On the Equivalence Between Eigen and Channel Inversion Based Precoders

Khalid W. Hameed[1], Yasir Al-Yasir[1], Naser O. Parchin[1], Raed A. Abd-Alhameed[1,2], and Peter S. Excell[1,2(✉)]

[1] Faculty of Engineering and Informatics, University of Bradford, Bradford BD7 1DP, UK
r.a.a.abd@bradford.ac.uk
[2] Wrexham Glyndwr University, Wrexham LL11 2AW, UK
p.excell@glyndwr.ac.uk

Abstract. Multi-user MIMO precoding is crucial in modern and next generation wireless communication systems. In this paper the equivalence between two linear precoding methods using closed form solutions is investigated. The first one is the regularized zero forcing (RZF) algorithm and the second one is signal to leakage and noise ratio (SLNR). Three studies are presented: (1) comparison between the regularized and non-regularized versions; (2) finding a good regularization factor that can fit with all methods; (3) to present the equivalence of the methods in certain cases and the superiority of SLNR over RZF for user cases with more than a single antenna. A simple mathematical proof of the equivalence between RZF and SLNR beamformer implementations for the single antenna user case in a multi-user transmission scenario is presented: this matches simulation results.

Keywords: Beamforming · Channel inversion · Eigenvalue decomposition Multi-user-MIMO · Regularized zero forcing · Signal to leakage and noise ratio

1 Introduction

To enhance the capacity of a communication system, various approaches have been applied, such as the use of multiple antennas or the smart antenna, where spatial diversity is used to mitigate the channel condition without increasing the transmitted power or bandwidth [1]. Increasing the capacity and reliability of wireless communications systems through the use of multiple antennas has been an active area of research for over 20 years [2] and modern multi-antenna systems can take several configurations [3]: multi-input single output (MISO), single-input multi-output (SIMO) and multi-input multi-output (MIMO). The MIMO configuration can operate in two modes, spatial diversity and spatial multiplexing. The first mode enhances the performance of the bit error rate (BER) while the latter mode is used to increase the capacity. A more advanced configuration is multi-user MIMO [4–6] (MU-MIMO). This configuration works in a spatial multiplexing mode. It differs from single user MIMO (SU-MIMO) or what is also called point-to-point MIMO in that it does not allow user co-operation in decoding, whereas cooperation between antennas is essential in detection with

M. H. Miraz et al. (Eds.): iCETiC 2018, LNICST 200, pp. 161–172, 2018.
https://doi.org/10.1007/978-3-319-95450-9_13

SU-MIMO. MU-MIMO has several advantages over SU-MIMO [7]: it allows for a direct gain in multiple access capacity, better immunity performance against system and environment impairments, and it can achieve high capacity with a single antenna at the users' terminals, meaning smaller, cheaper handsets. In the analysis of MU-MIMO there are two main scenarios studied in the literature [8]: the multiple-access channel (MAC) or the reverse link, where signals are transmitted from users' terminals simultaneously to the base station, and the broadcast channel (BC) where the base station transmits signals to the users using the same time-frequency resource.

The precoders that are used in MU-MIMO are divided into two categories, non-linear and linear. Although the non-linear category achieves higher data rates, it has a higher complexity in comparison with linear ones. This becomes a significant restriction in next generation networks. In these networks some large scale regimes (massive MIMO and dense small cells) are proposed to deliver the required capacity [9]. The lower complexity linear precoder categories include maximum ratio transmission (MRT) [10], channel inversion or zero forcing (ZF) [10, 11], regularized zero forcing (RZF) [11], which is also known as minimum mean square error (MMSE) [10], and signal to leakage ratio (SLR) [12] or its regularized form, the signal to leakage and noise ratio (SLNR) [13] which has also been adopted recently to support multiple streams per user [14].

