Physical-Layer Network Coding with High-Order Modulations

Xuesong Wang[1,2,3] and Lu Lu[1,2(✉)]

[1] Key Laboratory of Space Utilization, Technology and Engineering Center for Space Utilization, Chinese Academy of Sciences, Beijing, China
lulu@csu.ac.cn

[2] University of Chinese Academy of Sciences, Beijing, China

[3] Science and Technology on Communication Networks Laboratory, Shijiazhuang, China

Abstract. Physical-Layer Network Coding (PNC) can double the throughput of a Two-Way Relay Network (TWRN) by reducing packet exchanging timeslots. In a multi-user wireless communication system, time domain phase shift can inevitably lead to deterioration of PNC performance. In previous studies, there have been many studies result to enhance the performance of some low-order modulation techniques such as BPSK and QPSK, but fewer studies are designed for high-order modulation such as 16-QAM. It is known that high-order modulation is the only way to improve the spectrum utilization rate. This paper uses simulation to explain that the time domain phase shift will greatly affect the performance of 16-QAM PNC, and its' performance couldn't be improved even polar code is used. To address this phase penalty problem, we propose a half-symbol asynchronous algorithm to introduce correlations using belief propagation (BP). Simulation results show that the time domain phase shift problem of 16-QAM modulated PNC systems can be solved effectively using our proposed half-symbol asynchronous BP algorithm.

Keywords: Physical-Layer Network Coding · Phase asynchrony Symbol asynchrony · High-order modulation · Belief propagation

1 Introduction

The concept of Physical-Layer Network Coding (PNC) was first proposed in 2006 [1, 5]. PNC could increase throughput by reducing transmission slots. In wireless communications, various electromagnetic signals transmitted in space are superimposed within the channel, but only the signal sent by the specific terminal is useful to

The work of L. Lu was supported by the NSFC under Project 61501390. The work of X. Wang was supported in party by the Science and Technology on Communication Networks Laboratory under Project 614210401050217, and in part by the Special Presidential Foundation of Technology and Engineering Center for Space Utilization of the Chinese Academy of Sciences under Project CSU-ZDBS-201702.

P. H. J. Chong et al. (Eds.): SmartGIFT 2018, LNICST 245, pp. 173–191, 2018.
https://doi.org/10.1007/978-3-319-94965-9_18

the communication nodes, while other signals are considered as interferences. This interference has brought the obvious consequence to the multi-hop point-to-point network. For example, in the 802.11 protocol network, the throughput of a single hop network can reach 4 times which multi-hop network does theoretically [2]. One common solution is to use relay nodes to forward signals from other sources. The other solution is to specify a fixed communication protocol to prevent multiple nodes in the same channel sending messages at the same time. The latter way will result in a significant increase of the total transmission time slots in the whole communication process.

The PNC allows the sender to send information simultaneously to the relay, exploiting this “jamming” by implementing special information processing at the relay without affecting the reliability of the communication. As a result, PNC received widespread concern from the communications community.

Related Works. Since the PNC concept was proposed, a variety of PNC-based applications are also emerging. Zhang et al. discussed the feasibility of PNC in Galois Fields. Liew, Lu and Zhang performed a great deal of work on PNC, including channel coding PNC, asynchronous PNC, FPNC with OFDM and so on, and conducted a large number of experiments on Universal Software Radio Peripheral (USRP) [1, 3, 8]. Pan et al. conducted a study of 16-QAM with PNC combined with multiple access and MIMO techniques, but the solution will increase the cost by generating redundant information during demodulation, and it lacks the possibility of extending to higher order modulation [6].

With the application of channel coding, PNC can improve its code error correction capability to ensure the reliability of transmission. L. Chen combined Lattice coding with PNC to prove that Lattice physical layer network coding has superior performance, but the main result is hard to achieve due to its complexity [4]. Du et al. proposed a PNC scheme using LDPC under Gaussian Two-Way Relay Network (TWRN), and experimentally verified that it has an improvement on the Bit Error Rate (BER) at higher-order PSK [7]; however, QAM is more reliable than PSK at higher orders and there is a lack of QAM studies in it.

Due to the limited spectrum resources, high-order modulation is needed to improve the utilization of spectrum resources. However, the studies above are either based on simple modulation schemes such as BPSK, QPSK, or using high order modulation but lack of scalability. Moreover, many papers assume that the signal received by the relay has no phase offset. Even if there is phase offset, the study mainly focuses on simple modulation and does not involve high-order modulation. Therefore, this paper will mainly study the performance problems of PNC under high-order modulation, and a scheme of channel coding 16-QAM PNC will be introduce below. In addition, in this paper, and a scheme using half-symbol misalignment with belief propagation (BP), which could solve the performance problems, will be shown in detail. And this solution has the ability of extending to higher order modulation.

In summary, the contributions of this paper are as follows.

- We rigorously study a 16-QAM modulated PNC system in a TWRN under regular PNC mapping rules. We find that phase asynchrony can lead to tremendous system BER performance loss.

- We put forth a belief prorogation based decoding algorithm, by purposely introducing a half-symbol asynchrony to the 16-QAM modulated PNC system, to address the phase penalty issue.
- System performances of both non-channel-coded and channel-coded 16-QAM PNC system are discussed.

