Non-orthogonal Multiple Access for Similar Channel Conditions

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Abstract. Non-orthogonal multiple access (NOMA) is considered as promising multiple access (MA) scheme for upcoming fifth generation (5G) systems. The performance of NOMA is highly dependent upon having significant channel gain difference among users. In this paper, we focus on the situation of similar channel conditions and propose a channel gain stretching (CGS) strategy to apply NOMA more effectively under these conditions. In order to evaluate the performance, we derive a closed-form expression of the outage probability. Numerical results are also presented to validate the accuracy of the derived results and also to compare the performance of NOMA with and without CGS, and orthogonal MA (OMA).

Keywords: Non-orthogonal multiple access
Channel gain stretching · Power allocation

1 Introduction

In recent years, there has been a tremendous growth in the number of cellular subscribers coupled with the massive increase in portable devices to experience diverse services spanning from simple voice to high data rate real time multimedia applications. Moreover, this explosive growth in mobile devices requires a vigorous demand of seamless and ubiquitous connectivity. In order to meet the anticipated demands of future fifth generation (5G) communication systems, these trends pose a major challenge to network operators due to the scarcity of current spectrum resources [1].

Many potential solutions are proposed to realize the concept of 5G among which millimeter waves, massive multiple input multiple output (MIMO), full-duplex, heterogeneous deployments and software defined networks have received notable attention from both academia and industry [2]. Nevertheless, multiple access (MA) scheme always play a critical role to enhance spectrum efficiency in a cost-effective manner.

Non-orthogonal multiple access (NOMA) has been considered as a latest member of MA family and is proposed as a promising MA technology for 5G systems. The key idea of NOMA is that it superimposes multiple users into single resource (time/frequency/code) at the transmitter side by allocating different

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power levels to each user and applies successive interference cancellation (SIC) at the user's receiver to mitigate intra-user interference. Some of the key benefits of deploying NOMA include improved spectral efficiency, enhanced throughput and better fairness among users [3].

1.1 Related Work, Motivation and Contributions

The initial investigations on NOMA were conducted in [4] via system level simulations. The authors reported superior throughput and performance of NOMA over conventional orthogonal MA (OMA) scheme. The outage performance of NOMA with randomly deployed users is analytically derived and then evaluated in [5]. The application of MIMO systems to NOMA is explored in [6]. The authors presented novel design of precoder which is then utilized to suppress the inter-beam interference. The impact of user pairing on the performance of NOMA system is investigated in [7]. The authors discussed and evaluated the performance of two possible implementations of NOMA systems, namely fixed power allocation NOMA and cognitive-radio-inspired NOMA (CR-NOMA). A NOMA-based device-to-device (D2D) communication is proposed in [8] with underlay cellular network. The concept of group D2D communications is introduced in which D2D transmitter is communicating with multiple D2D receivers via NOMA protocol. In order to manage the interference from underlying uplink cellular communication, an optimal resource allocation strategy was proposed.

More recently, cooperative NOMA is proposed in [9] where strong user is equipped with full-duplex functionality. The authors proposed a scheme to improve the outage performance of a weak user using cooperative and direct transmissions by invoking D2D communications between strong and weak NOMA user pair. A large-scale D2D network is considered in [10], where the authors proposed a cooperative hybrid automatic repeat request assisted NOMA scheme to improve the outage and throughput performance of the D2D users.

In all the aforementioned works, the underlying assumption is to maintain a significant channel gain difference among NOMA users. However, this assumption may not always hold and under those scenarios, it may result in improper rate and power allocation that could result in complete outage [5]. This motivates us to propose a method that artificially generates a channel gain difference among different NOMA users for proper power allocation under situations of similar channel conditions. To this end, the main contributions of this work are summarized below:

- We find minimum optimal power allocation coefficients that would guarantee to meet the targeted rate of each user.
- We propose a channel gain stretching (CGS) scheme to apply NOMA effectively under comparable channel conditions.
- In order to evaluate performance, exact expressions for outage probability are derived.
- Numerical results are shown to validate the accuracy of the analysis, as well as compare outage performance of the NOMA under proposed CGS scheme to NOMA without CGS and OMA.

2 System Model

Consider a single-input single-output system with single source (S) located at the center of a disc D with radius R_D. We focus on a downlink scenario where S is communicating with M users via NOMA protocol. The users are randomly uniformly distributed inside disc D. The channel gain between user m and source S is given as, $h_m = g_m d_m^{-\alpha}$, where g_m is the power fading coefficient that follows exponential distribution with unit mean, d_m is the distance between user m and source S and α is the path loss exponent.

