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# Comparative Study of the Performances of Peak-to-Average Power Ratio (PAPR) Reduction Techniques for Orthogonal Frequency Division Multiplexing (OFDM) Signals

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Abstract. In this paper, two distortionless PAPR reduction techniques, Selected Mapping (SLM) and Partial Transmit Sequences (PTS), are compared in terms of PAPR reduction capability and computational complexity for equal number of candidate OFDM symbols. Using MATLAB simulation, it is shown that SLM outperforms PTS in PAPR reduction capability. For small values of the number of subblock partitions, the overall computational complexity of PTS is less than SLM. However, the required PAPR reduction level may not be achieved using small values of number of subblock partitions. Hence, for large values of number of subblock partitions used in PTS, the overall computational complexity of PTS is greater than SLM. In that case, SLM outperforms PTS both in PAPR reduction capability and computational complexity.

Keywords: Orthogonal Frequency Division Multiplexing Peak-to-Average Power Ratio · Selected Mapping · Partial Transmit Sequence

## 1 Introduction

International standards making use of OFDM for high-speed wireless communications are already established by IEEE 802.11, IEEE 802.16, IEEE 802.20, Digital Audio Broadcasting (DAB), and Digital Video Broadcasting (DVB) [[1,](#page-11-0) [2](#page-11-0)]. An OFDM based system can be of interest for wireless applications because it provides greater immunity to multipath fading and impulse noise, and simplifies equalization process. All the desirable attributes of OFDM do not come for nothing, but at the expense of large envelope variation, which is often cited as the major drawback of OFDM and is usually quantified through the Peak-to-Average Power Ratio (PAPR). Such signal envelope or power variations can be difficult for practical High Power Amplifiers (HPAs) and Digital to Analog Convertors (DACs)/Analog to Digital Converters (ADCs) of the OFDM system to accommodate, resulting in either low power efficiency or signal distortion, including signal clips [[1,](#page-11-0) [2\]](#page-11-0). The signal distortion in turn results in Bit Error Rate (BER) increase and Power Spectral Density (PSD) degradation. To avoid these

effects, the HPA can be made to work in its linear region with large back-off but this results in poor power efficiency. Similarly the DAC/ADC can be designed to accommodate the large dynamic range of the OFDM signal but this results in a reduced Signal to Noise Ratio (SNR), as the DAC/ADC already has significant amount of quantization noise. Thus, a better solution is to reduce the PAPR of the OFDM signal with some manipulation of the OFDM signal itself [\[1](#page-11-0), [2](#page-11-0)].

Several researches have been done to the development of PAPR reduction schemes for OFDM signals. An overview of the various PAPR reduction schemes can be found in [\[1](#page-11-0), [2](#page-11-0)]. In this paper, the performances of two distortionless PAPR reduction schemes are compared. The first technique is Selected Mapping (SLM), which was first presented in [[3\]](#page-11-0). The second method is Partial Transmit Sequences (PTS), which was introduced for the first time in [\[4](#page-11-0)].

Muller and Huber in [\[5](#page-11-0)] compared the PAPR reduction capabilities of the two schemes making the number of IFFTs used in the transmitters of both schemes equal, in which case the number of candidate OFDM symbols may not be equal. It is shown that PTS has better PAPR reduction capability than SLM. The simulation results also show that the PAPR reduction capability of SLM increases with the number of candidate OFDM symbols D. This shows that the PAPR reduction capability of SLM is sensitive to the number of candidate OFDM symbols generated. Similarly, the PAPR reduction capability of PTS increases with the number of subblock partitions V. For a given number of phase rotation factors Q, the number of candidate OFDM symbols  $Q<sup>V</sup>$ increases with V. This also shows that the PAPR reduction capability of PTS is sensitive to the number of candidate OFDM symbols. As a result, it is sensible to compare the PAPR reduction capabilities of the two schemes making the number of candidate OFDM symbols generated equal. That is why this comparison is chosen as one of the objectives of the paper.

Furthermore, the authors in [[5\]](#page-11-0) also stated that for equal number of transmitter IFFTs used in both schemes, PTS is more computationally complex than SLM. However, the work doesn't include computational complexity analysis and simulation results. In this paper, both the analysis and simulation are done.