The following sections are organized as follows. Section 2 presents the PNC system model with 16-QAM modulation. Section 3 points out the phase asynchrony problem in time-domain PNC systems. Section 4 studies and observes the performance of 16-QAM PNC with polar coding. Section 5 puts forward the solution to the time-domain phase shift and observes the results. Section 6 concludes the paper.

2 16-QAM PNC System Model

2.1 PNC Transmission Model

PNC is mainly used in Two-Way Relay Network (TWRN), which is a three-node communication model. In TWRN, node 1 and 2 transmit information to each other through relay node R. A real example could be that two base stations which are very far apart want to communicate via satellites. Assuming it is a half-duplex situation, in this case any node cannot send or receive data at the same time. In traditional scheme, the two communication nodes send messages to the node R in different timeslot to avoid conflict with each other, which needs four timeslots in total. The process of PNC transmission mode is shown in Fig. 1. In the uplink, two communication nodes 1 and 2 transmit message and at the same time. After the relay R receives this superposed "naturally generated" sum information $(M_1 + M_2)$, it uses the corresponding mapping rule to coding the message

$$M_R = g(M_1 + M_2) = M_1 \oplus M_2 \tag{1}$$

then the M_R will be sent back to both source nodes at the same time. In the downlink, the two source nodes receive the message and then recover the message sent by other side with the copy of the local message. With the "interference" being used here, the system will no longer limit whether the communication node can send message at the same time. The total use of the slot reduced to two, thereby the system throughput is increased in the same period of time.

We can see that PNC uses the expense of increasing local copy and increasing the processing overhead at the relay to exchange for a reduction of the time slot and an increase in throughput. However, this burden is negligible comparing to the benefit here, which is, the "interference" caused by the simultaneous transmission of information by nodes 1 and 2 becomes a part of the network coding calculation, thus the influence of interference is eliminated.

As can be seen from the process above, one of the key issues of PNC is how to achieve Eq. (1), which is, how to complete the mapping from $M_1 + M_2$ to $M_1 \oplus M_2$. This problem will be described below.

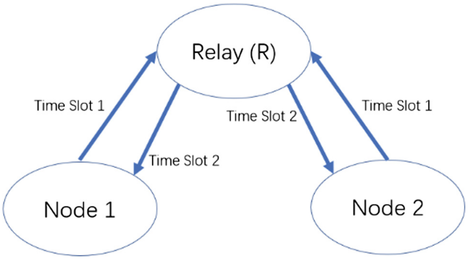


Fig. 1. PNC transmission mode

2.2 16-QAM PNC Mapping Rules

Here we establish the mapping rule under 16-QAM PNC system. In fact, any method who accomplishes a one-by-one mapping rule from to is available; but in this paper, a bit-by-bit mapping rule is established because it is one-to-one mapping on amplitude, and the nodes are easy to make a judgement.

When discussing "bit-by-bit mapping", it has to be mentioned that Gray coding is used in 16-QAM in this paper, since it is a kind of error minimization coding method because there is only one bit difference between the adjacent constellation points

The general model representation of 16-QAM PNC is consistent with the PNC model of Sect. 2.1, but it's more complicated comparing with BPSK or QPSK. 16-QAM involves I and Q components in different amplitudes. Section 2.1 states that the key issue with 16-QAM is the mapping of $M_1 + M_2$ to $M_1 \oplus M_2$. When discussing the mapping rule, only the mathematical realization is considered; which is, the factors in the actual communication is irrespective here. For 16-QAM, the mapping can be divided into I components and Q components separately, and the two mappings are the same.

When node 1 or 2 performs 16-QAM modulation and sends information, its I and Q components correspond to the mapping relationships in Table 1, where m is a bit sequence, s is the operator performing an eXclusive OR operation (XOR), and a is amplitude. Table 1 applies for any 16-QAM I/Q component involved in the overall system.

Let node 1 send the message M_1 with the expression

$$x_1 = a_1 \cos\theta + b_1 \sin\theta \tag{2}$$

and let node 2 send the message M_2 with the expression

$$x_2 = a_2 \cos\theta + b_2 \sin\theta \tag{3}$$

then the relay will receive the message M_R with the expression

$$x_R = (a_1 + a_2)\cos\theta + (b_1 + b_2)\sin\theta = a_R \cos\theta + b_R \sin\theta \tag{4}$$

where $a_1, a_2 \in \{-3, -1, 1, 3\}$, $a_R \in \{-6, -4, -2, 0, 2, 4, 6\}$.

Table 1. Correspondence of one component in 16-QAM.

Bit sequence m^I/m^Q	XOR operator s	amplitude a
00	0	−3
01	1	−1
11	2	1
10	3	3

After receiving x_R, the relay obtains the 16-QAM massage $\mathrm{M_R}$ through the network coding mapping function with the expression Eq. (1) and then sends it out. The I/Q components of the superposition signal y_R have seven kinds of amplitude, but components in $\mathrm{M_R}$ are still four kinds of amplitude. Here is an example using I component to specify the mapping process. The method is the same with Q component.