In this work, we consider a scenario of similar channel conditions. These situations may arise in practical scenarios that could include (1) Indoor deployments with source implementing NOMA and where channel conditions for users are expected to be very similar, (2) NOMA based group D2D communication forming a small local cell where the users exit in proximity of each other and are clustered around group head/transmitter [8] and (3) User-centric deployments of small-cell base stations where the users are clustered around small-cell base stations. Under these kind of situations, all M NOMA users have similar channel conditions i.e. $h_i \approx h_j, i \neq j, 1 \leq i, j \leq M$. It should be noted here that the channel gains of all users are not exactly the same and hence user ordering is still possible. Without loss of generality, the users are ordered as $h_1 \leq ... \leq h_M$. Consequently, the power allocation coefficients, denoted as, $a_m, 1 \leq m \leq M$, and are sorted as, $a_1 \geq ... \geq a_M$. The calculation of power allocation coefficients is discussed in the next sub-section.

2.1 Minimum Required Power Allocation Coefficients

Let us denote R_m and $\bar{R}_m$ by achievable and targeted rates of the user m respectively. Then, $\bar{R}_m$ of user m is met if:

$$R_m \geq \bar{R}_m. \tag{1}$$

Equation (1) can be further simplified as:

$$\log_2\left(1 + \frac{Ph_m a_m}{Ph_m \sum_{i=m+1}^{M} a_i + \sigma^2}\right) \geq \bar{R}_m$$
$$a_m \geq \tau_m \left(\sum_{i=m+1}^{M} a_i + \frac{1}{\Upsilon h_m}\right), \tag{2}$$

where $\tau_m = 2^{\bar{R}_m} - 1$, $\Upsilon = \frac{P}{\sigma^2}$ is the transmit signal-to-noise ratio (SNR), P is the maximum transmit power at the base station and σ^2 is the variance of additive noise. In order to proceed forward, we formulate the following optimization problem to obtain the optimal power allocation coefficients.

$$\min \sum_{m=1}^{M} a_m \tag{3}$$

$$s.t.(2) \tag{4}$$

To this end, the following lemma states the optimal power allocation coefficients that are sufficient to meet the users' targeted rates.

Lemma 1. *The optimal power allocation coefficients are obtained by solving the problem (3) and are given as:*

$$a_m = \tau_m \left(\sum_{i=m+1}^{M} a_i + \frac{1}{\Upsilon h_m} \right) \tag{5}$$

Proof. By inspecting problem (3), it can be observed that (3) is convex. Hence, a necessary and sufficient condition to obtain its optimal solution follows by the application of Karush-Kuhn-Tucker (KKT) conditions. The detailed proof follows a standard application of KKT conditions and hence is skipped. Curious reader is referred to see a Theorem 1 [11] for the detailed proof. □

3 Proposed Channel Gain Stretching Method and Outage Analysis

In this section, we first propose a CGS method that artificially generates channel gain difference among different NOMA users. Then, under the proposed CGS scheme, we analyse the outage probability of the considered system.

3.1 Channel Gain Stretching Method

Under situations of similar channel conditions, we propose a following transformation to artificially generate channel gain difference among NOMA users:

$$\bar{h}_m = k_{1,m} \left(h_m\right)^{k_{2,m}}, \tag{6}$$

where $\bar{h}_m$ is the transformed channel gain of user m and $k_{1,m} > 0, k_{2,m} > 0$ are positive constants for user m and are selected in such a way to achieve a significant difference among channel gains of users.

Example: Consider a case of two users with $(h_1, h_2) = (0.87, 0.9)$. Now applying (6) with $k_{1,1} = k_{2,1} = 0.5, k_{1,2} = 3, k_{2,2} = 3.5$ results in stretched coefficients as $\left(\bar{h}_1, \bar{h}_2\right) = (0.46, 2)$. The power allocation coefficients are then computed using (5) for a given SNR and targeted rate.

3.2 Outage Analysis

The outage occurs at the user m receiver whenever it fails to decode the message signal of any higher order user $j, 1 \leq j \leq m$. Then, the outage probability of user m in decoding user j can be expressed as:

$$
\begin{aligned}
P_{m\to j} &= \Pr\left(\frac{\bar{h}_j a_j \Upsilon}{\bar{h}_j \Upsilon \sum_{i=m+1}^{M} a_i + 1} < \tau_j\right) \\
&= \Pr\left[\bar{h}_j \Upsilon \left(a_j - \tau_j \sum_{i=m+1}^{M} a_i\right) < \tau_j\right] \\
&= \Pr\left(\bar{h}_j < \frac{\tau_j}{\Upsilon\left(a_j - \tau_j \sum_{i=m+1}^{M} a_i\right)}\right) \\
&= \Pr\left[h_j < \left(\frac{\varphi_j}{k_{1,j}}\right)^{k_{2,j}}\right] \\
&= F_{h_j}\left(\theta_j\right),
\end{aligned} \tag{7}
$$