The ZF/RZF category has simpler equations and is easier to implement, but it has a dimension restriction in that the number of antennas at the base station should be larger than the total number of active users' antennas. To mitigate this condition, optimization of criteria such as the signal to interference per user is desirable: however this is constrained by a problem with coupling of variables and gives no closed form. On the other hand, the SLR/SLNR category gives an optimized precoder with a closed-form solution.

The authors of [15] and [16] show, in two different approaches, the equivalence between the RZF (or MMSE) and the SLNR precoders. In the present work, a hybrid approach between the methods used in these two references was utilized to achieve the same result with simpler mathematics.

2 System Mathematical Model

Consider a communication system with K active users served by a base station with M antennas, as shown in Fig. 1.

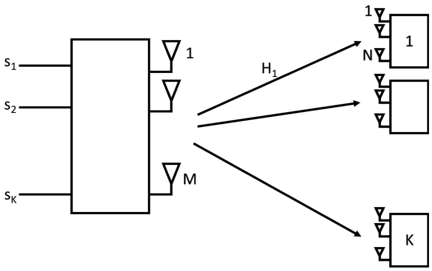

Fig. 1. The system model for MU-MIMO.

A time-frequency resource block is utilized to serve the active users simultaneously. The channel from the base station to user i is given by [4]:

$$H_i = \begin{bmatrix} h_{1,1,i} & \cdots & h_{1,M,i} \\ \vdots & \ddots & \vdots \\ h_{N,1,i} & \cdots & h_{N,M,i} \end{bmatrix} \tag{1}$$

Where $h_{n,m,i}$ is the channel from the mth transmitter antenna at the base station to the nth antenna at the ith user. The elements of H_i are assumed to be Rayleigh channels (i.e. unity variance with zero mean independently identically distributed (i.i.d.) complex Gaussian random variables), hence slow-flat fading channels. The aggregated channel for all users is given by [7]:

$$H = \left[H_1^T H_2^T \cdots H_K^T\right]^T \tag{2}$$

where H_i is the total channel matrix between the base station and the ith user. The leakage channel for user i (the channel from the base station to all other users except the intended user) is given by [13]:

$$\hat{H} = \left[H_1^T \cdots H_{i-1}^T H_{i+1}^T \cdots H_K^T\right]^T \tag{3}$$

The received signal by user i is

$$y_i = H_i X + n_i \tag{4}$$

Where n_i is the noise vector at user i with variance equal to σ^2, X is the transmitted vector from the base station and equals the sum of the transmitted vectors for all of the users:

$$X = \sum_{i=1}^{K} w_i s_i \tag{5}$$

Where $w_i \in \mathbb{C}^{M \times 1}$ is the precoder vector for user i and s_i is the data symbol for the same user.

3 Proposed Proof of Equivalence

For simplification of the mathematics, some definitions need to be established first. a is the user channel and b is the leakage channel. Now define A and B as follows:

$$A = a^H a \tag{6}$$

$$B = b^H b \tag{7}$$

$$Q = \left(B + \sigma^2 I\right) \tag{8}$$

From [16], the SLNR weights are given by

$$w_i^o \propto \max.EV\left(\left(B + N\sigma_i^2 I\right)^{-1} A\right) \tag{9}$$

which can be rewritten as:

$$w_{SLNR} \propto Q^{-1} a^H = \left(B + \sigma^2 I\right) a^H \tag{10}$$

From Lemma 1 in [15]:

$$C^{-1} d \propto \left(C + dd^H\right)^{-1} d \tag{11}$$

Where C is a matrix and d is a vector, then by letting $C = (\sigma^2 I + B)$ and $d = a^H$ we get:

$$\left(\sigma^2 I + B\right)^{-1} a^H \propto \left(\sigma^2 I + B + A\right)^{-1} a^H \tag{12}$$

which leads to:

$$\left(\sigma^2 I + B\right)^{-1} a^H \propto \left(\sigma^2 I + H^H H\right)^{-1} a^H \tag{13}$$