Let $m_1^I/s_1/a_1$ represent the bit sequence, the operator of the XOR operation, and the modulation amplitude of the I component of the node 1, respectively. Let $m_2^I/s_2/a_2$ represent the bit, the operator of the XOR operation, and the modulation amplitude of the I component of the node 2, respectively. If node 1 send the bit 1110, which means $m_1^I = 11$; node 2 send the bit 1001, which means $m_2^I = 10$, then according to the Table 1, the corresponding XOR operator should be $s_1 = 2$ and $s_2 = 3$. So, the relay does XOR calculation to get the operator

$$s_R = (s_1 + s_2) \bmod 4 = (2 + 3) \bmod 4 = 1 \tag{5}$$

As we can see that in Table 1, s_R is corresponded to bit $m_R^I = 01$ (when performing XOR by bit, we can get $m_1^I \oplus m_2^I = 11 \oplus 10 = 01 = m_R^I$), and the corresponding signal amplitude is $a_R = -1$. The signal amplitude previously received by R is $a_1 + a_2 = 1 + 3 = 4$. That is, if the amplitude of the component in the signal that the relay receives is $a_1 + a_2 = 4$, then it should be mapped $a_R = -1$ as its amplitude. The relay may not care about the value of s_R or m_R^I. The information contained can be given to node 1 or 2 to deal with. The relay's only mission is to get the correct transmission amplitude on it. Suppose that the relay sends a signal with one of the component is $a_R = -1$, and it is received by node 1. Node 1 maps it to $m_R^I = 01$ and matches the local copy bit m_R^I to perform XOR

$$m_1^I \oplus m_R^I = 11 \oplus 01 = 10 = m_2^I \tag{6}$$

then the message sent by node 2 is known by node 1.

When discussing the amplitude mapping method at the relay R, there are 16 cases for the component mapping at the relay R, since the two 16-QAM symbols' I and Q components have 4 amplitudes each [3]. Note that other corresponding solutions are also available. This article creates this mapping method because it is all one-to-one mapping on amplitude; using other methods may result in one-to-many mapping at some points, which in some cases judgment cannot be done. It can be obtained that the relay at the signal processing need to do the key mapping shown in Table 2.

Table 2. Key mapping at the relay.

$a_1 + a_2$	−6	−4	−2	0	2	4	6
a_R	−3	−1	1	3	−3	−1	1

Table 2 shows the key mapping, which means $a_1 + a_2$ to a_R. The previous example shows that the relay does not care about the value of s_R or the value of m_R, as long as the amplitude is mapped correctly. In this way, the mapping rule required under 16-QAM is obtained.

2.3 Establishment of 16-QAM PNC Model

This section describes the general mathematic models and the formulations of 16-QAM PNC systems involved in this dissertation for the remainder of the paper. There are many practical problems to consider about PNC, such as phase offset in time domain, phase offset in frequency domain, noise problem, channel coding problem, symbol synchronization problem, channel fading problem, and so on. Since the problem of phase shift in frequency domain can be solved by using Orthogonal Frequency Division Multiplexing (OFDM) [8], the problem of phase shift in time domain is mainly addressed in this dissertation. The phrase "phase offset" in this paper below refers to the time domain phase offset. Symbol synchronization is assumed here; in Sect. 5, symbolic asynchrony exists as a necessary condition, so the problem with asynchronous systems will be explained later. Channel fading is not a concern here. Additive White Gaussian Noise (AWGN) runs through the entire study. Section 4 will focus on the problem of 16-QAM PNC channel coding; for simplicity, channel coding is not considered here. Since the situation of PNC downlink is similar to that of Point to Point (P2P), this paper mainly considers the PNC uplink.

In general, the phase offset in the time domain is due to the asynchronous phase of the carrier frequency oscillator or to the different path delays of the two uplinks (node 1 to node R, node 2 to node R). The time-domain phase shift will cause the constellation points to twist when demodulating, resulting in the failure of demodulation and the increase of BER [10]. Assuming that two uplinks are symmetrical and constant-parameters channels, the phase offset due to the differences in path delays may not be considered. Then it can be assumed that the phase offset is mainly due to the frequency difference of the local oscillator of each node. Then, the relay receives the signal from node 1 and 2 to estimate the frequency difference in coherent demodulation between R and 1 (φ_1) or R and 2 (φ_2). Without loss of generality, assuming $\varphi_2 > \varphi_1$, then

$$\varphi = |\varphi_1 - \varphi_2| = \varphi_2 - \varphi_1 \tag{7}$$

is the frequency difference between the two uplinks obtained by the relay, which is the phase offset to be studied in this paper. It is assumed that specific value of φ can be estimated by relay, which can be solved by technical methods today. In this way, this article can focus on the impact of φ.

For a signal $x(t)$, it is mathematically possible to know $x(t) = x(t)e^{2N\pi}, N \in \mathbf{Z}$, which means the phase shift varies over each 2π period. For simplicity, this article mainly studies the situation of $\varphi \in [0, \pi]$.