Performance comparison of SLM & PTS and PAPR reduction techniques for OFDM signals are also done by the authors in [\[6](#page-11-0), [7\]](#page-11-0) respectively.

Hence, the objectives of this paper are

- i. To compare the PAPR reduction capabilities of SLM and PTS for the same number of candidate OFDM symbols generated per given OFDM symbol intended for transmission.
- ii. To compare the computational complexities of SLM and PTS for the same number of candidate OFDM symbols generated per given OFDM symbol intended for transmission.

In comparing SLM and PTS I have used an OFDM signal model such that for a given data vector  $S^{\mu} = [S_0^{\mu} S_1^{\mu} \cdots S_{N-1}^{\mu}]^T$ , a sequence of complex numbers drawn from a finite constallation (MBSV on MOAM) in the *ulh* cinnaling integral, the beschool a finite constellation (MPSK or MQAM), in the  $\mu^{th}$  signaling interval, the baseband OFDM symbol  $\left\{ s_{n/L}^{\mu} \right\}_{n=0}^{LN-1}$  is an oversampled IFFT output of  $S^{\mu}$ . That is [[8\]](#page-11-0)

$$
s_{n/L}^{\mu} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k^{\mu} e^{j\frac{2\pi}{LN}n}, n = 0, 1, \cdots, LN - 1
$$
  

$$
s_{n/L}^{\mu} = \frac{1}{\sqrt{N}} \left\{ \sum_{k=0}^{N/2-1} S_k^{\mu} e^{j\frac{2\pi}{LN}n} + \sum_{k=LN-\frac{N}{2}}^{LN-1} S_{k-N(L-1)}^{\mu} e^{j\frac{2\pi}{LN}n} \right\}
$$
(1)  

$$
s_{n/L}^{\mu} = IFFT \left\{ \sqrt{L} S_L^{\mu} \right\}
$$

where N is the number of subcarriers,  $L \ge 1$  is an integer and it is the oversampling factor,  $IFFT\{\cdot\}$  is NL-point oversampled IFFT indexed by  $n/L$ , and  $S_L^{\mu}$  $L =$  $S_0^{\mu} \cdots S_{\frac{N}{2}-1}^{\mu}$   $0 \cdots 0$   $S_N^{\mu} \cdots S_{N-1}^{\mu}$ generated by zero padding  $S^{\mu}$  with  $N(L-1)$  zeros at its middle [\[8](#page-11-0)]. The baseband PAPR is defined as [8]  $\left[S_0^{\mu} \cdots S_{N-1}^{\mu} \ 0 \cdots 0 \ S_N^{\mu} \cdots S_{N-1}^{\mu} \right]^{\text{T}}$  is the L times oversampled equivalent data vector PAPR is defined as [[8\]](#page-11-0)

$$
PAPR\left\{s_{n/L}^{\mu}\right\} = \frac{\max_{n\in[0, LN)} \left|s_{n/L}^{\mu}\right|^2}{E\left\{\left|s_{n/L}^{\mu}\right|^2\right\}},
$$
\n(2)

which is a random variable.

As the passband PAPR is roughly twice (3 dB higher than) the baseband PAPR, it is sufficient to consider only the PAPR of the baseband OFDM signal [\[8](#page-11-0)], pp. 22–23. In addition, the cyclic prefix of duration  $T_g$ , which is a repetition of part of the OFDM symbol, attached to the OFDM symbol to combat Inter-Symbol Interference (ISI) can be neglected for the purposes of PAPR analysis as the prefix will not produce a peak which is not already present in the OFDM symbol  $s^{\mu}_{n/L}$ . Theoretically, PAPR  $\{s^{\mu}_{n/L}\}$  $\int_{\alpha} \mu$   $\downarrow$ approaches PAPR $\{s^{\mu}(t)\}\$ as L becomes sufficiently large. However, it is has been shown in [\[9](#page-11-0), [10](#page-11-0)] that when  $L \ge 4$  the PAPR of  $s^{\mu}_{n/L}$  approximates the PAPR of  $s^{\mu}(t)$  and hence  $L = 4$  is used in the simulation results of this paper.

