



# Energy-Efficiency Maximisation in Random Cognitive Radio Networks

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**Abstract.** Energy-efficient cognitive radio network has received considerable attention recently because of improving spectrum and energy efficiency. In light of such observations, we present a model for cognitive radio network based on stochastic geometry theory where transmitters and receivers are distributed according to Poisson point process. In this paper, we focus on the optimization problem where energy efficiency is maximized under the constraint of outage probability for primary network and secondary network. We show that the energy efficiency is increased with the increment of threshold for primary network. However, the energy efficiency is maximum for a certain value of threshold in secondary network.

**Keywords:** Cognitive radio network · Stochastic geometry  
Poisson point process · Energy efficiency

## 1 Introduction

With the increasing demand for new wireless services and applications, as well as the increasing demand for higher capacity, wireless networks have become highly heterogeneous. In this context, it is essential to have updated information on the radio environment to enhance overall network performance. To support these increasing demands, researchers and engineers have made numerous attempts to introduce new technologies and different network deployments. The concept of cognitive radio (CR) is one of them, which facilitates the flexible usage of the radio spectrum [1]. Cognitive radios are being considered as an alternative to current wireless devices in many applications such as smart grid [2], public safety [3], cellular networks [4], and wireless medical networks [5]. Besides the technology, the topology and network deployment also play a significant role to meet these ever increasing demand. Therefore recent practice of cellular network, the base station (BS) deployment shifted from deterministic model (Wyner, hexagonal

and square lattice model) to a random model [6]. Another motivation of moving towards random deployment is energy efficiency. The authors in [7] reported that, the BS in a cellular network consumes the major portion of total energy consumption and contributes more than 70% of the electric bill. Moreover, higher energy consumption results not only in a rise in CO<sub>2</sub> emissions but also in a significant amount of harmful radiation. A recent study found that the global CO<sub>2</sub> share of the ICT is 5% and which is rising at a fast pace, due to the increasing number of network applications, services, and subscribers [8]. Therefore, an energy-efficient green wireless communication becomes a prerequisite.

In order to study the system performance such as energy efficiency, outage probability, coverage probability, and interference in CR networks, the stochastic method is commonly used in the literature [9–12]. In [9], the authors use the Poisson, Binomial, Hard Core point process and Poisson cluster process to evaluate the performance of multi-tier CR networks. However, the authors in [10] show that the use of Poisson point process (PPP) is impractical for assessing interference and outage probability due to the interaction between the primary and CR users through the exclusion region. The authors in [11] investigate the outage probability in two-tier cognitive heterogeneous cellular networks using the theory of homogeneous PPP. Following this line of thought, the authors show that outage probability can be decreased by the proper choice of the spectrum sensing in femto cells. In [12], the authors discuss the coverage probability and transmission rate in Rayleigh-lognormal fading.

In this work, we have analysed the performance of a CR network based on the theory of stochastic geometry. The main contribution of this work is two-fold: (a) We maximize the energy efficiency of the primary network under the constraint of outage probability of the primary receiver. (b) We maximize the energy efficiency of the secondary network under the constraint of outage probability of the secondary receiver. Finally, we present the numerical results of optimal transmission power and density. Also, we show the maximum energy efficiency for different thresholds. The rest of the paper is organized as follows. In Sect. 2, we present the network model and timeslot structure that has been considered in this work. In Sect. 3, we first describe the outage probability for the primary receiver (PR) and secondary receiver (SR) followed by energy efficiency for the primary network (PN) and secondary network (SN). An optimization problem is also discussed to maximize the energy efficiency for both PN and SN. In Sect. 4, the numerical results and discussion for a CR network are provided. Finally, we conclude our work in Sect. 5.

## 2 Network Model

Consider a downlink scenario of a CR network consisting of secondary users (SUs) and primary users (PUs) as shown in [13, 14]. In a CR network, two types of networks are comprised, such as the primary network (PN) and secondary network (SN) or cognitive radio network (CRN)<sup>1</sup>. Primary transmitters (PTs), primary receivers

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<sup>1</sup> cognitive radio and secondary user are used interchangeably.

