Mathematical Modeling and Dynamic Simulation of Gantry Robot Using Bond Graph

Tadele Belay Tuli(✉)

Department of Electromechanical Engineering, Addis Ababa Science and Technology University, Addis Ababa, Ethiopia
tademech2008@gmail.com

Abstract. This paper presents an initial mathematical modeling and dynamic simulation of gantry robot for the application of printing circuit on board. The classical modeling methods such as Newton-Euler, Kirchoff's law and Lagrangian fails to unify both electrical and mechanical system models. Here, bond graph approach with robust trajectory planning which uses a blend of quadratic equations on triangular velocity profile is modeled in order to virtually simulate it. In this paper, the algebric mathematical models are developed using maple software. For the sake of simulation, the model is tested on matlab by integrating robot models which are developed by using Solidwork.

Keywords: Robot · Bond graph · Dynamic simulation
Trajectory planning

1 Introduction

Robots are electromechanical devices that can perform different tasks that are difficult, dangerous, repetitive or dull for human beings [4]. They are programmable devices that follow a set of instructions to perform certain tasks. Now a day's robotics is so advanced that we have different kinds of robots, but all have something in common. First, all have mechanical structure to achieve a desired task. Second, all have a power supply and electrical units to control it. Third, robots have a programming aspect to help them decide how to interact with the environment [1,5,7].

A gantry robot consists of a manipulator mounted onto an overhead system that allows movement across a horizontal plane. Servo motors are used to deliver power to each axis by using a rack and pinion mechanism. They are usually large systems that are suitable for applications such as pick and place, cutting, welding and others.

Gantry robot systems provide the advantage of large work areas and better positioning accuracy which enables the robot to place a part correctly. They are easier to program with respect to motion, because they use with an X, Y, Z

F. Mekuria et al. (Eds.): ICT4DA 2017, LNICST 244, pp. 228–237, 2018.
https://doi.org/10.1007/978-3-319-95153-9_22

coordinate system. Another advantage is that they are less limited by floor space constraints [8]. Though they seem cheap compared to other robots like SCARA or articulated robots; they are not affordable to be implemented in emerging small scale industries that are flourishing in developing countries like Ethiopia. So, mathematical modeling and dynamic simulation of gantry robot using bond graph contributes a step toward the design and manufacture of gantry robot.

2 Kinematic Model of Gantry Robot

Kinematic modeling of gantry robot refers the study of link motions without considering the causing forces. The study takes place by assigning frames on links and joints. **Denavit-Hartenberg** (DH) convention simplifies the matters of frame assignment and creates a common language to fix frames in certain convection. In this paper modern DH-parameter convection is used.

2.1 Link Description

A link is a mechanical structure that connects two joint axes in space. Joint axes are vector that show direction of motion of link i with respect to $i-1$. The perpendicular distance between axes of $i-1$ and i is called link length a_{i-1} to joint axis i about a_{i-1} in right hand rule sense.

2.2 Intermediate Links in the Chain

Other parameters are defined based on common axes of consecutive links are link off set and joint angle. Link off set d_i is a parameter that describes the distance between two links (link lengths) along a joint axis. The off set on link i is d_i. Joint angle θ_i is the angle from link $i-1$ to i in right hand rule sense about these common axes.

2.3 Conventions on Modern DH - Parameter

Always assign Z_i-axis along axes of motion of joint i. While frame i is located at the intersection of a_i and joint i axis or Z_i-Axis. The axis of x_i points along a_i in the direction from joint i to joint $i+1$. The remaining axis will be completed using right hand rule [3].

The Homogeneous transformation matrix for modern DH-Matrix for link i is given by Eq. 1.

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

Where $c\theta_i$ and $s\theta_i$ are the short hands for $cos\theta_i$ and $sin\theta_i$ respectively.

Table 1. DH-parameters of Gantry robot

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	q_1	0
2	90	0	q_2	90
3	90	d	q_3	0

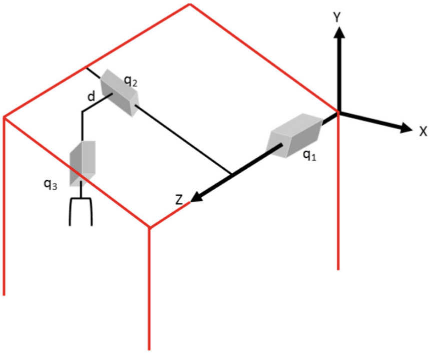

Fig. 1. Assignment of DH-reference frames

The DH parameters for this robot is shown on the Table 1.

Substituting the DH-parameters given on Table 1 in Eq. 1, we obtain reference frames for each links Fig. 1 as they are presented following.