Where $H = A + B$. Now from [17] the RZF precoder is given by:

$$w_{RZF} \propto \left(H^H H + \alpha I\right)^{-1} H^H \tag{14}$$

4 Results

In this section results are presented to give a general perspective and to prove the equality between RZF and SLNR in a certain case. The non-regularized version of the methods is that where the effect of the channel only is considered in the optimization of the beamformer weights, while the regularized version takes the effect of the additive white Gaussian noise into account by adding a factor related to this noise. The first three figures, Figs. 2, 3 and 4, present a comparison between two approaches of zero forcing, the first one by using the pseudo-inverse (pinv) function in Matlab: this is equivalent to $H^H * (HH^H)^{-1}$. The second one is $(H^H H)^{-1} H^H$. Three observations can be made from these figures. Firstly, there is equivalence between ZF1 and RZF2. The second observation is that this equivalence still holds for multi-antenna users, especially in the low SNR region. The third observation is that the capacity tends to saturate when we use 8 antenna elements per user for the same scenario. The next two sets, including the figures from Figs. 5, 6, 7, 8, 9 and 10, present the performance of RZF

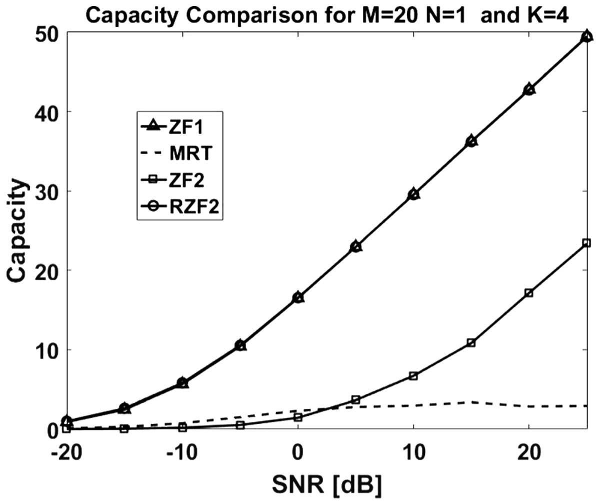

Fig. 2. Performance comparison between MRT, two versions of ZF, and RZF for a single antenna at user's location.

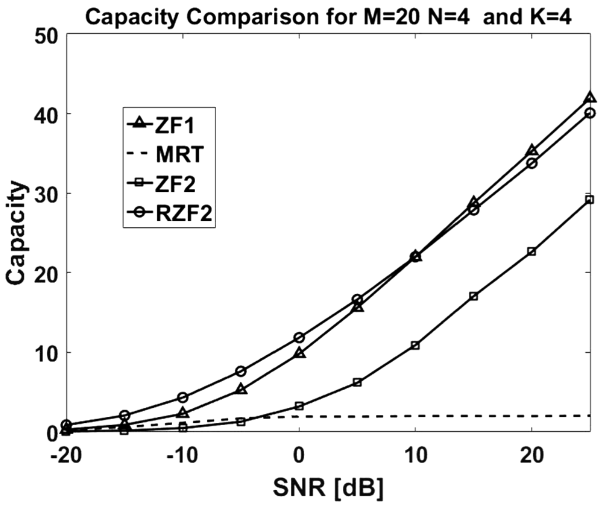

Fig. 3. Performance comparison between MRT, two versions of ZF and RZF for 4 antennas at users' location.

and SLNR with different regularization terms ($\sigma^2 I$, $\sigma^2 MI$, $\sigma^2 KI$, $\sigma^2 NI$ and I) in each figure for different numbers of antennas at the users' ends. The same behavior mentioned in the first set when the number of antennas per user was changed was also noticed with different regularization factors. The conclusion from these figures is that the regularization factor $\sigma^2 I$ is the best choice as it gives better performance compared with the others, for both beamformers.

