Implementation and Performance Analysis of Trellis Coded Modulation in Additive White Gaussian Noise

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Abstract. Band-limited channels face a serious challenge in modern communication due to the need to increase transmission rate while bandwidth remains the same. Mobile communications including mobile satellite, wireless mobile and indoor wireless communication all need to transmit at a higher rate. Trellis-coded Modulation (TCM) is a modulation and error control scheme that allow band-limited and/or power limited channels to attain an efficient data rate without an increase in bandwidth. It is an advanced mode of convolutional coding that combines digital modulation and error correction coding in a single stage. This project sought to investigate the performance of TCM in Additive White Gaussian Noise, modelling a mobile wireless channel. To do this, trellis codes are digitally modulated with M-ary PSK and M-ary QAM using computer simulation (MATLAB) and the performance is evaluated when the modulated signals propagates through additive white Gaussian Noise. Experiments were also carried out on hardware equipment using Telecommunication Instructional Modeling System (TIMS). The plot of bit error rate against E_b/N_0 showed that there is a significant coding gain at no extra cost of bandwidth expansion.

Keywords: TCM · Bandwidth · AWGN · Coding gain · Power

1 Introduction

Trellis coded modulation (TCM) have developed over the last three decades as the modulation scheme that allows efficient data transmission for additive white Gaussian noise (AWGN) channel where bandwidth and power are limited. Lately, mobile radio and indoor mobile channels now make use of TCM and such channels are modelled as a fading channel. Trellis coded modulation was first proposed by Gottfried Ungerboeck in 1976 and following a more detailed publication in 1982 which led into thorough study of the subject to a point where a good knowledge and understanding of the theory and capabilities are achieved and then implemented [2].

The traditional forward error control (FEC) codes that were in use before TCM provides coding gain at the expense of increased bandwidth and power, for example block codes and convolutional codes etc. which is not an option for channels whose bandwidth is fixed and power limited like the microwave channels. The main attraction is that the technique combines forward error control coding and digital modulation in a

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single stage to provide a significant coding gain over the traditional un-coded M-level modulation schemes. With channel bandwidth the same and the need for more data transmission increases, there exists an urgent need to either expand the bandwidth by finding an error correction code and a digital modulation scheme that would increase the data rate or switch to a whole new system that will provide the required bandwidth. In the late 1980s prior to the invention of TCM, modems operate over plain old telephone service, it is an analogue service with bandwidth between 56 and 64 kbps and can typically achieve a data rate of 9.6 kbps when employing QAM modulation with symbol rate of 2400 baud. Despite the efforts of researchers to improve data transmission the best that could be achieved was 14 kbps for a two-way communication which was approximately 40% of the bit rate Shannon theoretically predicted for this kind of communication medium.

1.1 Areas of Application

Trellis coded modulation has found application in telecommunication systems and it is commonly used in satellite communication where power is limited. Due to the distance signal has to travel to get to the earth station and attenuation that the signal suffers, only a fraction of the original signal is detected at the earth station, 10 pW at the best. This power might be too small for the receiver to be able to decode properly and can even make the error controlling scheme already implemented (if any) to insert more errors than it is correcting. TCM is also used on telephone lines to maximize the capacity of the channel and increases data rate and if an Asymmetry Digital Subscriber Line (ADSL) is used instead of the conventional voice modem then even faster bit rate can be achieved by making use of the frequencies that are not utilized by the telephone voice service. TCM has also found application in personal and mobile wireless communication which involves transmission of signals at microwave frequencies, these signals are severely affected by multipath propagation. It is preferred because of its ability to function at low power and delivers high data throughput while making use of the available bandwidth which are the characteristics of microwave transmission.

2 TCM Encoder

The coding techniques used prior to the advent of TCM provided error improvement at the expense of bandwidth because of the addition of redundant bits to the information bits. Examples are block code and convolutional codes where k information bits are inputted into the encoder and larger n code words are outputted requiring more bandwidth. For this singular reason, error control coding was not common especially for channels where bandwidth expansion is impracticable e.g. telephone lines. Unlike convolutional coding, TCM schemes uses redundant non-binary modulation in combination with a finite state encoder, the term finite state encoder refers to a device that has memory (Shift register) which can save information about the past signals and has limited and distinguishable number of these memories. The state of the encoder is the smallest bit of information that when combined with the present input can predict the output of the device. The state of this device carries information about the past signals and also restricts output to a set of possibilities limited by the past state [2]. This redundant non-binary modulation and finite state encoder governs the selection of modulation signals to generate the coded signal sequence. To improve error performance of the system, when k bits are to be coded, a convolutional encoder with a code rate of $\frac{k}{k+1}$ is used followed by a constellation mapper that maps k + 1 bits into 2^{k+1} signal sets (waveforms). Of course the individual symbols will be closer to one another on the signal space and one will tend to think that the signal is prone to error. One thing to keep in mind is that error performance is no longer measured by the Hamming distance but rather by the Euclidean distance. The encoder should be designed to achieve maximum free Euclidean distance.