The remaining part of the paper is arranged such that Sect. 2 presents the SLM and PTS PAPR reduction schemes. In Sect. [3](#page-5-0), the computational complexity analysis of both schemes is presented. Section [4](#page-7-0) provides comparative simulation results and discussions. Finally, the concluding remarks are provided in Sect. [5.](#page-10-0)

#### 2 Selected Mapping and Partial Transmit Sequence

Selected Mapping (SLM): SLM is a distortionless PAPR reduction technique [[3\]](#page-11-0). In the SLM technique, the transmitter generates a set of sufficiently different candidate data vectors, all representing the same information as the original data vector, for each data vector intended for transmission by rotating the phase of each data symbol and selects the candidate data vector with the lowest PAPR for transmission [\[3](#page-11-0)].

<span id="page-3-0"></span>A set of D markedly different, distinct, pseudorandom but fixed phase rotation vectors [\[3](#page-11-0), [10\]](#page-11-0)

$$
\mathbf{P}^{(d)} = \left[ P_0^{(d)} P_1^{(d)} \cdots P_{N-1}^{(d)} \right]^T, \tag{3}
$$

with  $P_k^{(d)} = e^{j\phi_k^{(d)}}, \phi_k^{(d)} \in [0, 2\pi), k = 0, 1, \dots, N - 1, d = 1, 2, \dots, D$  must be defined and available both at the transmitter and receiver. The data vector  $S^{\mu}$  is multiplied element-wise with each one of the D phase rotation vectors  $P^{(d)}$ , resulting in a set of D different phase rotated data vectors  $S^{(\mu,d)}$  given by [\[3](#page-11-0), [10](#page-11-0)]

$$
\mathbf{S}^{(\mu,d)} = \mathbf{S}^{\mu} \circ \mathbf{P}^{(d)}.
$$
 (4)

Then, all the D data vectors are transformed into time domain to get D candidate OFDM symbols [\[3](#page-11-0), [10](#page-11-0)]

$$
s_{n/L}^{(\mu,d)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k^{\mu} P_k^{(d)} e^{j \frac{2\pi k n}{L} n}, n = 0, 1, \cdots, LN - 1
$$
  

$$
s_{n/L}^{(\mu,d)} = IFFT\left\{\sqrt{L} S_L^{(\mu,d)}\right\}
$$
 (5)

Among the D candidate OFDM symbols, the transmitter selects the lowest PAPR sequence,  $s_{n/L}^{(\mu, \bar{d})}$ , for transmission where [[3,](#page-11-0) [10](#page-11-0)]

$$
\bar{d} = arg \min_{1 \le d \le D} PAPR \left\{ s_{n/L}^{(\mu,d)} \right\} \tag{6}
$$

The SLM-OFDM transmitter is shown in Fig. 1.



Fig. 1. Block diagram of SLM-OFDM transmitter.

<span id="page-4-0"></span>It is assumed that the transmitter and the receiver have the D phase rotation vectors  $P<sup>(d)</sup>$ . However, in order to recover an OFDM symbol the receiver has to know which phase rotation vector  $P^{(\bar{d})}$  has actually been used by the transmitter. The simplest method is to transmit  $\bar{d}$  as side information which requires log<sub>2</sub> D bits. As side information transmission decreases the information throughput, another method to determine  $\bar{d}$  is the blind technique [[11,](#page-11-0) [12\]](#page-11-0) where it is determined based only on the received OFDM symbol and the known phase rotation vectors.

The original data vector  $S^{\mu}$  is recovered by multiplying the received data vector  $\widetilde{S}^{\mu}$ element-wise by  $\mathbf{P}^{(\bar{d})} = \left[ e^{-j\phi_0^{(\bar{d})}} e^{-j\phi_1^{(\bar{d})}} \cdots e^{-j\phi_{N-1}^{(\bar{d})}} \right]$ . In the paper, it is assumed that the blind technique is used.

It is illustrated by simulation in  $[10]$  $[10]$  (p. 50 and 66) and  $[13]$  $[13]$  that for a given D, choosing  $P_k^{(d)}$ ,  $d = 1, 2, ..., D$  such that the corresponding  $\phi_k^{(d)}$  are uniformly distributed in  $[0, 2\pi)$ , then the PAPR reduction capability of SLM is the same whether  $P_{k}^{(d)}$ are chosen from set  $\{\pm 1\}$  or  $\{\pm 1, \pm j\}$ . Note that  $\mathbf{P}^{(d)}$  with elements  $\{\pm 1\}$  is generated from a randomly generated binary data mapped onto BPSK symbols and hence number of bits per symbol m = 1. Similarly  $P^{(d)}$  with elements  $\{\pm 1, \pm i\}$  is generated from a randomly generated binary data mapped onto QPSK symbols i.e.  $m = 2$ .

