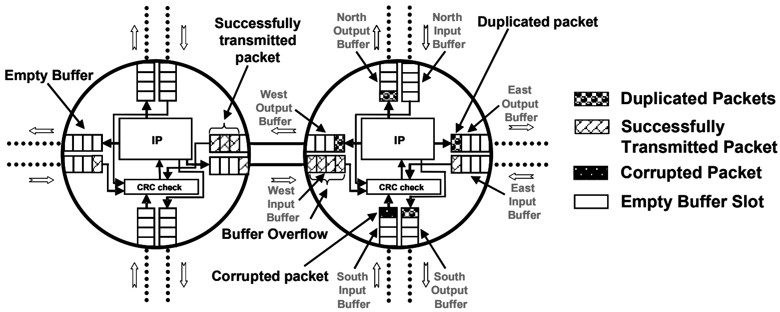

Fig. 1. Possible events between two neighboring nodes under stochastic communication.

The paper is organized as follows: Section 2 presents an overview of the hitting time theory and motivates the hitting time analysis for multiple random walks. Section 3 describes the Markov chain model associated with the stochastic communication and how the mean hitting time can be estimated. Section 4 presents our experimental results, while Section 5 outlines possible directions for future work.

2 Related Work

The hitting time concept for random walks was developed in connection with electrical networks [7][10] and attracted significant attention due to its potential applications such as the design of distributed computation [5][7][11], estimation of the complexity of distributed algorithms [6], search in peer-to-peer networks [12], estimation of Web size [3]. While the evolution of a random walk on graphs is extensively studied, only recently there has been an increased interest in the study of multiple, yet finite number of random walks [1][9]. However, many natural phenomena (*e.g.*, epidemics [2]) and human made processes cannot be modeled by imposing a single or finite number of random walks. Thus, similarly to epidemics, we model the stochastic communication as a *collection* of random walks which increase or decrease in cardinality based on the forwarding probability or the packet corruption probability.

3 Hitting Time Analysis

We consider a stochastic communication scenario in an $N \times N$ mesh network starting at node $(i,j)_S$. The protocol evolves as follows: If the packet is successfully received at any given node, it is first CRC checked for information integrity. If the packet is not already a duplicate, the current node copies and probabilistically sends it to a set of its neighbors. Two design cases can be considered: Either the packets are stored at each node and the transmission between two neighbors happens only when there is a free slot (*e.g.*, the West input buffer of right hand side node in Fig. 1 is full and no new packets can be stored - a buffer overflow is flagged), or the protocol allows the packets stored in the buffer to be overwritten by the incoming packets. For the sake of simplicity, we assume that packets can be overwritten and discuss only how the buffer overflow can be modeled. Besides these events, the packets can be lost due to node failures (*e.g.*, the incoming packet on the South input buffer of the right hand side node of Fig. 1 can be corrupted during the routing decision). The packet diffusion is a branching process, while the reverse process, in which packets are corrupted or overwritten, is an *annihilating* process. The number of received copies at a given node (i,j) can be described via a Markovian process $\{O_{ij}(t) | 1 \le i, j \le N,\ t \ge 0\}$ as follows:

a) Packet duplication: In a short time interval δt, a node (i,j) can duplicate a packet (see the right hand side node in Fig. 1) according to the next relation:

$$Pr\{O_{ij}(t+\delta t)=k+1 | O_{ij}(t)=k\} = \lambda_{ij} k \delta t + O(\delta t) \tag{1}$$

where λ_{ij} is the packet duplication rate for each node (i,j), k is the number of received copies, and $O(\delta t)$ is a negligible term. The duplication starts only if the node (i,j) received at least 1 copy ($k > 0$), otherwise the probability is zero.