TCM encoder consists of a convolutional encoder with code rate of $k/k+1$ and an M-ary constellation mapper combined as a single function. The M-ary signal mapper maps M = 2^k into a larger constellation of $M = 2^{k+1}$ points [4]. It expands the signal points in order to accommodate coding bits rather than increasing the bandwidth, for instance, to code a 4-QAM, k = 2, there are two signal points on the un-coded system and M = 4. The constellation point will have to expand to k = 3 and M = 8, so the output of the encoder will be 8-QAM and code rate 2/3 (Fig. 1).

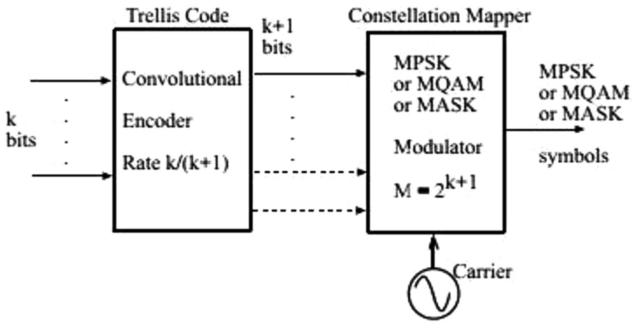

Fig. 1. TCM encoder

A convolutional encoder is characterized and can be fully described by the following parameters *n, k, K* and the generator polynomial. K is the constraint length and it specifies the number of shift register stages in the encoder. The encoder also contains modulo-2 adder whose output are interleaved to obtain the output code words. The connection of these modulo-2 adders to the shift registers fully describes the characteristics and behavior of a TCM encoder and this connection is generally represented by generator polynomial and are formally denoted in octet format. The input bits are streamed into the shift registers k bits at a unit time, k − 1 bit (which is the past output of the K − 1 shift register) together with the next input determines the next output (Fig. 2).

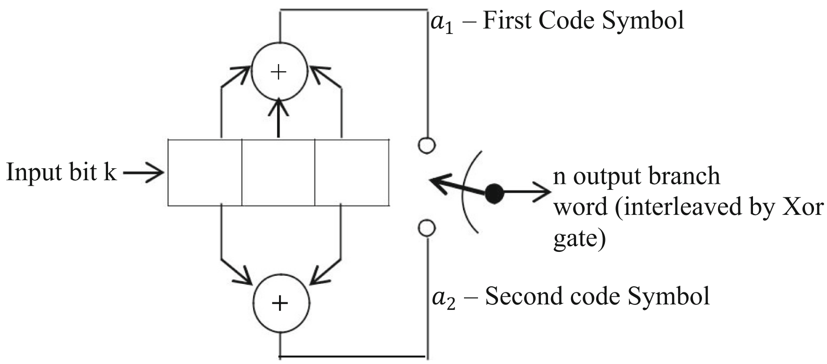

Fig. 2. Convolutional encoder, code rate, K = 3.

Ungerboeck in [3] pursed another TCM design method that seeks to maximize the free Euclidean distance based on a mapping rule called mapping by set partitioning. The rule follows that a modulation signal constellation is partitioned into signal subsets having an increasing minimum distance $\Delta_0 < \Delta_1 < \Delta_2 \ldots$ between the signals of these subset. Figure 3 shows Ungerboeck's set partitioning of a 8-PSK modulation, if the average signal power is unity then the minimum distance Δ_0 between any two adjacent signal is $2\sin(\pi/8) = 0.765$. The first partitioning results in subset B_0 and B_1 with distance $\Delta_1 = \sqrt{2}$ between them, the next partition is C_0 and C_1 which differ in distance by $\Delta_2 = 2$.

TCM is classified as a waveform coding technique it only requires a suitable trellis and a set of modulation waveform to describe and develop a working TCM code. To maximize the ED, the signal points from the extended $M = 2^{k+1}$ are assigned to the trellis transition.