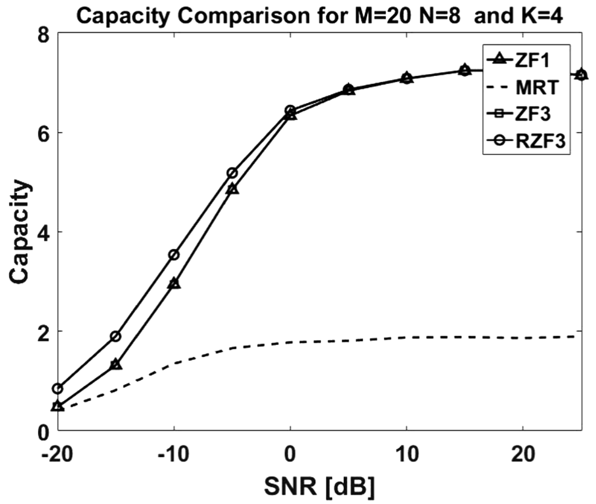

Fig. 4. Performance comparison between MRT, two versions of ZF and RZF for 8 antennas at users' location.

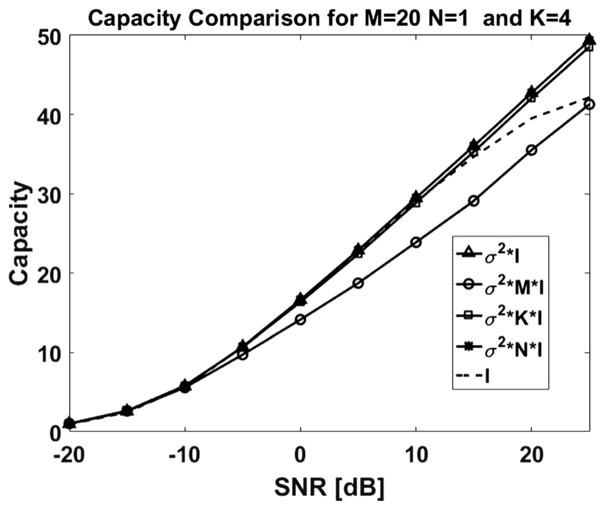

Fig. 5. Performance comparison for RZF with different regularization term for single antenna users.

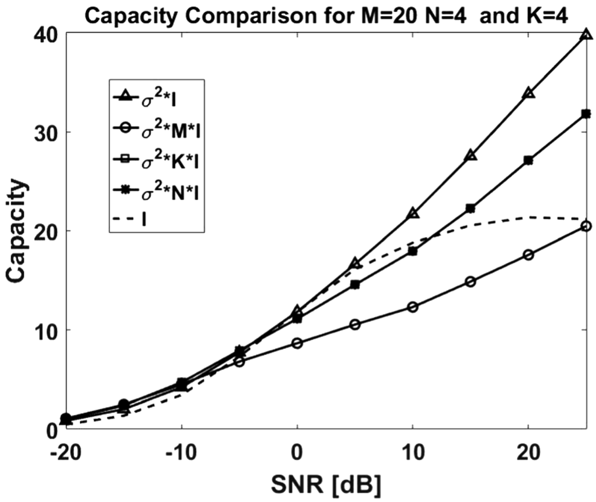

Fig. 6. Performance comparison for RZF with different regularization term for users with 4 antennas.

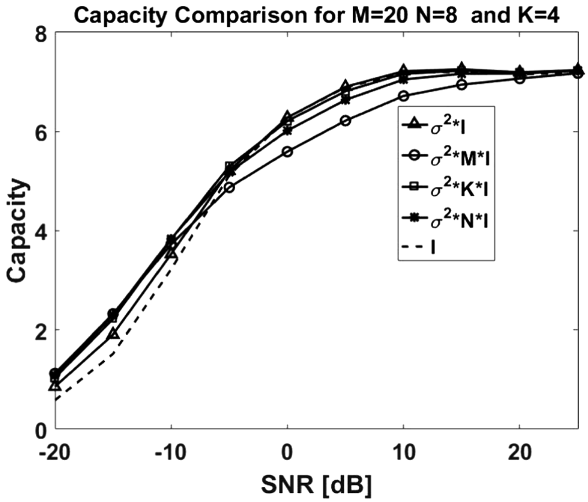

Fig. 7. Performance comparison for RZF with different regularization term for users with 8 antennas.