Since this paper only considers the phase offset φ in the time domain and the noise n in AWGN channel, it can be assumed that a certain symbol with its length N sent by node 1 is $m_1[n](n = 1, 2, \ldots, i, \ldots, \mathrm{N})$, and another certain symbol with length N sent by node 2 is $m_2[n]$. Which is, the symbol is the signal sent by node 1 is

$$x_1(t) = \sum_{n=1}^{\mathrm{N}} m_1[n] \tag{8}$$

and node 2 send the message

$$x_2(t) = \sum_{n=1}^{\mathrm{N}} m_2[n] \tag{9}$$

then the relay will receive

$$x_R(t, \varphi) = h_1 x_1(t) + h_2 x_2(t) + n(t) \tag{10}$$

where $h_1 = \mathrm{P}_1 e^{\varphi_1}$ and $h_2 = \mathrm{P}_2 e^{\varphi_2}$ with $\mathrm{P}_1 = \mathrm{P}_2$ when symmetrical channel is assumed. Then the message received at the relay can be rewritten as

$$x_R(t, \varphi) = P_0(x_1(t) + x_2(t)e^{\varphi}) + n(t) \tag{11}$$

where $\mathrm{P}_0 = \mathrm{P}_1 e^{\varphi_1}$ is a constant value.

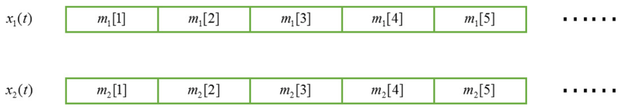


Fig. 2. Sequence sent by node 1 and 2

After the relay receives $x_R(t, \varphi)$, it performs the mapping according to the mapping rule obtained in Sect. 2.2 to get

$$y_R(t) = x_1(t) \oplus x_2(t) \tag{12}$$

then sent back to node 1 and node 2. Note that the signal before transmission has already resolved the phase offset and noise at the relay, which requires the half-symbol asynchronization with belief propagation involved in Sect. 5. The mapped signal $y_R(t)$ is also composed of 16-QAM symbols, denoted as

$$y_R(t) = \sum_{n=1}^{N} m_R^y[n] \tag{13}$$

where

$$m_R^y = m_1 \oplus m_2 \tag{14}$$

3 Time Domain Phase Offset Problem for 16-QAM PNC

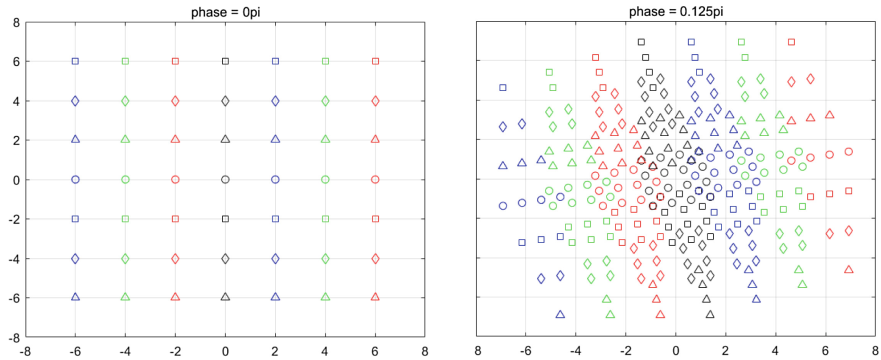
3.1 Constellation Pattern at the Relay in 16-QAM PNC

The following is based on the time-domain phase offset model of the 16-QAM PNC. Since this paper mainly studies the uplink, so we determine the BER by comparing the data received at the relay node with the data sent by the nodes 1 and 2. Therefore, it is of crucial importance whether the relay can receive the data correctly or not. In this case, we study the constellation pattern at the relay to illustrate the impact of phase offset.

Then the constellation at the 16-QAM PNC relay will be described. First, let us focus on the standard constellation at the relay, where only involves the phase offset and without noise. When performing PNC for 16-QAM, since there are 16 kinds of symbols sent by nodes 1 and 2 respectively, there are $16^2 = 256$ kinds of M_R involved at the relay. From Sect. 2.2 it can be seen that when $\varphi = 0$, one component of the signal at the relay makes a mapping of seven amplitudes to four amplitudes, so the overall signal at the relay should be mapped from 49 amplitudes to 16 amplitudes.

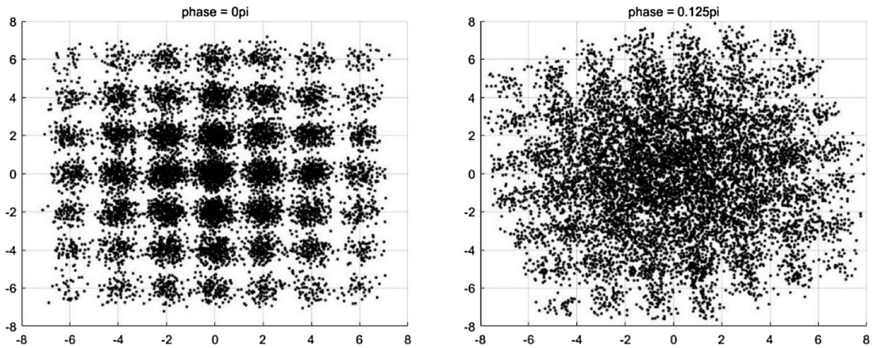
The constellation at the relay is shown in Fig. 3. In Fig. 3(a), the numbers in the upper right corner of each symbol represent the amount of the points overlapped in the same place, and the total sum is 256. The points in the figure are with various colors and shapes corresponding to the 16 types of 16-QAM symbol. It can be seen that when $\varphi = 0$, constellation points are of the same mapping coincide, and the demodulation can be the best at this time. When $\varphi = \pi/8$, the constellation pattern at the relay is shown in Fig. 3(b). It can be seen that some points belonging to different mappings are overlapped together, so it cannot be judged which symbol the constellation point at the overlapping position should be mapped to. Moreover, the middle part of the constellation point is too dense, which causes the small Euclidean distance between adjacent points, so it is prone to decode wrong.