Partial Transmit Sequence (PTS): PTS is also a distortionless PAPR reduction technique [\[4](#page-11-0)]. In this technique the data vector  $S<sup>\mu</sup>$  is partitioned into V pair-wise disjoint subblocks  $S^{(\mu, v)}$ ,  $v = 1, 2, ..., V$ . That is, data symbol positions in  $S^{(\mu, v)}$ , which are already represented in another subblock are set to zero so that  $[4, 10]$  $[4, 10]$  $[4, 10]$  $[4, 10]$ 

$$
\mathbf{S}^{\mu} = \sum_{\nu=1}^{V} \mathbf{S}^{(\mu,\nu)} \tag{7}
$$

where  $S^{(\mu,\nu)} = \begin{bmatrix} S_0^{(\mu,\nu)} & S_1^{(\mu,\nu)} & \cdots & S_{N-1}^{(\mu,\nu)} \end{bmatrix}$  $\left[S_0^{(\mu,\nu)}\ S_1^{(\mu,\nu)}\ \cdots\ S_{N-1}^{(\mu,\nu)}\right]^T$ , such that  $S_k^{(\mu,\nu)} = S_k^{\mu}$  or  $0, k = 1, 2, ..., N-1$ .

The time domain representation of the subblocks can be generated using LN-point IFFT, i.e. [\[4](#page-11-0), [8](#page-11-0), [10\]](#page-11-0)

$$
s_{n/L}^{(\mu,\nu)} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k^{(\mu,\nu)} e^{j\frac{2\pi}{LN}n},
$$
  
\n
$$
s_{n/L}^{(\mu,\nu)} = IFFT\left\{\sqrt{L}S_L^{(\mu,\nu)}\right\}
$$
\n(8)

Each  $\left\{ s_{n/L}^{(\mu,\nu)} \right\}_{n=0}^{LN-1}$  is called Partial Transmit Sequence (PTS). Then each one of these PTSs are independently rotated by a phase rotation factor  $b_d^{(\mu,\nu)} = e^{j\theta_d^{(\mu,\nu)}}$ ,  $\theta_d^{(\mu,\nu)} \in [0, 2\pi)$ and then combined to form a candidate OFDM symbol, i.e. [\[4](#page-11-0), [10\]](#page-11-0)

$$
s_{(n/L,d)}^{\mu} = \sum_{\nu=1}^{V} b_d^{(\mu,\nu)} \cdot IFFT\left\{\sqrt{L}S_L^{(\mu,\nu)}\right\},\tag{9}
$$

<span id="page-5-0"></span>where  $d = 1, 2, \ldots, Q^V$  and Q is the number of phase rotation factors. The objective is to optimally combine the V PTSs to minimize the PAPR of the transmit OFDM symbol  $s^{\mu}_{(n/L,\bar{\rm d})}$  $\left\{ s_{(n/L,\bar{d})}^{\mu} \right\}_{n=0}^{(LN-1)}$  by a suitable combination of the free phase rotation factors  $\left\{b_d^{(\mu,\nu)}\right\}_v^V$  $v=1$ . The  $b_d^{(\mu,\nu)}$  may be chosen with a continuous valued phase rotation angle  $\theta_d^{(\mu,\nu)}$ , but more appropriate in practice is to restrict on a finite set of Q allowed phase angles to reduce the search complexity. Hence, we have  $Q<sup>V</sup>$  possible combination of phase rotation factors where all of them need to be searched exhaustively to find one that results in minimum PAPR transmit OFDM symbol. In the paper, this method is used to find the optimum combination so that the best PAPR reduction capability of PTS can be achieved.