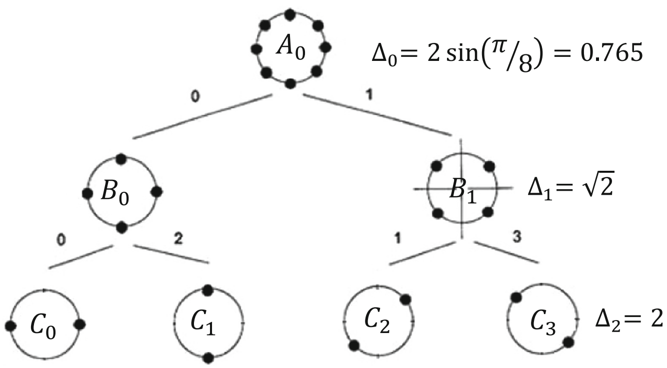

Fig. 3. Partition of 8-PSK channel signals into subsets with increasing minimum distance [1]

The design of trellis code can be done arbitrarily (theoretically) using a trellis diagram and Euclidean distance analysis. Practically, the code designed theoretically needs a circuitry if it is going to be implemented to actually encode and modulate information bits. Depending on the encoder, the circuitry may be easily designed where the choice of code words assignment gives rise to a simple encoder design. But a situation can arise where the choice of the code words assignment may not give rise to a simple encoder design and this may dictate an unwieldy encoder design [1].

3 TCM Decoder Using Maximum Likelihood Soft Decision Decoding

The major objective of a convolutional decoder is to track the path that the message received had traversed through in the encoding trellis. A convolutional encoder is a finite state machine that has memories, hence the optimum decoder for this type of encoder is a Maximum Likelihood sequence estimator which involves a search of the trellis for the most probable sequence. Following the rules for assigning trellis transition branches to waveforms, then all input message sequence should be equally likely. A decoder that will achieve minimum probability of error is the one that will compare the probability of the received sequence waveform with the most probable sequence waveforms and choose the maximum – this decision making criterion is known as maximum likelihood; it is a way of making decision when there is a statistical knowledge of the possibilities.

Significant coding gain is achieved by trellis coded modulation schemes at the expense of encoder and decoder complexities, complexities means extra processing time at both the encoder and the decoder which is interpreted as delay at the receiver. There is a processing delay when the encoder accumulates the input bit to generate the code word and another delay at the receiver when the decoder inspects a certain number of possible code words to arrive at a decision. Quality of service can be affected especially for services that are sensitive to delay such as telephone or video transmission if the delay is large enough to be perceived [5]. Therefore, the degree of TCM code complexities should be carefully chosen based on the kind on service it is intended for. A satellite link can endure the long delay introduced and also have large bandwidth to support higher code rates as an added advantage.

Figure 4 shows a transmitted sequence $U = \cdots U_1, U_2, U_3, \ldots$ path through the trellis and one probable sequence $V = \cdots V_1, V_2, V_3 \ldots$ The path of V is seen to be diverging from U and then remerge. For a binary sequence of L branch code word, there are 2^L number of possible sequences that could have been transmitted. Therefore, decoder chooses a sequence waveform as the transmitted sequence if its likelihood is greater than the likelihood of the other possible transmitted sequences. Assuming soft decision decoding is employed then an error event will occur if the received symbol is closer in Euclidean distance to some alternate V rather than U. The larger the free Euclidean distance between the signal waveforms, the lower the probability of error.

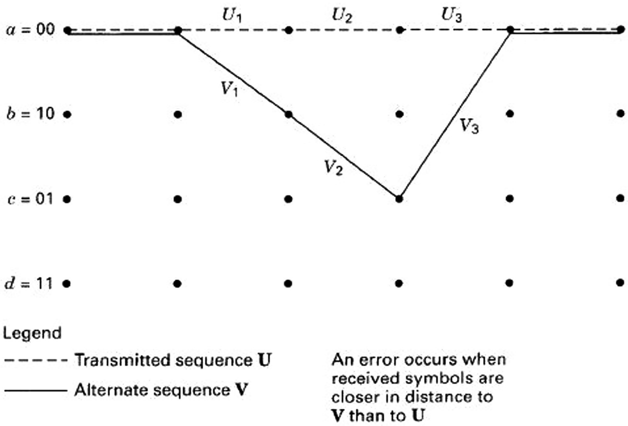

Fig. 4. Illustration of error event at the decoder.