When considering the noise ($snr = E_b/N_0 = 15\text{dB}$), the actual constellation at the relay is as shown in Fig. 4. It can be seen that when $\varphi = \pi/8$, the constellation points become even more dense and disorderly when $\varphi = 0$. The degree of point chaos in the middle part is very high, and the degree of agglomeration in the periphery points also decreases, thus a lot of code errors happen.



(a) (b)

Fig. 3. Constellation pattern at the relay with no noise and (a) $\varphi = 0$ (b) $\varphi = \pi/8$



(a) (b)

Fig. 4. Constellation pattern at the relay with $snr = 15\,\text{dB}$ and (a) $\varphi = 0$ (b) $\varphi = \pi/8$

3.2 16-QAM PNC Under Maximum Posterior Probability

Next, the Bit Error Rate (BER) curve will be drawn by MATLAB simulation in different SNR and different phase offsets. MAP decoding method is used in the simulation, that is, the Euclidean distance is determined between each received point and each point in the standard constellation map, and the point with the smallest Euclidean distance is selected as the mapping target. The SNR is sequentially incremented from 0 to 20 in steps of 1. In the same phase offset with different SNRs, each time node 1 and a node 2 send 1,000 data packets in sequence, and each data packet includes 10,000 16-QAM symbols.

Figure 5 shows the 16-QAM PNC BER performance curves for different phase shifts φ. It can be seen that when there is a small phase offset ($\varphi = \pi/8$), its performance is very poor already, and completely unable to meet the communication needs. When the phase shift increases to $\varphi = \pi/4$, the performance becomes even worse.

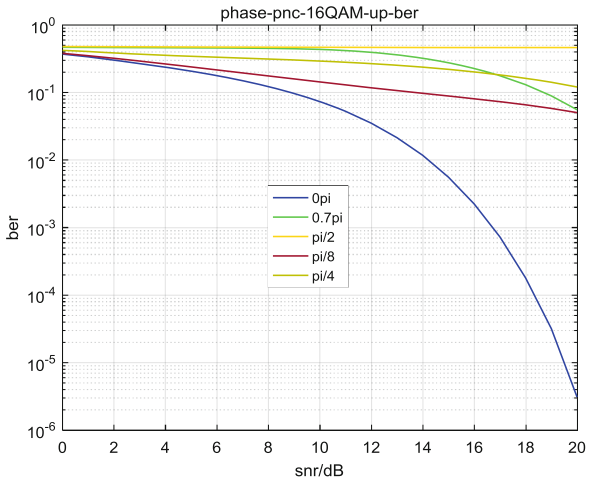


Fig. 5. BER curve at the relay in 16-QAM PNC

Thus, it can be seen that when there is phase offset, the performance of the system will deteriorate sharply and the BER will increase greatly. Even at large SNR, it cannot improve well and cannot meet the communication requirements. Although the MAP decoding can achieve the best reception, it cannot solve the phase shift decoding problem by itself.

4 Linear Channel Coding on 16-QAM PNC

4.1 Channel Coding Model Establishment

In general, channel coding has the ability of forward error correction. Using appropriate channel coding in Point to Point (P2P) channel can solve part of the transmission errors effectively and enhance the ability of noise or interference resisting, which can improve system reliability. The following will try to use linear channel coding in 16-QAM PNC and observe its performance changes.

As mentioned above, due to the assumption of symmetrical channels, the same channel coding scheme is used in all the links. Let the original bit information at nodes 1 and 2 be channel-coded before being sent out. As described in Sect. 2.3, one of the original symbol sent by node 1 or node 2 is $m_1[i]$ ($n = 1, 2, \ldots, i, \ldots, \mathrm{N}$) or $m_2[i]$, respectively. Let a linear channel coding method (function) be C. Symbol $m_1[i]$ and $m_2[i]$ are coded to $C(m_1[i])$ and $C(m_2[i])$, then sent out at the same time. Then the relay will receive the signal

$$x_R(t) = x_1(t) + x_2(t) = C(\sum_n m_1[n]) + C(\sum_n m_2[n]) \tag{15}$$

Now there are two ways to deal with this $x_R(t)$.

(a) Due to

$$C(m_1[i]) + C(m_2[i]) = C(m_1[i] + m_2[i]) \tag{16}$$

the relay will receive the signal, channel decode it to $m_1[i] + m_2[i]$, and then perform mapping mentioned above to get

$$m_R^y[i] = m_1[i] \oplus m_2[i] \tag{17}$$

Then channel code it to $C(m_R^y[i])$ and send it to node 1 and 2.

(b) Due to linear channel coding method being used here, the relay can perform PNC on $C(m_1[i]) + C(m_2[i])$ to get $C(m_1[i]) \oplus C(m_2[i])$. And obviously,

$$C(m_1[i]) \oplus C(m_2[i]) = C(m_1[i] \oplus m_2[i]) \tag{18}$$

So $m_1[i] \oplus m_2[i]$ is get after channel decoding.