Accordingly, link 1 is defined as,

$$ {}^{0}_{1}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

Reference frame for link 2 is given as;

$$ {}^{1}_{2}T = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & -q_2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3} $$

Reference frame for link 3 is given by

$$ {}^{2}_{3}T = \begin{bmatrix} 1 & 0 & 0 & d \\ 0 & 0 & -1 & -q_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

The Transformation of end effector frame to the base frame (reference frame) is the matrix product of each transformation matrices, Hence

$$ {}^{0}_{i}T = {}^{0}_{1}T \times {}^{1}_{2}T \times {}^{2}_{3}T \ ... \ {}^{i-1}_{i}T \tag{5} $$

where i is the number of links.

Hence, the description of end effector of gantry robot position relative to base frame is,

$$ {}^{0}_{3}T = \begin{bmatrix} 0 & 0 & 1 & q_3 \\ 0 & -1 & 0 & -q_2 \\ 1 & 0 & 0 & d+q_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6} $$

Since no joints are revolute all rotational variables are set to zero. Therefore velocity of end effector relative to base frame is;

$$ V_3^0 = \dot{q}_3 i - \dot{q}_2 j + \dot{q}_1 k \tag{7} $$

Acceleration of links is calculated as the same as velocity but effect of gravity is introduced by assuming the base is accelerating up ward at $g = 9.8\,\mathrm{m/s^2}$. Finally a_3 with respect to base is given as;

$$ a_3^0 = (\ddot{q}_3 i - g)i - \ddot{q}_2 j + \ddot{q}_1 k \tag{8} $$

3 System Modeling Using Bond Graph Method

Classical system modeling uses Newton's equations of motion to model mechanical systems and the Kirchoff's law is applicable for modeling the electrical systems behavior. If we consider hydraulic system, it is common to consider electric circuit analogy [6].

However, Bond graph method utilizes conservation of energy principle with a logical approach for studying a dynamic systems. It is a logical way to deal with multidisciplinary problems. They are used to map flow of power from one part of a system to another. It consists of subsystems linked together by lines representing power bonds.

3.1 Dynamic Model Formulation

Once we have defined the basic components of a system, the following is the modeling approach of dynamic model of gantry robot.

Bond graph of each motion axes are shown below.

Link 3
Link three has two effort sources; one for applied force and one for gravitational force (Fig. 2).
Note: p and q are state variables.

Here, we will attempt to derive the governing equations using questions and answers.
Question: What does the elements give to the system?
Bond 1

$$ e_1 = F_3 \tag{9} $$

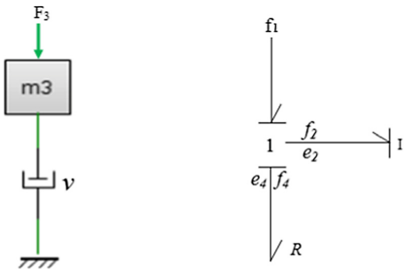

Fig. 2. Bond graph for Link 3

Bond 2

$$e_2 = G \tag{10}$$

Bond 3

$$f_1 = \frac{p_3}{m_3} \tag{11}$$

Bond 4

$$e_4 = v_3 f_4 \tag{12}$$

Question: What does the system give to storage elements?
It gives e_3 to bond 3.
Where

$$e_3 = \dot{p_3} = e_1 + e_2 - e_4 \tag{13}$$

Because we said that,

$$e = \dot{p} \tag{14}$$

$$f = \dot{q} \tag{15}$$

And

$$\begin{aligned} e_3 &= m_3\ddot{q_3} \\ e_1 &= F_3 \\ e_2 &= G \\ e_4 &= v_3\dot{q} \end{aligned} \tag{16}$$

From the substitution of the values of e_1, e_2, e_3 and e_4 into Eq. 13, we arrive at;

$$m_3\ddot{q_3} = F_3 + G - v_3\dot{q} \tag{17}$$

However, since

$$G = m_3 g \tag{18}$$

Finally, the governing system equation of link 3 is;

$$F_3 = m_3\ddot{q_3} + v_3\dot{q_3} - m_3 g \tag{19}$$

Note: (F_3) represents force applied on the system.