3.1 Fading Channels

Communication channels linking one geographical location to another are bound to experience two major channels perturbations namely; Noise and fading due to multipath propagation. The TCM codes introduced earlier are designed to have a robust performance against Additive White Gaussian Noise but not equipped enough to overcome signal attenuation due to multipath fading.

Fading channels for mobile GSM are usually modelled using Rayleigh statistical model with its local mean following lognormal statistics and its phase distributed uniformly. This models a transmission that is basically multipath with no dominant line of sight. Another fading channel is Rician fading, here there is a dominant line of sight between the transmitter and the receiver. This can also be modelled statistically by using two Gaussian random variables – one with zero mean and the other with non-zero mean.

4 Results and Analysis

If a 2 level signal (ASK) is to be coded and transmitted, the code rate for the convolutional encoder will be ½ and the signal will be mapped into 2^2 signal sets, expanding the signal space to allow redundant bits and therefore transmitting a 4-ASK when a 2 level signal is received. A significant improvement to the error performance can be observed with a robust digital transmission against additive noise by 3 dB as seen in Fig. 5.

In Fig. 5 the error performances of an un-coded binary ASK is compared to that of trellis-coded modulated 4-ASK in an experiment (using Emona TIMS equipment). When the BER decreases the coding gain increases. The best that the bit error rate can go is 10^{-3} because TIMS equipment can only send a maximum of 10^6 pulses for evaluation and that explains why the results at higher power close to 5 dB are not reliable due to the fact that the pulses are simply not enough to reliably evaluate the corresponding BER. This places limits on the results obtained from the experiment.

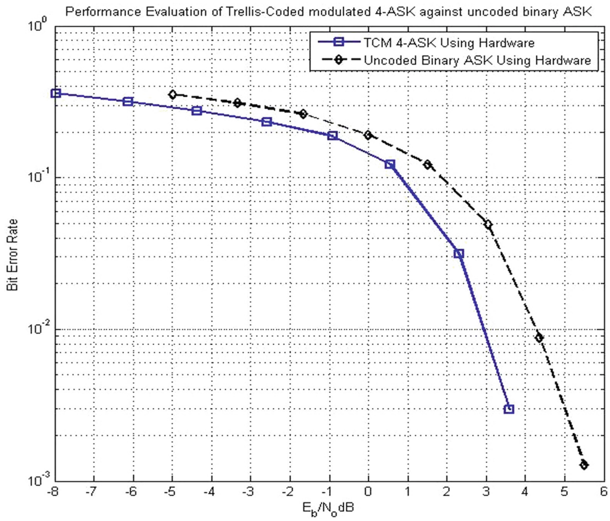

Fig. 5. Comparison of the performance of TCM coded 4-ASK with un-coded binary ASK

The maximum coding gain obtainable from this experiment is 1.35 dB at = 3 × 10^{-3}. At = 3 × 10^{-4} the coding gain as compared to an un-coded binary ASK reference system is 2.4 dB which is close to the finding of Ungerboeck [2].

4.1 Simulation Results

The curve in Fig. 6 depicts that a system that is seeking to decrease the bandwidth of transmission will need to increase the transmission power and vice versa.

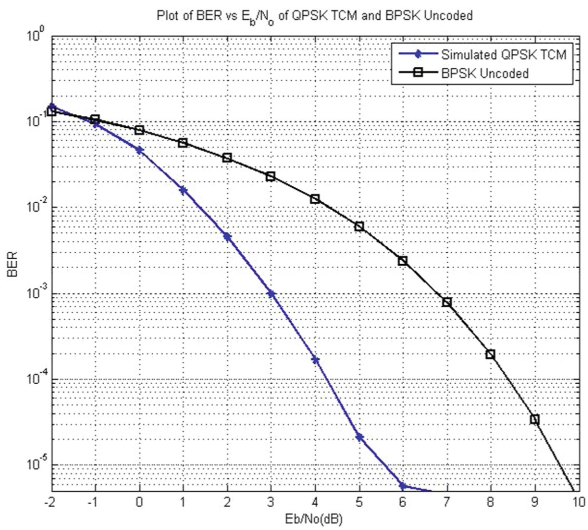

Fig. 6. Simulation results for BPSK un-coded and QPSK TCM systems

Typically, an un-coded BPSK requires E_b/N_0 of 9.6 dB at bit error rate of 10^{-5} which is 11.2 dB away from Shannon's limit [1]. With the aid of trellis coded modulation as much as 4 dB coding gain is obtained as show in Fig. 5.