(c) Multi-User Detection (MUD) technology can be used to achieve the detection of the original signal (such as $x_1(t)$ or $x_2(t)$), and then the corresponding symbol finish XOR operation to get $\sum_n m_1[n] \oplus m_2[n]$. This will increase the system burden, because relays do not need to know what $x_1(t)$ or $x_2(t)$ is. The relay is only concerned about getting $m_1[n] \oplus m_2[n]$ correctly.

It has been pointed out in the paper [1] that the method (a) is better because channel decoding can solve some of the channel mapping errors. In the method (b) which is directly perform PNC mapping on $C(m_1[i]) + C(m_2[i])$, the error brought by the channel will be passed on and cannot be resolved; that is, if $C(m_1[i]) + C(m_2[i])$ is wrong, then the mapping will be wrong. This kind of mistake cannot be corrected by channel decoding. Therefore, this section uses method (a) for 16-QAM PNC to achieve channel coding.

In this way, after the steps in (a), the terminal node (with node 1 as an example) receives the signal $C(m_1[i] \oplus m_2[i])$, then calculates

$$m_1[i] \oplus (C^{-1}(C(m_1[i] \oplus m_2[i])) = m_2[i] \tag{19}$$

to get the message sent by node 2.

4.2 Polar Coding on 16-QAM PNC

Different channel coding methods also have different error correction capabilities. Since this paper does not regard channel coding as a variable, this paper fixes a representative channel coding method to do related research. Polar code was first proposed by Arikan in 2008 [11]. Its practicability is very high and meets the requirements of this article. In the following, a polar code with the code rate $R = 0.5$ and the code length of 8 is described.

“Polar code” is called because it deriving the channel polar to construct a codeword that can achieve symmetric channel capacity. Note that for an N bit channel W_{N}, the symmetric channel capacity of a binary discrete memoryless channel is given by

$$I(W_{\mathrm{N}}) = \sum_{y \in Y} \sum_{x \in X} \frac{1}{2} W(y|x) \log\left(\frac{2W(y|x)}{W(y|0) + W(y|1)}\right) \tag{20}$$

where x is the channel input, y is the channel output, the value of $I(W_{\mathrm{N}})$ is $[0, 1]$. Bhattacharyya parameter in the W_{N} is given by

$$Z(W_{\mathrm{N}}) = \sum_{y \in Y} \sqrt{W(y|0)W(y|1)} \tag{21}$$

and its value is between 0 and 1. $Z(W_{\mathrm{N}})$ measures the channel reliability.

When various circumstances’ probability of the input W_{N} is equal, $I(W_{\mathrm{N}})$ reach the maximum value. It can be intuitively obtained if and only if there is $Z(W_{\mathrm{N}}) \approx 0$ then $I(W_{\mathrm{N}}) \approx 1$, and only when $Z(W_{\mathrm{N}}) \approx 1$ then $I(W_{\mathrm{N}}) \approx 0$.

An important process of polar coding is to choose a reliable channel to convey useful information. In general, the generator matrix of the polar codes with of the block size $\mathrm{N} = 2^n$ is represented by

$$\mathbf{G}_N = \mathbf{F}^{\otimes n} \tag{22}$$

and $\mathbf{F} = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} \end{pmatrix}$, where $\mathbf{F}^{\otimes n}$ represents the Kronecker inner product. The channel codeword with bit length N is given by

$$\mathbf{w} = \mathbf{u}\mathbf{G}_N \tag{23}$$

where $\mathbf{u}$ is the input of the channel coder. In this paper the polar code’s length is $\mathrm{N} = 8 = 2^3$ and code rate is $R = 0.5$, so the input $\mathbf{u}$ is an 1×8 vector, which contains a 4 bit higher $I(W)$ and lower “pureness” channel as the channel to send information, in which the arrangement of information bits; and 4 bit higher $Z(W)$ and lower “clutter” channel as the noise channel, in which the fixed bit arranged.

In order to select a 4-bit reliable channel, it is necessary to calculate the value of the channel capacity $I(W)$. According to the correlation calculation method of the polar coding [11], it can be obtained that

$$I(W_{\mathrm{N}=8}) = \{0.0039, 0.1211, 0.1914, 0.6836, 0.3164, 0.8086, 0.8789, 0.9961\}$$

It can be seen that the corresponding values of the $I(W)$ at the fourth, sixth, seventh and eighth bits are larger, and their corresponding positions should be placed in the information bits. Then, according to the specific process of the general polar code coding, the channel arrangement should be done according to the following method.

In the transmitter, an 8-bit encoding input polar code should be arranged the fourth, sixth, seventh, and eighth bits used as the information bits. In the simulation of this paper, the fixed position is all placing 0; it can be proved that in the symmetric channel, the value of the fixed bits has no effect on decoding. At the receiving end, the codeword is decoded through Successive Cancellation (SC) decoding to get specific information whose length is 8, of which the seventh, sixth, fourth, and eighth bits correspond to first, second, third and fourth bits of the effective decoding information, respectively. For example, for a 4-bit codeword $\{1, 1, 0, 1\}$, the coded codeword is $\{1, 1, 0, 0, 0, 0, 1, 1\}$. In this way, the coding and decoding process of the entire polar code is completed.