(PRs) and secondary transmitters (STs) are included in PN and also STs, secondary receivers (SRs) and PTs are included in SN. All active STs and PTs are assumed to be spatially distributed in accordance with homogeneous PPPs  $\Phi_{st}$  and  $\Phi_{pt}$  with densities  $\lambda_s$  and  $\lambda_p$ , where  $\lambda_s$  and  $\lambda_p$  are the average numbers of STs and PTs per unit area, respectively. Consequently, the associated SRs and PRs are located in accordance with independent PPPs  $\Phi_{sr}$  and  $\Phi_{pr}$  with densities  $\lambda_{sr}$  and  $\lambda_{pr}$ , respectively. We assume a scenario of constant transmit power, and denote by  $P_s$  and  $P_p$  the transmit power of STs and that of PTs, respectively.

In this work, we have considered time slotted network operation on licensed channel. Predominately, PUs are the legitimate users if the licensed channel and SUs can only use it when it is not used by PUs. We assume that the activity of PUs follows the two state Markov model which are ON and OFF period. The ON period indicates the presence of PUs, hence channel is busy. The OFF periods refers the absence of PUs, thus channel free and can be used by SUs. In order to detect the free slots the SUs, first perform spectrum sensing at the beginning of each frame during the time interval of  $[0, T_s]$ . This sensing result is used to create a list of available channels that can be used as a potential data channel. An SU will select the best channel from the available channel list based on QoS requirement and transmit data during the time interval of  $[T_s, T_s + T_p]$ .

### 3 Analysis of Performance Metric

In this section, the outage probability and energy efficiency have provided based on [13, 14] where outage probability and energy efficiency have discussed in details. This paper focuses on the optimization problem. We maximize the energy efficiency under the constraint of outage probability. In the downlink of CR networks, the outage probability depends on the signal to interference plus noise ratio (SINR) of each receiver (i.e., PR or SR).

#### 3.1 Outage Probability

In wireless communication, the outage refers to a significant performance degradation if the SINR is less than a certain threshold. The outage probability can be defined as the probability of SINR being less than the threshold.

**Outage Probability of PR.** A PR is successfully decoded a packet when it's SINR is above the threshold  $\theta_p$ . Therefore, the outage at the PR occurs whenever the SINR at the PR declines below a threshold  $\theta_p$ . According to [13, 14], the outage probability for PR can be expressed as

$$\epsilon_p = 1 - \frac{\lambda_p}{\lambda_p - \lambda'_p - p_{im}\lambda_s\beta^2 + 0.5\nu}, \quad (1)$$

where  $\nu = \mathbb{L}_p V(\theta_p, 4) + \mathbb{L}_s W_b(\theta_p, 4, \beta)$ ,  $V(\theta_p, 4) = \Gamma(-1/2, \mu_p \theta_p g_p) - \Gamma(-1/2)$ ,  $W_b(\theta_p, 4, \beta) = \Gamma(-1/2, \mu_p \theta_p g_s P_s P_p^{-1} \beta^{-4}) - \Gamma(-1/2)$ ,  $\mathbb{L}_p = \lambda'_p \sqrt{\mu_p g_p \theta_p}$ ,

$\mathbb{L}_s = \lambda_s p_{im} \sqrt{\mu_p g_s \theta_p P_p^{-1} P_s}$ , and  $p_{im} = p_{ic, im} p_{st, im}$ .  $P_p$  is the transmission power of PT,  $P_s$  is the transmission power of ST,  $\theta_p$  is the threshold,  $\lambda'_p$  is the density of PTs located in the area  $\mathbb{R}^2 \setminus b(0, r_p)$ ,  $b(x, y)$  represents a sphere of radius  $y$  centered at point  $x$ ,  $g_p$  is the interference channel gain between the tagged PR and the interfering PT,  $g_s$  is the interference channel gain between the tagged PR and the interfering ST,  $\beta$  is a constant parameter that depends on the cell radius,  $p_{ic, im}$  and  $p_{st, im}$  are the *probability of unoccupied channel selection* and the *probability of successful transmission* for imperfect detection, respectively.  $p_{im}$  is related with  $N$ ,  $M$ ,  $N_s$  described in [14] where  $N$  is the total number of PU channels,  $M$  is the number of unoccupied channels and  $N_s$  is the sensed channels from the unoccupied channels  $M$ , respectively.