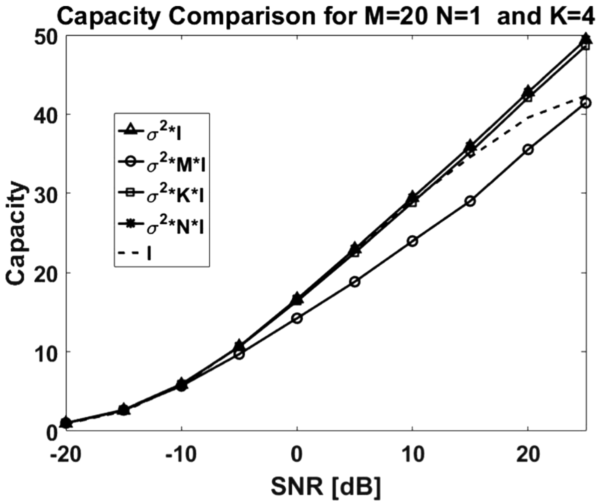

Fig. 8. Performance comparison for SLNR with different regularization term for single-antenna users.

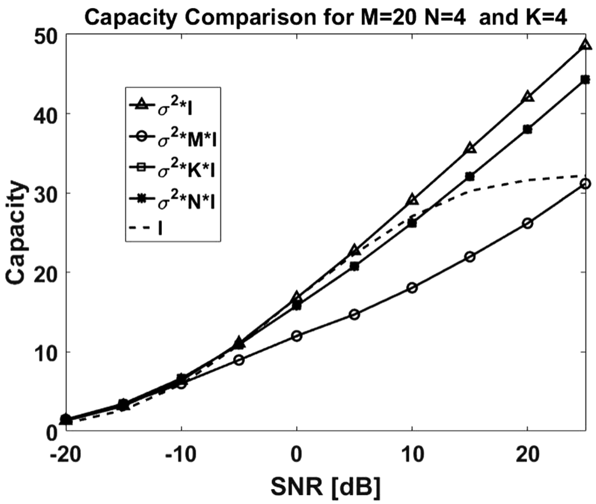

Fig. 9. Performance comparison for SLNR with different regularization term for users with 4 antennas.

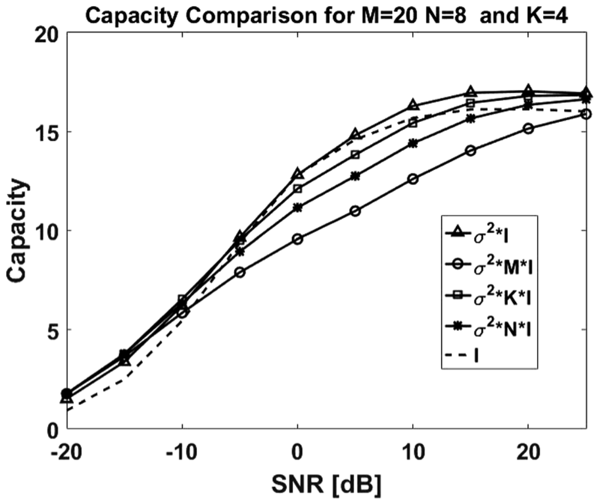

Fig. 10. Performance comparison for SLNR with different regularization term for users with 8 antennas.