Using polar coding in the case of different phase migration φ, the system performance of 16-QAM PNC is shown in Fig. 6. In this simulation, the SNR is from 0 to 20 in 1 step length. Under each SNR condition with different phase offset, nodes 1 and node 2 send 500 packets in turn, and each packet contains 2500 16-QAM symbols. From Fig. 6, it can be seen that the polar code channel coding does not improve the system performance of 16-QAM in PNC phase shift. As a result of encoding a single species here, it cannot be concluded like "channel encoding of 16-QAM PNC is useless", but in view of the polar code is a powerful error correcting capability of encoding, channel encoding itself is lacking the possibility to the problem of the phase shift in 16-QAM PNC.

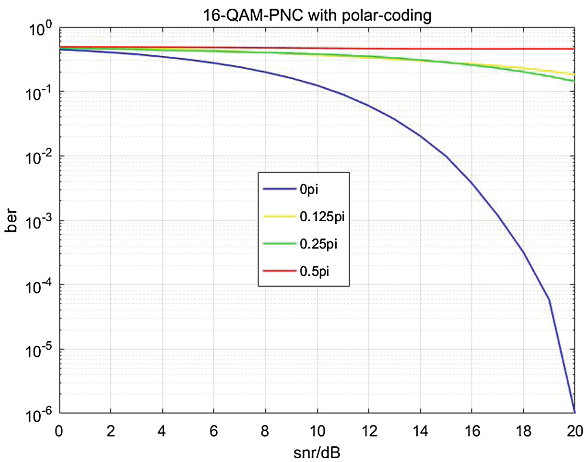


Fig. 6. BER of 16-QAM PNC using polar coding

5 Half Symbol Asynchronization with Belief Propagation

5.1 The Proposal of the Half-Symbol Asynchronous Belief Propagation

Section 3 summarizes the general case of 16-QAM PNC performance. It can be seen in Fig. 3(b) that, in the case of 256 points scattered unevenly within the constellation pattern, where the inner constellation points are relatively dense and the Euclidean distance between them is relatively small. If two symbols' sum $m_1[a] + m_2[b]$ is mapped to the inside, it is more difficult to make the right judgment. In general, there is a barrel effect in the system's BER [1], that is, the constellation point at which the system's SNR receives the worst-case error, as if the ability to hold water in the barrel was constrained by the shortest plank. Therefore, the decision of the inner mapping is difficult and error prone, resulting in a high system error and cannot easily be improved.

However, the outer constellation points are relatively scattered, and it is relatively easy to make a correct judgment to $m_1[a] + m_2[b]$ who is mapped to the outside. Notice that for every constellation symbol in Fig. 3(b), once the relay knows that it should map to a specific point, its corresponding $m_1[a]$ and $m_2[b]$ sent by node 1 or node 2 will be known. Then, as the point mapping to the outside of the constellation has a larger probability to make a correct decision, there is a larger probability of getting the two information at the same time. If this information can be utilized to help demodulate the points that are mapped to the inside side, it will be beneficial to improve the overall BER [12].

Belief Propagation (BP) is an algorithm for the implementation of the probabilistic computing problem containing the message transfer process. An undirected graph model is constructed as Markov random field, where each point represents a random variable, and the belief propagation algorithm is a probabilistic inference method based on it. If some information of the points in the random field is known, it is necessary to get some other information using this information. For every node in the field, the probability state of a node is transmitted to another adjacent node by message propagation, and the probability state at another node is updated. Information is transmitted repeatedly and updated at all points through repeated iterations. After several iterations, the probability distribution at each point will tend to a steady state. That is to say, the random field has reached the convergence state, and each point has the best confidence. In general, it is to find the edge probability distribution of each point in the random field. In this way, the overall performance of the system can be promoted.

To realize the previous idea, this paper proposes a design of two nodes in the uplink sending the information with half symbol duration asynchronous between each other in purpose. Then the relay receives the asynchronous signal and performs the belief propagation algorithm to obtain the correct codeword mapping. The simulation shows that it can solve the problem of phase shift in 16-QAM PNC system.

5.2 Half-Symbol Asynchronous System and the BP Algorithm

As described in Sect. 2.3, the signal sent by node 1 is $x_1(t) = \sum_{1}^{\mathrm{N}} m_1[n]$, and node 2 is $x_2(t) = \sum_{1}^{\mathrm{N}} m_2[n]$, where m is a 16-QAM symbol. Assuming that the sequence of symbols transmitted by node 1 arrives at relay R a half-symbol earlier than node 2. For the convenience of discussion, the end of the $x_1(t)$ is complemented of a half-symbol $m_1[\mathrm{N}+1]$ which does not actually exist, and in the front of the $x_2(t)$ a half-symbol