It is illustrated through simulation in [[10\]](#page-11-0), page 69, that the choice  $b_d^{(\mu,\nu)}$  $d$  $\{\pm 1, \pm j\}$  gives better PAPR reduction capability than  $b_d^{(\mu,\nu)} \in \{\pm 1\}$ .<br>There are three types of subblock partitioning schemes: adjacent

There are three types of subblock partitioning schemes: adjacent, interleaved, and pseudo-random. It is shown in  $[10]$  $[10]$  (p. 67) and  $[14]$  $[14]$  that pseudo-random subblock partitioning scheme gives the best PAPR reduction and hence used in this paper. The PTS-OFDM transmitter is shown in Fig. [2](#page-6-0).

The blind technique is used to determine the optimum phase factor combination  $b^{(\mu,\nu)}_{\bar{d}}$  $\left\{b_{\overline{d}}^{(\mu,\nu)}\right\}^V$  at the receiver [\[12](#page-11-0)]. Assuming that the receiver knows the subblock parthe  $\int_{v=1}^{u}$  is  $\int_{v=1}^{v=1}$  tition scheme used by the transmitter, the received data vector is partitioned into subblocks. Then transmitted data block  $S<sup>\mu</sup>$  can be recovered by independently derotating each received subblock by  $\left\{b_{\bar{d}}^{(\mu,\nu)}\right\}^* = e^{-j\theta_{\bar{d}}^{(\mu,\nu)}}$  and then combining them to get  $S^{\mu}$ [\[10](#page-11-0)].

# 3 Computational Complexity Analysis of SLM and PTS

The computational complexity difference that may exist between SLM and PTS-OFDM systems mainly arises in the process of generating the candidate OFDM symbols and the subsequent selection of the symbol with the least PAPR.

For SLM, as it is shown in ([4\)](#page-3-0), ND complex multiplications are required to generate  $S^{(\mu,d)}$ ,  $d = 1, 2, ..., D$ . Then, D length LN IFFTs are required to generate  $s^{(\mu,d)}_{n/\lambda}$ . Each LN-point IFFT requires  $(LN/2) \log_2(LN)$  complex multiplications and LN log<sub>2</sub> $(LN)$ complex additions [\[14](#page-11-0), [15](#page-11-0)]. Finally,  $s_{n/L}^{\mu}$  $\begin{array}{c} \begin{array}{c} \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \end{array} \end{array}$  $\begin{array}{c} \begin{array}{c} \begin{array}{c} \end{array} \\ \begin{array}{c} \end{array} \end{array} \end{array}$ <sup>2</sup> = Re  $\left\{ s_{n/L}^{\mu} \right\}$  $\left\{ s_{n/L}^{\mu} \right\}^2 + Im \left\{ s_{n/L}^{\mu} \right\}$  $\left\{ s_{n/L}^{\mu} \right\}^2$  need to be calculated for each  $n$  in determining the PAPR which requires 2DLN real multiplications and DLN additions.

<span id="page-6-0"></span>To generate the PTSs in ([8\)](#page-4-0), V length LN IFFTs are used. For pseudo-random subblock partitioning scheme this can be achieved at the expense of  $V\frac{LN}{2}\log_2(LN)$ <br>complex multiplications and  $V\frac{N}{2}\log_2(LN)$  complex additions. In generating the  $QV$ complex multiplications and  $VLN \log_2(LN)$  complex additions. In generating the  $Q<sup>V</sup>$ candidate OFDM symbols in [\(9](#page-5-0)), we need  $Q<sup>V</sup> VLN$  complex multiplications to create  $b_d^{(\mu,\nu)} s_{n/L}^{(\mu,\nu)}$  which are combined through  $Q^V(V-1)LN$  complex additions. Finally, to calculate the PAPR of  $\left\{ s^{\mu}_{(n/L,d)} \right\}$  $\int_{R} \mu$   $\lambda^{LN}$  $n=1, 2, \ldots, Q^V$ , we need  $2Q^VLN$  real multiplications and  $Q<sup>V</sup>LN$  real additions.



Fig. 2. Block diagram of PTS-OFDM transmitter.