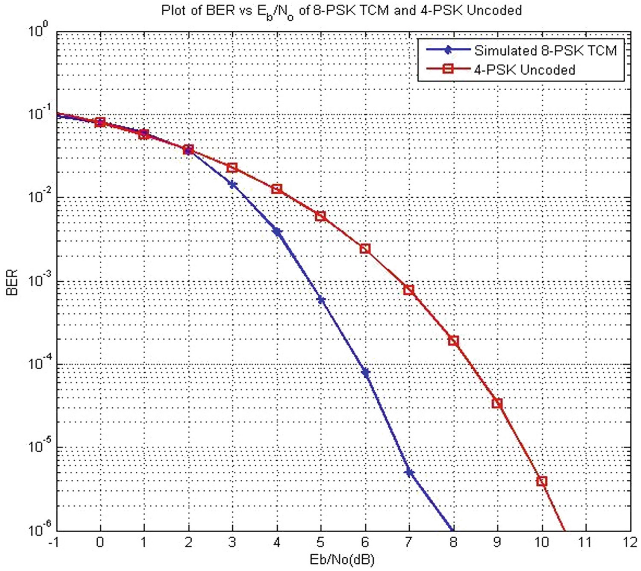

Fig. 7. Plot of BER versus E_b/N_0 of 8-PSK TCM (4 states) and 4-PSK un-coded system

From Fig. 7 above, it is seen that at higher values of BER, the power requirement is low; also the curve of the reference system requires more power to attain a certain BER on the coded 8-psk curve. This is due to the addition of coding which helps to reduce the power requirement. The TCM scheme used above accepts two symbols and outputs three symbols of information bits i.e. k = 2 and n = 3, i.e. code rate is 2/3 and it has N numbers of parallel transition signals with maximum distance between them.$N = 2^{m-2} = 2$. The free distance in this four-state 8-PSK is found to be $d_{free} = 2$ which when expressed in decibel gives an improvement of 3 dB over the un-coded 4-PSK whose $d_{free} = 2$ (Fig. 8).

The results achieved in figure agrees with several literatures, Ungerboeck [2] plotted a graph of simulated 8-PSK TCM and 4-PSK un-coded and got a similar results, also Sklar [1] obtained Asymptotic coding gain results for 4-state 8-PSK TCM by considering the expanded free distance which is 2 for coded system and $\sqrt{2}$ for un-coded system. The asymptote Coding gain $G = 10\log_{10}\left(\frac{2^2}{\sqrt{2}^2}\right) = 3dB$, the coding gain of the above system at $BER = 10^{-5}$ is 2.7 dB.

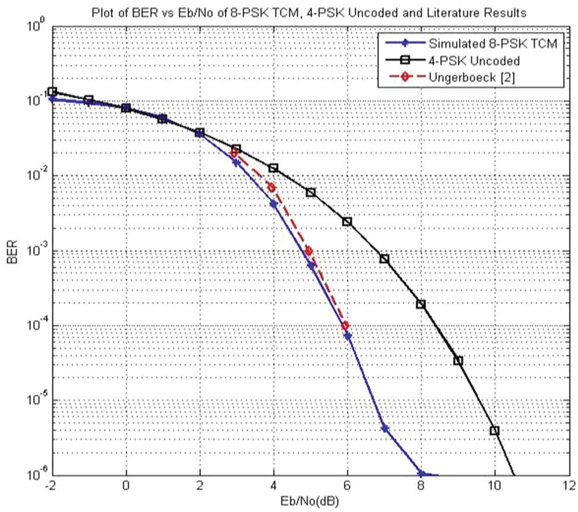

Fig. 8. Plot of BER versus E_b/N_0 of 8-PSK coded, 4-PSK un-coded and literature results

5 Conclusion and Further Work

The main contribution of this paper was to analyse trellis coded modulation in AWGN. A significant improvement can be observed using a simple 4-ASK four state TCM schemes that easily achieved coding gain of 3 dB and with more complex coding schemes, 6 dB is achievable. Signal can now be transmitted at a considerable lower *power* while maintaining the same *information rate* and even higher. *Bandwidth* can also be saved and trade-offs can be made between the choice of these three properties when designing a communication link (Link Budget).

Implementation of TCM in Rayleigh fading channel follows the same concepts as that of AWGN only that the signal gets to the receiver through a number of paths.

Further work should include the combination of TCM with other error correction codes e.g. block codes and turbo codes, this bring about further increase in coding gains and can also prove effective in fading channels. Further work will also include proper analyses of bandwidth efficiency and coding gain of TCM in fading channel and designing of optimum trellis codes for fading channel.

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