where $\varphi_j = \frac{\tau_j}{\Upsilon\left(a_j - \tau_j \sum_{i=m+1}^{M} a_i\right)}$, $\theta_j = \left(\frac{\varphi_j}{k_{1,j}}\right)^{k_{2,j}}$ and F_{h_j} is the cumulative distribution function (CDF) of h_j. Now let us define $\theta_m^{\max} = \max\{\theta_1, ..., \theta_m\}$. The outage probability at user m is then given as:

$$
P_m = F_{h_m}\left(\theta_m^{\max}\right). \tag{8}
$$

In order to obtain outage probability P_m of user m, we require CDF of h_m which is obtained by analyzing order statistics [12] and is given as:

$$
P_m = \mu_m \sum_{l=0}^{M-m} \binom{M-m}{l} (-1)^l \int_0^{\theta_m^{\max}} \left(F_{\hat{h}}(x)\right)^{m+l-1} f_{\hat{h}}(x) dx, \tag{9}
$$

where $F_{\hat{h}}$ and $f_{\hat{h}}$ are the CDF and probability density function (PDF) of the unordered channel gain $\hat{h}$ respectively. The CDF $F_{\hat{h}}$ of the unordered channel gain is given as [13]:

$$
\begin{aligned}
F_{\hat{h}}(x) &= \frac{2}{R_D^2} \int_0^{R_D} \left(1 - e^{-(1+z^{\alpha})x}\right) z dz \\
&\overset{(a)}{=} \frac{\delta}{R_D^2} \int_0^{R_D^{\alpha}} \left(1 - e^{-(1+y)x}\right) y^{\delta-1} dy \\
&\overset{(b)}{=} 1 - \delta e^{-x} \mathrm{B}(1,\delta) \Phi(\delta, 1+\delta; -x R_D^{\alpha}),
\end{aligned} \tag{10}
$$

where (a) and (b) are obtained by a change of variable from $z^{\alpha} \to y$ and applying Eq. 3.383 of [14] respectively, $\delta = \frac{2}{\alpha}$, $\mathrm{B}(\cdot,\cdot)$ is the beta function and $\Phi(\cdot,\cdot;\cdot)$ is the confluent hypergeometric function. Now, take the derivative of (10) to obtain $f_{\hat{h}}$ and substitute $F_{\hat{h}}$ and $f_{\hat{h}}$ in (9), P_m can be expressed as:

$$P_m = \mu_m \sum_{l=0}^{M-m} \binom{M-m}{l} (-1)^l \int_0^{\theta_m^{\max}} \delta \mathrm{B}(1,\delta) e^{-x} \left[\Phi(\delta, 1+\delta; -xR_D^{\alpha}) + \rho \Phi(1+\delta, 2+\delta; -xR_D^{\alpha})\right] \times \left[1 - \delta e^{-x} \mathrm{B}(1,\delta) \Phi(\delta, 1+\delta; -xR_D^{\alpha})\right]^{m+l-1} dx. \quad (11)$$

The analytical solution of (11) is difficult to obtain and hence we apply Gaussian-Chebyshev quadrature to approximate the outage probability of user m as follows:

$$P_m = \mu_m \sum_{l=0}^{M-m} \binom{M-m}{l} (-1)^l \left\{ \sum_{n=1}^{N} \Psi_n \left[\Phi(\delta, 1+\delta; -b_n) + \rho \Phi(1+\delta, 2+\delta; -b_n)\right] \times \left[1 - \delta e^{-\theta_m^{\max} s_n} \mathrm{B}(1,\delta) \Phi(\delta, 1+\delta; -b_n)\right]^{m+l-1} \right\}, \quad (12)$$

where $b_n = \theta_m^{\max} s_n R_D^{\alpha}$, $s_n = \frac{1}{2}(1+\vartheta_n)$, $\vartheta_n = \cos(\frac{2n-1}{2N}\pi)$, $\Psi_n = \delta \omega_n \sqrt{1-\vartheta_n^2}\ \mathrm{B}(1,\delta)\, \theta_m^{\max} e^{-\theta_m^{\max} s_n}$, $\omega_n = \frac{\pi}{N}$ and N is the complexity-accuracy trade-off parameter.