Generally, a complex multiplication requires four real multiplications and two real additions. On the other hand, a complex addition requires two real additions. Hence, the total number of real additions A and multiplications M required for each scheme can be summarized as

$$
A_{SLM} = DLN(3 \log_2(LN) + 1) + 2ND \tag{10}
$$

$$
M_{SLM} = 2DLN(\log_2(LN) + 1) + 4ND \tag{11}
$$

$$
A_{PTS} = 3VLN \log_2(LN) + Q^V(4V - 1)LN
$$
\n(12)

$$
M_{PTS} = 2VLN \log_2(LN) + 2Q^V(2V+1)LN
$$
\n(13)

The computational complexity of PTS and SLM is quantified through a parameter  $f$ which is the number of addition instructions required for each multiplication operation. Therefore, the overall computational complexity of each scheme for  $f = 4$  is

$$
C_{SLM} = A_{SLM} + fM_{SLM}
$$
  
\n
$$
C_{SLM} = [11LN \log_2(LN) + 9N(L+2)]D
$$
\n(14)

$$
C_{PTS} = A_{PTS} + fM_{PTS}
$$
  
\n
$$
C_{PTS} = 11 \, VLN \log_2(LN) + LN(20V + 7)Q^V.
$$
\n(15)

<span id="page-7-0"></span>If the two schemes are made to have equal number of candidate OFDM symbols, i.e.  $D = Q^V$ , then Eq. (14) becomes.

$$
C_{SLM} = [11LN \log_2(LN) + 9N(L+2)]Q^V.
$$
 (16)

#### 4 Comparative Simulation Results

This section presents comparative simulation results for PAPR reduction capability and computational complexity which are produced using MATLAB version 7.5. For the comparative PAPR reduction simulations,  $10<sup>5</sup>$  randomly generated OFDM symbols each containing 128 QPSK modulated data symbols (subcarriers) are generated.

As stated in Sect. [1,](#page-0-0) the authors in [[5](#page-11-0)] mentioned that for equal number of transmitter IFFTs used in SLM and PTS schemes, PTS is more computationally complex than SLM. However, the work doesn't include computational complexity analysis and simulation results. In this paper, the analysis, Eqs.  $(10)$  $(10)$ – $(15)$ , and the simulation (Fig. [3](#page-8-0)) are done. Figure 3 shows the plot of the ratio  $C_{\text{PTS}}/C_{\text{SIM}}$  in dB versus the number of subblock partitions V of the PTS scheme. The simulation was done for different number of subcarriers N,  $L = 4$ ,  $f = 4$ , and the elements of the phase rotation vectors chosen from the set  $\{\pm 1\}$  as a compromise for the large values of N, like  $N = 131,072$  to decrease the long simulation time required had the set  $\{\pm 1, \pm i\}$  been used. In the plots it can be seen that the ratio  $C_{\text{PTS}}/C_{\text{SIM}}$  increases with V, in fact the increment gets faster for large values of V. The plot clearly shows that the ratio  $C_{\text{PTS}}/C_{\text{SIM}}$  is always greater than 1 for all values of V. This in turn shows that the computational complexity of PTS is greater than SLM for all values of V, which is the cost to be paid for its better PAPR reduction capability.

Simulation results for comparisons in PAPR reduction capability and computational complexity for equal number of candidate OFDM symbols generated, i.e.  $D = Q^V$ , are shown in the following paragraphs. Hence, Fig. [4](#page-8-0) shows a plot of the Complementary Cumulative Distribution Function (CCDF) of PAPR of PTS and SLM-OFDM signals for  $D = Q^V$ . For every OFDM symbol intended for transmission equal number of candidate OFDM symbols is generated in both techniques. In the simulation, pseudo-random subblock partition scheme is used to partition the data blocks in PTS.

<span id="page-8-0"></span>

Fig. 3. Plot of ratio of computational complexity of PTS to SLM (C\_PTS<sup>/C\_SLM)</sup> versus number of subblock partitions V for  $L = 4$ ,  $f = 4$ , equal number of IFFTs,  $D = V$ , and the elements of the of phase factor vectors chosen from the set  $\{\pm 1\}$ .



Fig. 4. CCDF of PAPR of SLM, PTS, and OFDM signal with,  $L = 4$ , the elements of the phase rotation vectors chosen from the set  $\{1, -1\}$ .