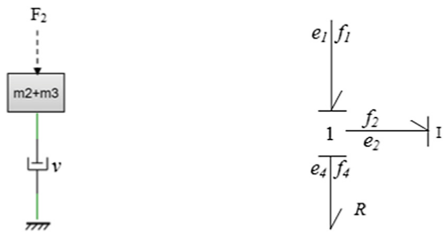

Fig. 3. Link 2

Link 2

Link two has one effort sources as it is given on Fig. 3.
Question: What does the elements give to the system?

$$\begin{aligned} e_1 &= F_2 \\ f_2 &= \tfrac{p_2}{m_2+m_3} \\ e_3 &= v_2\, f_3 \end{aligned} \tag{20}$$

Note: F_2 represents force applied on the system.
Question: What does the system give to storage elements?

$$e_2 = e_1 - e_3 \tag{21}$$

$$e_2 = \dot{p_2} = e_1 - e_3 \tag{22}$$

Further simplifying, we obtain the governing equation as;

$$F_2 = (m_2 + m_3)\,\ddot{q_2} + v_2\,\dot{q_2} \tag{23}$$

Link 1

Link two has one effort sources.
Question: What does the elements give to the system?

$$e_1 = F_1 \tag{24}$$

$$f_1 = \frac{p_1}{(m_3 + m_2 + m_1)} \tag{25}$$

$$e_3 = v_1\, f_3 \tag{26}$$

Note: F_1 represents force applied on the system.
Question: What does the system give to storage elements?

$$e_2 = \dot{p_1} = e_1 - e_3 \tag{27}$$

Finally by simplifying further, we will arrive at the final governing equation which is given by Eq. 28.

$$F_1 = (m_1 + m_2 + m_3)\,\ddot{q_1} + v_1\,\dot{q_1} \tag{28}$$

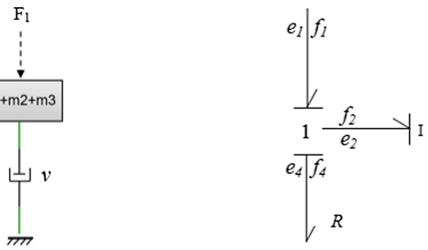

Fig. 4. Link 1

The dynamic system equation which unifies all the motion of the links of the gantry robot can be given in state space equation form as it is shown in equation,

$$F=\begin{bmatrix} m_1+m_2+m_3 & 0 & 0 \\ 0 & m_2+m_3 & 0 \\ 0 & 0 & m_3 \end{bmatrix}\begin{bmatrix} \ddot{q_1} \\ \ddot{q_2} \\ \ddot{q_3}+g \end{bmatrix}+\begin{bmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \end{bmatrix} \tag{29}$$

where

$$v=\begin{bmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_3 \end{bmatrix} \tag{30}$$

And

$$F=\begin{bmatrix} F_1 \\ F_2 \\ F_3 \end{bmatrix} \tag{31}$$

4 Direct Dynamics

Direct dynamics involves determining joint accelerations, velocity as well as position of end effector resulted by an applied force of link actuator.

$$\ddot{q}=M^{-1}\left[F-G(q)-F(\dot{q})\right] \tag{32}$$

M is the mass matrix which its inverse is given by Eq. 33.

$$M^{-1}=\begin{bmatrix} \frac{1}{m_1+m_2+m_3} & 0 & 0 \\ 0 & \frac{1}{m_2+m_3} & 0 \\ 0 & 0 & \frac{1}{m_3} \end{bmatrix} \tag{33}$$

The gravitational force $G(q)$ and it is represented with Eq. 34.

$$G(q)=m_3\begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} \tag{34}$$

The non conservative forces which is due to frictional forces is given as $F(q)$ by Eq. 35.

$$F(q) = \begin{bmatrix} v_1 & 0 & 0 \\ 0 & v_2 & 0 \\ 0 & 0 & v_3 \end{bmatrix} \begin{bmatrix} \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \end{bmatrix} \tag{35}$$

5 Trajectory Planning

In order to simulate the dynamic system model of the robot, it is necessary to plan a trajectory for the motion of the robot links. As a result, a quadratic blend of triangular velocity with a known time of maximum velocity is computed by considering three control points of the quadratic function [2].

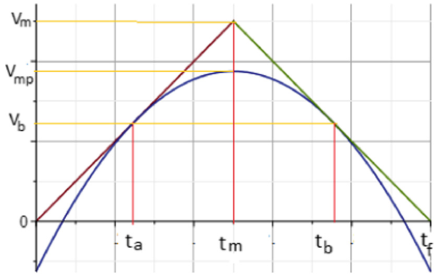

Fig. 5. Planning a triangular velocity profile with a quadratic blend

In order to study the motion profile, we approach by classifying the velocity motion in three regions. These are linearly accelerating, quadratic blending and linearly decreasing.

Case I: Linearly accelerating velocity profile

For all t, $t \in [t_0, t_a]$ the linear velocity for this case is forward one, which is given by;

$$v_a = a * t \tag{36}$$

where a is the constant linear acceleration absorbed by the links, t is the instantaneous time and v is the linear instantaneous velocity.