b) Packet successful transmission: A packet duplicated at node (i,j) can be successfully sent to its neighbor $(i,j\text{-}1)$ (thus increasing its $O_{ij\text{-}1}$) as follows:

$$Pr\{(O_{ij}, O_{ij\text{-}1})(t+\delta t)=(k\text{-}1, n+1) | (O_{ij}, O_{ij\text{-}1})(t)=(k, n)\} = \alpha_{ij}^{W} k\left(1-\frac{n}{B_{ij\text{-}1}}\right)\delta t + O(\delta t) \tag{2}$$

where α_{ij}^{W} is the link successful transmission rate from node (i,j) to its West neighbor $(i,j\text{-}1)$ (*i.e.*, the router at (i,j) sends a copy to $(i,j\text{-}1)$ node and the packet is successfully received). This probability is strictly positive only if the sending node (i,j) has received at least one copy. The excessive packet duplication can cause buffer overflow situations, which can be captured by inserting the $(1 - O_{ij\text{-}1}/B_{ij\text{-}1})$ term, $B_{ij\text{-}1}$ being the buffer size at the $(i,j\text{-}1)$ node. If the buffer at node $O_{ij\text{-}1}$ is empty, then this term has no effect on the transition probability. If the number of received copies $O_{ij\text{-}1}$ increases, then this probability decreases. Similarly, we can describe the transmission events from (i,j) to the North ($O_{i\text{-}1j}$), East (O_{ij+1}), and South (O_{i+1j}) neighbors.

c) Packet corruption while routing: The probability that a node corrupts the received packet during the routing decision, causing it to be discarded, is:

$$Pr\{O_{ij}(t+\delta t)=k-1 | O_{ij}(t)=k\} = \mu_{ij} k \delta t + O(\delta t) \tag{3}$$

where μ_{ij} is the packet corruption rate at node level. This transition is activated with rate μ_{ij} only if the node (i,j) already received a positive number of packets ($k >$ 0). This accounts for potential errors in computation at node-level.

The packet diffusion over the entire network is seen as a *collection* of Markov processes, where the occurrence probability of any transition in an interval $(t,t+\delta t]$ relies on the number of packets received at time t in each node. The nodes evolution can be described via a master equation of the multivariate probability distribution:

$$P(o_{11}\ldots, o_{ij}\ldots, o_{NN};t)=Pr\{O_{11}(t)=o_{11}, \ldots O_{NN}(t)=o_{NN} | O_{11}(0)=m_{11},\ldots O_{NN}(0)=m_{NN}\} \tag{4}$$

which shows that the stochastic process $O_{ij}(t)$ received o_{ij} packets ($O_{ij}(t) = o_{ij}$). The evolution of probability function (*i.e.*, Eq. 4) is given by the following equation:

$$\frac{dP(\ldots o_{ij}\ldots;t)}{dt} = \sum_{i,j=1}^{N}[\lambda_{ij}(o_{ij}\text{-}1)P(\ldots o_{ij}\text{-}1,\ldots;t)+\mu_{ij}(o_{ij}+1)P(\ldots o_{ij}+1\ldots;t)$$

$$+\alpha_{ij}^{N}(o_{ij}+1)\left(1\text{-}\frac{o_{i\text{-}1j}\text{-}1}{B}\right)P(\ldots o_{i\text{-}1,j}\text{-}1,\, o_{ij}+1;t)+\alpha_{ij}^{E}(o_{ij}+1)\left(1\text{-}\frac{o_{ij+1}\text{-}1}{B}\right)P(\ldots o_{ij}+1,\, o_{ij+1}\text{-}1;t)$$

$$+\alpha_{ij}^{E}(o_{ij}+1)\left(1\text{-}\frac{o_{ij+1}\text{-}1}{B}\right)P(\ldots o_{ij}+1,\, o_{ij+1}\text{-}1;t)+\alpha_{ij}^{W}(o_{ij}+1)\left(1\text{-}\frac{o_{ij\text{-}1}\text{-}1}{B}\right)P(\ldots o_{ij}+1,\, o_{ij\text{-}1}\text{-}1;t) \quad (5)$$

$$+\alpha_{ij}^{S}(o_{ij}+1)\left(1\text{-}\frac{o_{i+1j}\text{-}1}{B}\right)P(\ldots o_{ij}+1,\, o_{i+1,j}\text{-}1\ldots;t)\Bigg] -$$