**Outage Probability of SR.** The SUs are unlicensed users in this network and only transmit a signal if the channel is free. The outage probability of a SR can be defined as one minus coverage probability of the SR. The coverage probability for a SR is the probability that the SINR of the tagged SR is greater than the threshold  $\theta_s$ . According to [13, 14], the outage probability for SR can be expressed as

$$\epsilon_s = 1 - \frac{\lambda_s}{\lambda_s - p_{im} \lambda'_s + \nu_1}, \quad (2)$$

where  $\nu_1 = p_{ic, pt} \lambda_p \Gamma(1/2) \sqrt{\mu_s \theta_s G_p P_s^{-1} P_p} + 0.5 p_{im} \lambda'_s \sqrt{\mu_s \theta_s G_s} X(\theta_s, 4)$ ,  $X(\theta_s, 4) = \Gamma(-1/2, \mu_s \theta_s G_s) - \Gamma(-1/2)$ , and  $p_{im} = p_{ic, im} p_{st, im} \cdot \theta_s$  is the threshold,  $\lambda'_s$  is the density of STs located in the area  $\mathbb{R}^2 \setminus b(0, r_s)$ ,  $G_s$  is the interference channel gain between the tagged SR and the interfering ST,  $G_p$  is the interference channel gain between the tagged SR and the interfering PT and  $p_{ic, pt}$  is the probability of selecting an unoccupied channel from the total number of PU channels.

### 3.2 Energy Efficiency

In order to improve the energy consumption, it is important to understand the relationship between energy efficiency and area spectral efficiency. In general, the energy efficiency can be defined as the amount of transmitted bits per unit energy consumption and the amount of transmitted bps per unit bandwidth is known as area spectral efficiency. A different definition is presented in [15] where energy efficiency is defined as the ratio of area total throughput to area total power consumption and the area spectral efficiency is defined as the area total throughput per unit bandwidth per unit area. Hence the energy efficiency ratio ( $\eta$ ) can be written as the ratio of area spectral efficiency over average network power consumption.

**Energy Efficiency of PN.** The energy efficiency for the downlink channel is expressed as

$$\begin{aligned}\eta_{PN} &= \frac{(1 - \epsilon_p) \log_2(1 + \theta_p)}{p_{ic,pt} P_p + P_{pc}} \\ &= \frac{\lambda_p \log_2(1 + \theta_p)}{\{\lambda_p - \lambda'_p - p_{im} \lambda_s \beta^2 + 0.5\nu\} (p_{ic,pt} P_p + P_{pc})},\end{aligned}\quad (3)$$

where  $P_{pc}$  is the constant circuit power consumption in the PT. For simulation purposes, we assume  $P_{pc} = 50$  kW [16].

**Energy Efficiency of SN.** It is considered that the SN consists of ST, SR and PT. The energy efficiency can be expressed as

$$\begin{aligned}\eta_{SN} &= \frac{(1 - \epsilon_s) \log_2(1 + \theta_s)}{\mathbb{P}_{pc}} \\ &= \frac{\lambda_s \log_2(1 + \theta_s)}{\{\lambda_s - p_{im} \lambda'_s + \nu_1\} \mathbb{P}_{pc}}, \quad (\text{b/J/Hz})\end{aligned}\quad (4)$$