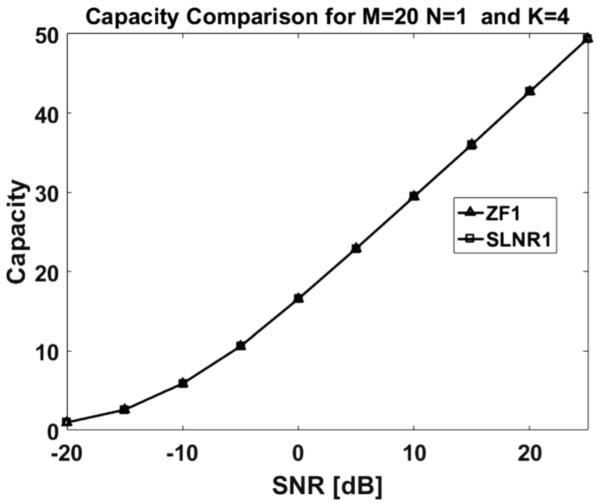

Fig. 11. Equivalence between RZF and SLNR with single-antenna users and regularization factor = $\sigma^2 I$.

Figures 11 and 12 show the relation between RZF and SLNR. Figure 11 reveals the equivalence between RZF and SLNR for the case of single-antenna users, using the regularization factor $\sigma^2 I$, however when the numbers of antennas at the user are increased the two methods start to diverge. It should noticed that the regularization factor used in Fig. 13 is $\sigma^2 NI$, which leads to lower performance than that of RZF with regularization factor $\sigma^2 I$ (Fig. 13 and Table 1).

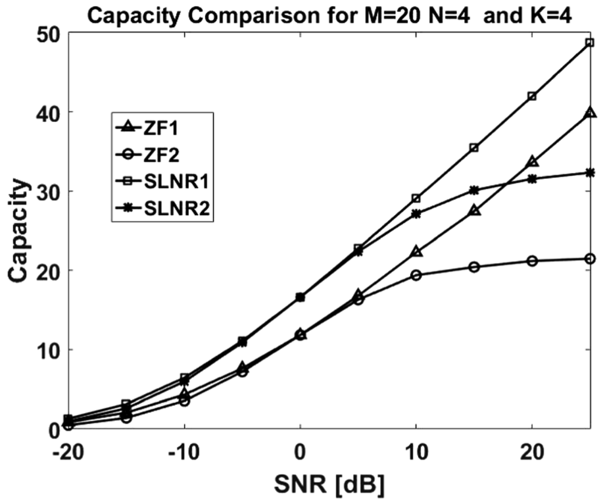

Fig. 12. Performance comparison between RZF and SLNR for 4 antennas at user side and two regularization factors.

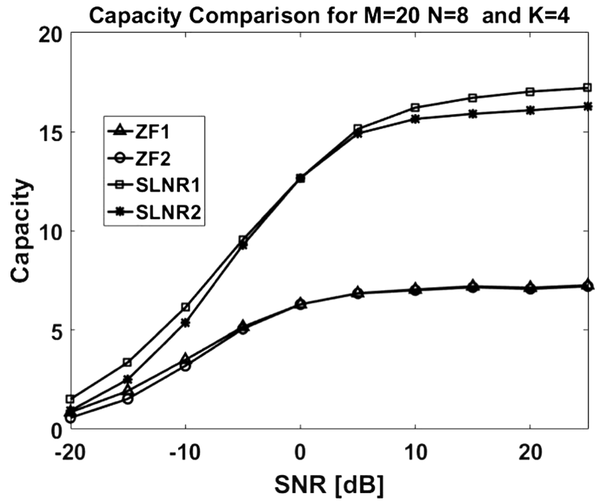

Fig. 13. Performance comparison between RZF and SLNR for 8 antennas at user side and two regularization factors.

Table 1. Capacity saturation for different cases

Method	Antennas per user		
	1	4	8
ZF	No	Factor dependent	Yes
SL	No	Factor dependent	Yes

5 Conclusion

In this paper the equivalence between RZF and SLNR precoders for the MU-MIMO transmission scheme has been presented. The equivalence between the two methods was proven by simulation. It was observed that the equivalence was intrinsic with single antenna users, meaning that channel inversion is another form of Eigenvector for simple cases where the user channel is a vector rather than a matrix. For more complex cases, where each user channel is a matrix, the SLNR performed better than RZF for the same regularization factor and the maximum performance was attained through the usage of ($\sigma^2 I$) as a regularization factor.