$$-\sum_{i\,j=1}^{N}\left[\lambda_{ij}+\mu_{ij}+\alpha_{ij}^{N}\left(1\text{-}\frac{o_{i\text{-}1j}}{B}\right)+\alpha_{ij}^{E}\left(1\text{-}\frac{o_{ij+1}}{B}\right)+\alpha_{ij}^{W}\left(1\text{-}\frac{o_{ij\text{-}1}}{B}\right)+\alpha_{ij}^{S}\left(1\text{-}\frac{o_{i+1j}}{B}\right)\right]o_{ij}P(\ldots o_{ij}\ldots;t)$$

with an initial condition $P(o_{11}=0\ldots,\, o_{(ij)_S}=1,\ldots o_{NN}=0;t=0) = 1$.

Traditional methods of solving Eq. 5 (*e.g.*, the moment generating function, the transition matrix approach) do not offer a scalable solution. We approximate the solution of Eq. 5 via nonhomogeneous Poisson distribution [13] and express the mean hitting time between source $(i,j)_S$ and destination $(i,j)_D$ as follows:

$$\langle t\rangle = \int_0^{\infty} t \sum_{o_{(ij)_D}=0}^{1} P(o_{(ij)_D}, t; o_{(ij)_S}=1, t=0)dt \quad (6)$$

where the right hand side accounts for the mean time during which the destination node did not receive any packet and the time needed to receive exactly one packet.

4 Experimental Results

Next, we evaluate the proposed model by considering a stochastic communication scenario between node $(1,1)_S$ and node $(10,10)_D$ on a 10×10 mesh NoC. First, we estimate the probability for node (10,10) to receive exactly 0, 1, 2, 3 and 4 packets (see

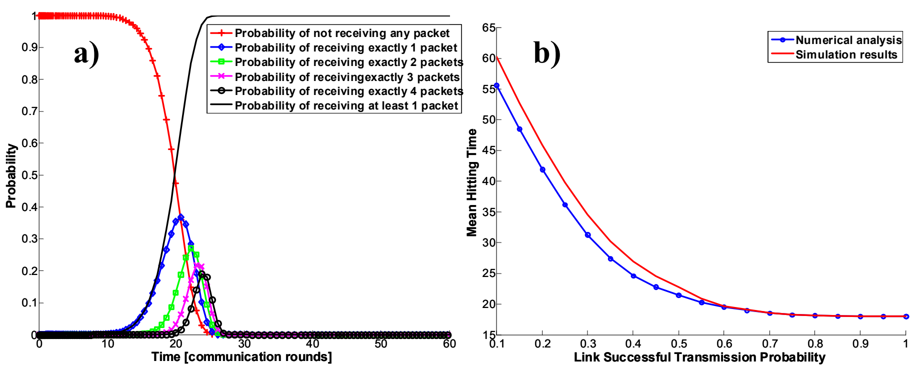

Fig. 2. a) Time-dependent probabilities for destination (10,10) to receive 0, 1, 2, 3 and 4 packets from source (1,1). The probability of receiving at least 1 copy is almost 1 after 25 communication rounds. b) Comparison between analytical and simulation results of the mean hitting time between (1,1) and (N,N) nodes on a 10×10 mesh.

Fig. 2.a. We also report the time-dependent probability of receiving at least 1 packet. For this experiment, we use a 0.15 packet injection rate, and a 0.8 link successful transmission probability for all directions. The probability of receiving *k* or more copies at destination can be used for designing various voting strategies to improve the error correcting methods (*e.g.*, the destination can recover the packet out *k* received copies). Fig. 2.b shows the mean hitting time between nodes (1,1) and (*N*,*N*) obtained via Eq. 6 and simulation in a 10×10 mesh, a 0.15 packet duplication rate, and zero probability of overflow. The mean hitting time results were obtained by averaging over 500000 runs. We note that the numerical analysis becomes more inaccurate compared to the simulation as the link successful transmission probability decreases.

5 Conclusions

This paper presented a framework for computing the mean hitting time of a branching process running on a finite graph and discussed its application to stochastic communication. As future work, we plan to model the traffic burstiness and the effects of finite buffers so that the analysis provides accurate estimates of the node-to-node latency.

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