where  $\mathbb{P}_{pc} = N_{trx} P_o T_t + p_{im} \Delta_s P_s T_p + P_{ss} T_s + P_s T_p$  and  $T_t = T_s + T_p$ .  $T_s$  is the sensing period,  $T_p$  is the transmission period,  $P_{ss}$  is the power consumption during the sensing period,  $N_{TRX}$  is the number of transceivers,  $P_o$  is the load-independent power consumption at a nonzero output power, which depends on the circuit power consumption of the four types of BS (i.e., macro, micro, pico, and femto).  $\Delta_s$  is the slope of the load-dependent power consumption, which is the transmission power consumed in the RF transmission circuits for the four types of BS. From the power consumption expression, we observed that  $P_o$  and  $\Delta_s$  are fixed for any given BS. We can verify two aspects (i.e.,  $N_{TRX}$  and  $P_s$ ) for use in the design of an energy-efficient network.

### 3.3 Optimization Problem

In this section, we introduce the optimization problem of PN and SN which are discussed below:

**Maximization of Energy Efficiency of PN.** In PN, the energy efficiency is maximized where outage probability of PR is constraint. In this optimization problem, we determine the  $P_p$ ,  $\lambda_p$ ,  $\lambda'_p$  that maximize the energy efficiency. The optimization problem can be formulated as

$$\max_{P_p, \lambda_p, \lambda'_p} \eta_{PN}(P_p, \lambda_p, \lambda'_p) \quad (5a)$$

$$\text{s.t.} \quad \text{C1} : \epsilon_p(P_p, \lambda_p, \lambda'_p) \leq \epsilon_{pp} \quad (5b)$$

$$\text{C2} : \lambda'_p \leq \lambda_p \quad (5c)$$

$$\text{C3} : P_p \geq 0, \lambda_p \geq 0, \lambda'_p \geq 0. \quad (5d)$$

In (5), constraint C1 ensures that the outage probability  $\epsilon_p$  does not exceed the particular value of  $\epsilon_{pp}$ . C2 verifies that  $\lambda'_p$  is less than  $\lambda_p$ . C3 confirms the non-negative value of  $P_p$ ,  $\lambda_p$ , and  $\lambda'_p$ .

The optimization problem is difficult to solve mathematically. However, we can solve it numerically using the *fmincon* function in MATLAB software [17]. The objective function in (5a) is an increasing function and the feasibility condition is satisfied. Moreover, there are optimal solutions (e.g.,  $P_p$ ,  $\lambda_p$ , and  $\lambda'_p$ ) that satisfy the constraints (5b)–(5d). The numerical findings are discussed in detail in Sect. 4.

**Maximization of Energy Efficiency of SN.** In SN, the energy efficiency is maximized where outage probability of SR is constraint. In this optimization problem, we determine the  $P_s$ ,  $\lambda_s$ ,  $\lambda'_s$  that maximize the energy efficiency. The optimization problem can be formulated as

$$\max_{P_s, \lambda_s, \lambda'_s} \eta_{SN}(P_s, \lambda_s, \lambda'_s) \quad (6a)$$

$$\text{s.t. C1 : } \epsilon_s(P_s, \lambda_s, \lambda'_s) \leq \epsilon_{ss} \quad (6b)$$

$$\text{C2 : } \lambda'_s \leq \lambda_s \quad (6c)$$

$$\text{C3 : } P_s \geq 0, \lambda_s \geq 0, \lambda'_s \geq 0. \quad (6d)$$

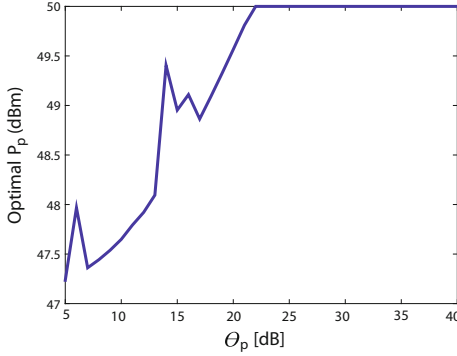
In (6), constraint C1 ensures that the outage probability  $\epsilon_s$  does not exceed the constant value of  $\epsilon_{ss}$ . C2 verifies that  $\lambda'_s$  is less than  $\lambda_s$ . C3 confirms the non-negative value of  $P_s$ ,  $\lambda_s$ , and  $\lambda'_s$ . Following the above optimization, the optimal solution (i.e.,  $P_s$ ,  $\lambda_s$ , and  $\lambda'_s$ ) is found by the same procedure. The details of this optimization are presented in Sect. 4.