4 Numerical Results

This section presents the numerical simulations to validate the accuracy of derived outage results as well as to compare the performance of NOMA system under proposed CGS scheme with no CGS applied and OMA by considering similar channel conditions for all users. In all simulations, we consider $M = 2$, $\bar{R}_1 = \bar{R}_2 = 1$ bits per channel use, $R_D = 20\,\mathrm{m}$, $\Upsilon = [10 - 50]\,\mathrm{dB}$ and $N = 5$. Further, as a representative case, the parameters $\left\{h_m, \bar{h}_m, k_{1,m}, k_{2,m}\right\}_{m=1}^{M}$ are taken from Example (Sect. 3.1).

The impact of varying R_D on the outage performance of the users is presented in Fig. 1. Following observations are drawn from the results. First, increasing the radius R_D increases the outage probability of the users due to the higher path loss. Second, user $m = 1$ has lower outage probability than user $m = 2$ because under similar channel conditions scenario, the application of CGS results in $\bar{h}_1 < \bar{h}_2$ (see Example in Sect. 3.1). As a consequence, $a_1 > a_2$ for all SNR values which results in better performance of user $m = 1$. Moreover, Monte-Carlo simulations are also performed to validate the accuracy of derived results in (12). It can be observed that the analytical and simulation results are in good agreement.

The outage performance among NOMA system with CGS, without CGS and OMA is presented in Fig. 2. It can be observed that NOMA under proposed CGS scheme outperforms NOMA without CGS and OMA. Further, it can be noted that the performance of NOMA without CGS is badly impacted. These results can be explained as follows: The scenario of similar channel conditions result in

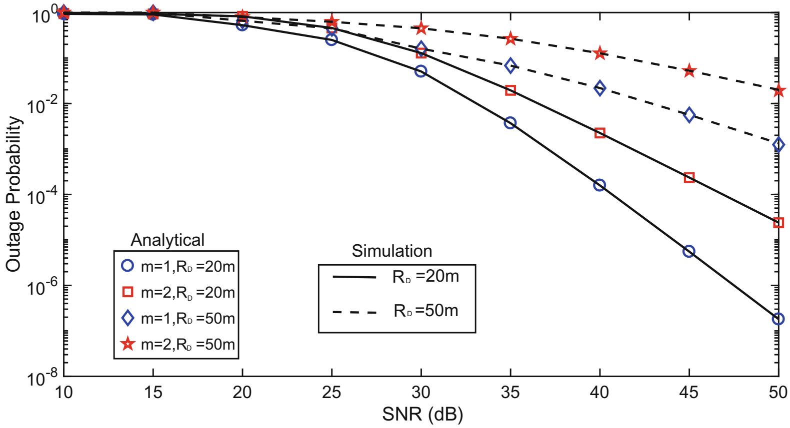

Fig. 1. Impact of varying R_D on outage performance.

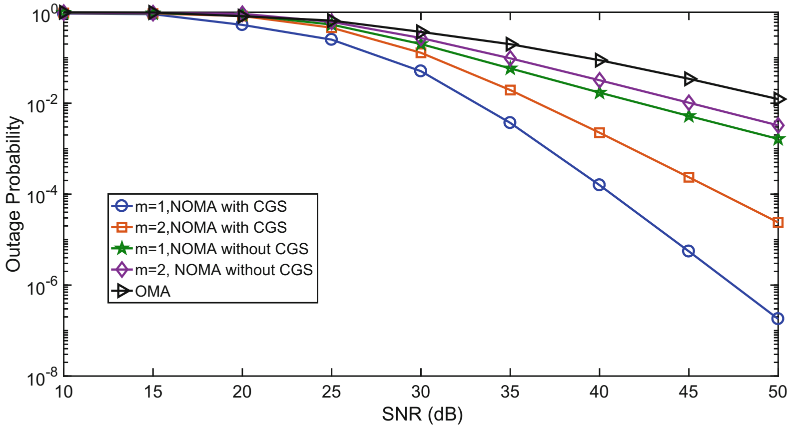

Fig. 2. Outage performance comparison among NOMA with and without CGS and OMA.

very comparable power allocation coefficients, which then increase the signal-to-interference-plus-noise ratio (SINR) threshold required for SIC decoding. By applying proposed CGS using (6) produces significant difference in channel gains resulting in significantly different power allocation coefficients and hence reducing the SINR threshold for SIC decoding. Further, both $m = 1, 2$ users have similar channel conditions, therefore, application of OMA results in same performance for both users, and hence we presented only one result for OMA scheme.

5 Conclusion

In this work, we consider a scenario of similar channel conditions for NOMA. In order to apply NOMA more effectively under these situations, we propose CGS method to artificially generate a channel gain difference among users. Closed-form expression for outage probability is derived to characterize the performance. It can be observed from the results that the NOMA under proposed CGS method outperforms NOMA without CGS and OMA under situations of comparable channel conditions. As future extension of this work, we plan to extend the proposed scheme for MIMO systems.

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