Case II: Quadratic blending velocity profile

For all t, $t \in [t_a, t_b]$ the linear velocity is not forward. Instead, we derive by using control points shown on Fig. 5 which are (t_a, v_b), (t_m, v_{mp}), and (t_b, v_b). And solving the quadratic equations, we obtain;

$$v_{ab} = \frac{-(t - t_m)(t - t_a - t_b + t_m)v_b + v_{mp}(t - t_b)(t - t_a)}{(t_b - t_m)(t_a - t_m)} \tag{37}$$

Considering v_b as the velocity where the linear motion starts to be in the state of quadratic motion, and v_{mp} is the rescaled maximum velocity of the motion.

From Fig. 5, we can understand that v_m is computed as;

$$v_m = a * t_m \tag{38}$$

And we can solve v_b and v_{mp} as

$$v_{mp} = \frac{v_m + v_b}{2} v_b = a * ta \tag{39}$$

Finally, we can simplify Eq. 37 as;

$$v_{ab} = -a\left(\frac{(-t - tb + 2 * tm) * ta + t * (t - tb)}{2 * tb - 2 * tm}\right) \tag{40}$$

Case III: Linearly decelerating velocity profile
For all t, $t \in [t_b, t_f]$, there is the linearly decelerating motion profile;

$$v_{bf} = v_b + a * (t - t_b) \tag{41}$$

However, v_b is computed by equating $t = t_b$ in Eq. 40. And finally come with;

$$v_{bf} = -(-ta + t - tb) * a \tag{42}$$

6 Simulation and Discussion

By considering motion characteristics of motor 1, we observe that the applied linear force to the motor from the stepper motor is linear (Fig. 7) for the quadratic portion of the velocity profile and remains constant where the velocity is in linear motion behavior. Accordingly, power required by the motor exhibits non linear property for the quadratic portion of the velocity profile (Fig. 6).

Now, by programming the mathematical models on matlab, we can simulate the robot in order to study the behavior of the dynamics. In this paper, matlab and solidworks software's are used as it is shown on Figs. 8 and 9

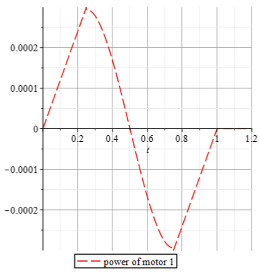

Fig. 6. Power required by Motor 1

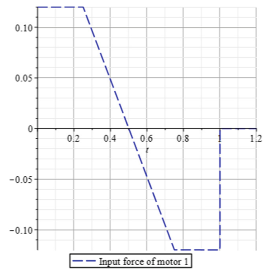

Fig. 7. Input force to Motor 1

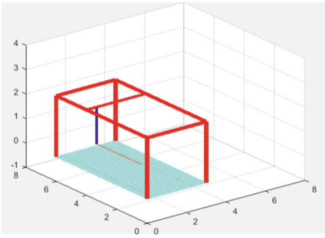

Fig. 8. Simulation of the robot on matlab.

Fig. 9. The simulation of gantry robot using CAD tools

7 Conclusion

In this paper, mathematical modeling of gantry robot for the application of printing on circuit board, is performed using bond graph modeling method. In order to simulate the dynamics of the robot model, I integrated matlab software with solidwork. Here, solidwork replicates the physical model of the robot where matlab controls the trajectory plan of the robot. The trajectory of the robot is planned by using a blend of quadratic function with a triangular velocity profile which is suitable for small interval of time in between subsequent points.

References

1. Sciavicco, L., Villani, L.: Robotics: Modelling, Planning and Control. Springer, Heidelberg (2009). https://doi.org/10.1007/978-1-84628-642-1
2. Corke, P.: Robotics, Vision and Control Fundamental Algorithms in MATLAB, vol. 73. Springer, Heidelberg (2011). https://doi.org/10.1007/978-3-319-54413-7
3. Craig, J.J.: Introduction to Robotics: Mechanics and Control. Pearson/Prentice Hall, Upper Saddle River (2005)
4. Williams, B.: An Introduction to Robotics. http://www.ohio.edu/people/williar/html/PDF/IntroRob.pdf
5. Dietz, T., et al.: Programming system for efficient use of industrial robots for deburring in SME environments. In: Proceedings of 2012 7th German Conference on Robotics ROBOTIK. VDE (2012)
6. Das, S.: Mechatronic Modeling and Simulation Using Bond Graphs. CRC Press, Boca Raton (2009)
7. Cubero, S.: Industrial Robotics: Theory, Modelling and Control. Pro Literatur Verlag, Augsburg (2006)
8. www.robots.com/faq/show/what-are-gantry-robots