Fig. 5. CCDF of PAPR of SLM, PTS, and OFDM signal with  $L = 4$ , and the elements of the phase rotation vectors chosen from the set  $\{\pm 1, \pm j\}.$ 

As can be seen from the plot, at CCDF of  $10^{-3}$  the PAPR difference is more than 1 dB when V = 1 for PTS and D =  $Q<sup>V</sup>$  = 2 for SLM. In fact, this difference decreases as  $D = Q<sup>V</sup>$  increases to 16. Therefore, the plot shows that SLM outperforms PTS in PAPR reduction capability for equal number of candidate OFDM symbols generated per given OFDM symbol in both schemes.

Figure 5 is a similar plot as Fig. [4](#page-8-0), but in this case the elements of the phase rotation vectors are chosen from the set  $\{\pm 1, \pm j\}$ . The plot illustrates the same result as Fig. [4.](#page-8-0)

It is shown above, Fig. [4,](#page-8-0) that for  $D = Q^V$ , SLM has a better PAPR reduction capability than PTS. Figure [6](#page-10-0) shows comparative computational complexities of SLM and PTS for equal number of candidate OFDM symbols, i.e.  $D = Q<sup>V</sup>$ . It shows the plot of the ratio  $C_{PTS}C_{SIM}$  in dB versus the number of subblock partitions V. The simulation is done for different number of subcarriers N,  $L = 4$ ,  $f = 4$ , and phase rotation factor set  $\{\pm 1\}$ .

Though SLM has a better PAPR reduction capability than PTS, it is more computationally complex than PTS for small values of V, all values of V that make  $C_{PTS}/$  $C_{SLM}$  < 1, and less computationally complex than PTS for large values of V, all values of V that make  $C_{\text{PTS}}/C_{\text{SLM}} > 1$ 

<span id="page-10-0"></span>

Fig. 6. Ratio of computational complexities of PTS to SLM (CPTS/CSLM) versus number of subblock partitions V for L = 4, f = 4, equal number of candidate OFDM symbols (D =  $Q^V$ ), and the elements to the phase vectors chosen from the set  $\{\pm 1\}$ .

## 5 Conclusion

In this paper two distortionless PAPR reduction techniques, namely SLM and PTS, are compared for their PAPR reduction capability and computational complexity. Both schemes produce a number of candidate OFDM symbols per given OFDM symbol intended for transmission and then choose one, with the least PAPR, among them for transmission. In SLM, D length LN candidate OFDM symbols are generated using D parallel LN-point IFFTs, one for each candidate OFDM symbol. Whereas in PTS the candidate OFDM symbols are constructed from V PTSs and hence V LN-point IFFTs are required to generate the LN samples of each PTS. The receiver of each scheme uses one FFT, the same as the OFDM system without any PAPR reduction scheme, to detect the transmitted OFDM symbol from the received OFDM symbol. Hence, the additional system complexity in using the two PAPR reduction schemes is found in their transmitters.

In [[5\]](#page-11-0), the two schemes have been compared for equal number of IFFTs used in the transmitters of both schemes, and found that PTS has a better PAPR reduction capability than SLM. But the authors didn't do a thorough computational complexity comparison other than stating that PTS is more computationally complex than SLM. However, this comparison is done in this paper and hence it is found that for any number of subblock partitions V used in the PTS scheme, PTS is always more computationally complex than SLM.

In this work the two schemes are compared through simulation in terms of PAPR reduction capability and computational complexity for a given number of candidate

<span id="page-11-0"></span>OFDM symbols generated per OFDM symbol in both schemes. Hence, it is shown through simulation that SLM outperforms PTS in PAPR reduction capability, which is illustrated in Fig. [3](#page-8-0). When the number of subblock partitions V used in PTS are small, all values of V that result in  $C_{PTS}/C_{SIM}$  < 1, SLM is more computationally complex than PTS, which is the cost to be paid for its better PAPR reduction capability, which is illustrated in Fig. [4.](#page-8-0) Hence, if a PTS scheme using small number of subblock partitions V can achieve the required PAPR reduction, PTS is better than SLM; otherwise SLM is better than PTS. Whereas for relatively large number of subblock partitions V used in PTS, all values of V that make  $C_{PTS}/C_{SLM} > 1$ , PTS is more computationally complex than SLM, which is illustrated in Fig. [4.](#page-8-0) Therefore, SLM is better than PTS in both PAPR reduction capability and computational complexity for all values of V that result in  $C_{\text{PTS}}/C_{\text{SLM}} > 1$ .

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