Acknowledgment. This work is partially supported by the EU Innovation Programme under grant agreement H2020-MSCA-ITN-2016 SECRET-722424 and financial support from the UK Engineering and Physical Sciences Research Council (EPSRC) under grant EP/E022936/1.

References

1. Wong, K.-K., Cheng, R.-K., Letaief, K.B., Murch, R.D.: Adaptive antennas at the mobile and base stations in an OFDM/TDMA system. IEEE Trans. Commun. **49**, 195–206 (2001)
2. Swindlehurst, A.L., Ayanoglu, E., Heydari, P., Capolino, F.: Millimeter-wave massive MIMO: the next wireless revolution? IEEE Commun. Mag. **52**, 56–62 (2014)
3. Paulraj, A., Nabar, R., Gore, D.: Introduction to Space-Time Wireless Communications. Cambridge University Press, Cambridge (2003)
4. Choi, L.-U., Murch, R.D.: A transmit preprocessing technique for multiuser MIMO systems using a decomposition approach. IEEE Trans. Wirel. Commun. **3**, 20–24 (2004)
5. Serbetli, S., Yener, A.: Transceiver optimization for multiuser MIMO systems. IEEE Trans. Sig. Process. **52**, 214–226 (2004)
6. Spencer, Q.H., Peel, C.B., Swindlehurst, A.L., Haardt, M.: An introduction to the multi-user MIMO downlink. IEEE Commun. Mag. **42**, 60–67 (2004)
7. Gesbert, D., Kountouris, M., Heath Jr., R.W., Chae, C.-B., Salzer, T.: Shifting the MIMO paradigm. IEEE Sig. Process. Mag. **24**, 36–46 (2007)
8. Goldsmith, A., Jafar, S.A., Jindal, N., Vishwanath, S.: Capacity limits of MIMO channels. IEEE J. Sel. Areas Commun. **21**, 684–702 (2003)
9. Rajoria, S., Trivedi, A., Godfrey, W.W.: A comprehensive survey: small cell meets massive MIMO. Phys. Commun. **26**, 40–49 (2018)
10. Björnson, E., Bengtsson, M., Ottersten, B.: Optimal multiuser transmit beamforming: a difficult problem with a simple solution structure [lecture notes]. IEEE Sig. Process. Mag. **31**, 142–148 (2014)

11. Peel, C.B., Hochwald, B.M., Swindlehurst, A.L.: A vector-perturbation technique for near-capacity multiantenna multiuser communication-part I: channel inversion and regularization. IEEE Trans. Commun. **53**, 195–202 (2005)
12. Tarighat, A., Sadek, M., Sayed, A.H.: A multi user beamforming scheme for downlink MIMO channels based on maximizing signal-to-leakage ratios. In: Proceedings of the IEEE International Conference on Acoustics, Speech, and Signal Processing, 2005 (ICASSP 2005), vol. 3, pp. iii/1129–iii/1132 (2005)
13. Sadek, M., Tarighat, A., Sayed, A.H.: A leakage-based precoding scheme for downlink multi-user MIMO channels. IEEE Trans. Wirel. Commun. **6**, 1711–1721 (2007)
14. Lopes, P.A., Gerald, J.A.: Leakage-based precoding algorithms for multiple streams per terminal MU-MIMO systems. Digit. Sig. Proc. **75**, 38–44 (2018)
15. Yuan, F., Yang, C.: Equivalence of SLNR precoder and RZF precoder in downlink MU-MIMO systems. arXiv preprint arXiv:1202.1888 (2012)
16. Seo, B., Shin, J.: Equivalence between SLNR and MMSE precoding schemes in the K-user MISO interference channel. WSEAS Trans. Sig. Process. **11**, 23–28 (2015)
17. Sung, H., Lee, S.-R., Lee, I.: Generalized channel inversion methods for multiuser MIMO systems. IEEE Trans. Commun. **57**, 3489–3499 (2009)