## 4 Numerical Results and Discussion

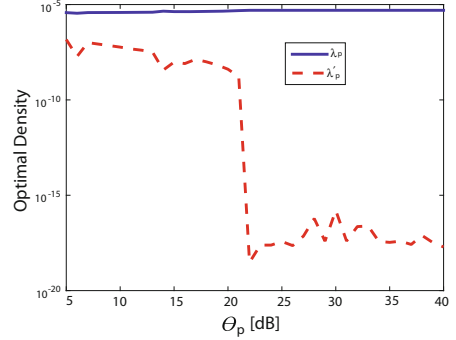
In this section, based on the previous analysis of optimization problem, we present numerical results to evaluate the performance of energy efficiency. In the following numerical results, simulation parameters are provided as  $N = 25$ ,  $M = 10$ ,  $N_s = 15$ ,  $T_s = 1$  ms,  $T_p = 4$  ms,  $\Delta_s = 2.8$ ,  $P_o = 84$  W,  $P_{ss} = 0.2$  W,  $\mu_p = 0.5$ ,  $\mu_s = 0.5$ ,  $\beta = 1$ ,  $\epsilon_{ss} = 0.1$  and  $\epsilon_{pp} = 0.1$ .

Figure 1 shows the optimal value of  $P_p$  in dBm for different value of threshold  $\theta_p$  considering the value of densities  $\lambda_s = 25/\text{km}^2$ ,  $\lambda'_s = 20/\text{km}^2$  and  $P_s = 30$  dBm. The optimal  $P_p$  is fluctuated from  $\theta_p = 5$  dB to  $\theta_p = 22$  dB. Then the optimal  $P_p$  is stable. As shown in Fig. 2, as  $\theta_p$  increases, the optimal value of  $\lambda_p$  slightly increases, but  $\lambda'_p$  decreases.

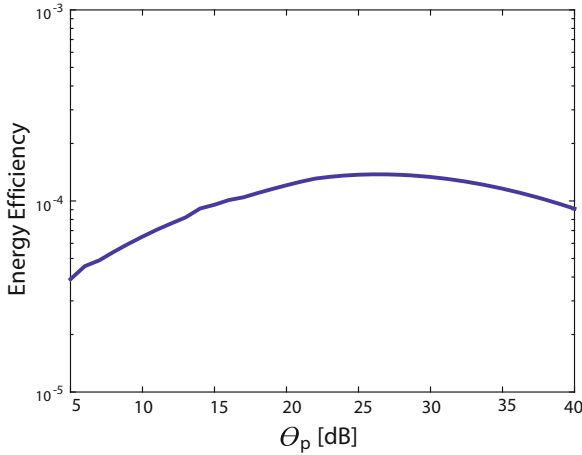
Figure 3 illustrates that the energy efficiency increases with the increment of  $\theta_p$  at  $\theta_p = 27$  dB and then energy efficiency decreases with the increment of  $\theta_p$ . The maximum energy efficiency is  $1.3 \times 10^{-4}$  at  $\theta_s = 27$  dB. Obviously, as  $\theta_p$  increases, power consumption reduces so energy efficiency is maximized.



**Fig. 1.** Optimal  $P_p$  for different  $\theta_p$  in Eqs. (5a)–(5d).



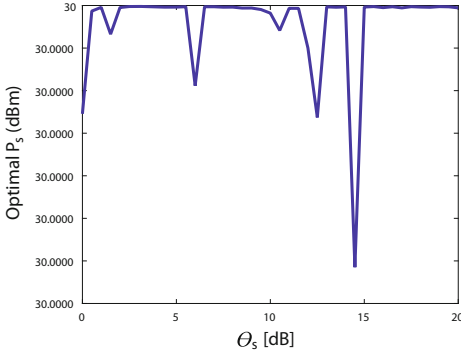
**Fig. 2.** Optimal density (i.e.,  $\lambda_p$  and  $\lambda'_p$ ) for different  $\theta_p$  in Eqs. (5a)–(5d).



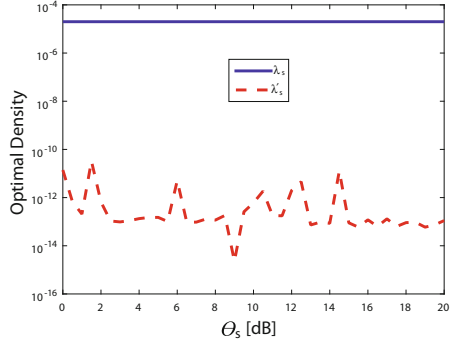
**Fig. 3.** Energy Efficiency for different  $\theta_p$  in Eqs. (5a)–(5d).

Now we explore the energy efficiency of SN considering the value of densities  $\lambda_p = 15/\text{km}^2$ ,  $\lambda'_p = 5/\text{km}^2$  and  $P_p = 50$  dBm. A careful observation of Fig. 4 indicates that the optimal value of  $P_s$  is dramatically changing. In Fig. 5, similar behaviors can be observed for density ( $\lambda_s$  and  $\lambda'_s$ ). The optimal  $\lambda_s$  is constant for different  $\theta_s$ . But, the optimal value of  $\lambda'_s$  is fluctuated.

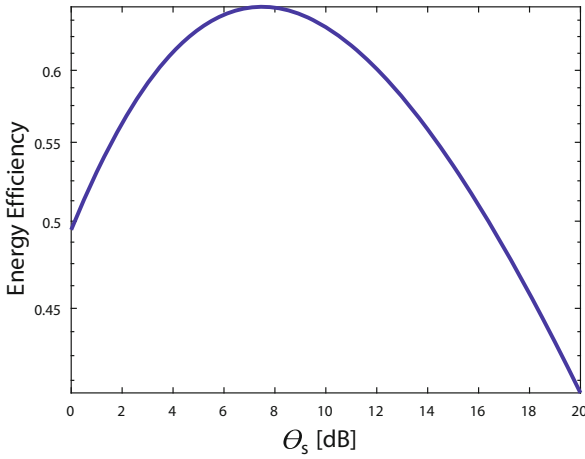
Figure 6 shows the relationship between energy efficiency and  $\theta_s$ . Interestingly, we observe that the energy efficiency is increased at  $\theta_s = 7.5$  dB then energy efficiency is decreased with the increment of  $\theta_s$ . For the optimal setting of  $P_s$ ,  $\lambda_s$  and  $\lambda'_s$ , the maximum energy efficiency is achieved for different value of  $\theta_s$ . The maximum energy efficiency is 0.65 at  $\theta_s = 7.5$  dB.



**Fig. 4.** Optimal  $P_s$  for different  $\theta_s$  in eqs. (6a)–(6d).



**Fig. 5.** Optimal density (i.e.,  $\lambda_s$  and  $\lambda'_s$ ) for different  $\theta_s$  in eqs. (6a)–(6d).



**Fig. 6.** Energy Efficiency for different  $\theta_s$  in eqs. (6a)–(6d).

## 5 Conclusion

In this paper, we have investigated the optimization problem for PN and SN in CR network by using the theory of stochastic geometry. In particular, we maximize the energy efficiency for PN and SN in term of outage probability. Also, we find the optimal transmission power and density of PN and SN for the network that can achieve the maximum energy efficiency. Numerical results demonstrate that the energy efficiency increases in an increase of threshold. For the SN, the energy efficiency is maximum for a particular value of threshold.



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