$m_2[0]$ is complemented which actually does not exist either. Now the length of $x_1(t)$ and $x_2(t)$ is equal. Then $x_R(t)$ at this time will be

$$m_R[a+b] = m_1[a] + m_2[b] \tag{24}$$

where $a \in \{1, 2, 3, \ldots, \mathrm{N}+1\}$, $b \in \{0, 1, 2, 3, \ldots, \mathrm{N}\}$ with $a - b \leq 1$; then

$$x_R(t) = \sum_{n=1}^{2N+1} m_R[n] \tag{25}$$

Figure 7 shows the details of a half-symbol asynchrony system. In Fig. 7, due to asynchrony, the sample value has been increased to $2\mathrm{N}+1$, which the relay can obtain more information comparing the synchronization case. For $a = b(m_R[4] = m_1[2] + m_2[2]$ as example), it corresponds to the sampling information at the synchronous situation, which is also the key information needed to derive $m_R^y = m_1 \oplus m_2$; for $a - b = 1$ ($m_R[7] = m_1[4] + m_2[3]$ as example), it corresponds to the sampling information at the asynchronous situation, which is the "more" information get here. Whether $m_R^y = m_1 \oplus m_2$ is correct or not can be judged through the posterior probability

$$P(m_R^y = m_1 \oplus m_2 | m_R) \tag{26}$$

This probability is the confidence level to be calculated. In this way, we construct the Markov random field needed by this paper, as shown in Fig. 8. The right-hand side of the $x_R(t)$ shows the $2\mathrm{N}+1$ values sampled by the relay after receiving, and these values are used to calculate the confidence probability. η means check node which provides external information decoding process, whose verification rule is

$$\eta(a,b) = \begin{cases} 1, m_R[a] = m_R[b] \\ 0, m_R[a] \neq m_R[b] \end{cases} \tag{27}$$

Where $a \in \{1, 2, 3, \ldots, \mathrm{N}+1\}$, $b \in \{0, 1, 2, 3, \ldots, \mathrm{N}\}$, $a - b \leq 1$.

The general process of information update in BP algorithm is to update the probability information iteratively to calculate the best confidence of each sample point (i.e., the maximum value of Eq. (26)), and make corresponding mapping according to the confidence judgment result to get the corresponding $m_R^y(t)$ on the top.

BP algorithm is a relatively mature algorithm, its information iterative updating process is basically fixed [9].

(a) information initialization, including the likelihood function of all hidden nodes, and potential energy and message value of each neighbor node;
(b) update of the check node;
(c) update of the value of the bit node;
(d) making judgments, verification. If correct, stop decoding; otherwise return to step (a).

The specific details of the BP algorithm are not the focus of this article, so we do not do in-depth discussion here.

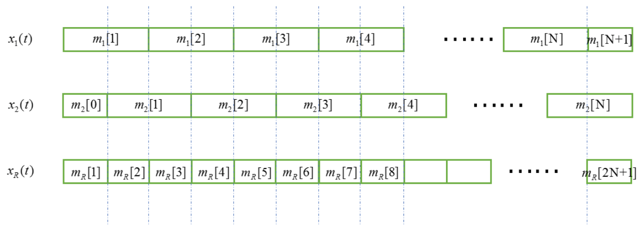


Fig. 7. Half-symbol asynchronous system

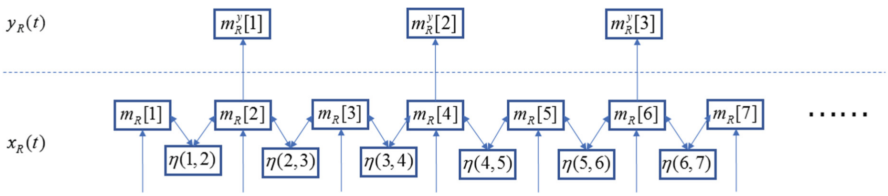


Fig. 8. Confidence propagation diagram

5.3 The Capability of Half-Symbol Asynchronous BP Algorithm

The half-symbol asynchronous BP algorithm proposed in this paper can reflect its good performance in the following two aspects.

(a) Half-symbol asynchronous. In the synchronization case of Fig. 2, the symbols $x_1(t)$ and $x_2(t)$ are corresponded one by one. When the relay receives $x_R(t)$, it samples and obtain N samples. In Fig. 7, the sample values for each are doubled because of asynchrony, and the original information obtained at the relay is doubled ($2N+1$ samples in total), allowing for more accurate decoding and mapping decisions.

(b) Information transfer implied in BP algorithm (Fig. 9). As mentioned in Sect. 5.2, the BP decoding algorithm is the finding the maximum of the probability of Eq. (26); in all sample values of $x_1(t) + x_2(t)$, values like

$$m_R[2i] = m_1[i] + m_2[i] \tag{28}$$

is the key to launching the required mapping. If it is mapped to the inner position in Fig. 3(b), it will be difficult to determine the constellation attribution correctly, because

$$P(m_R^y[i] = m_1[i] \oplus m_2[i] | m_R[2i]) \tag{29}$$

its maximum is small and with low confidence. However, the sample values adjacent to it, which are

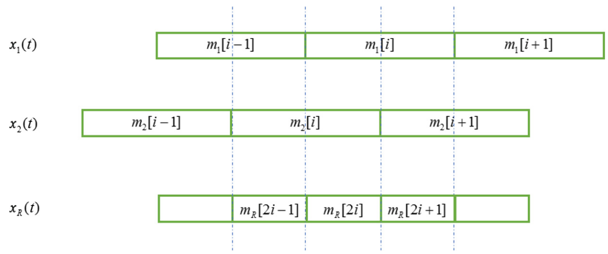


Fig. 9. Information transfer in BP algorithm

$$m_R[2i+1] = m_1[i+1] + m_2[i] \tag{30}$$

or

$$m_R[2i-1] = m_1[i] + m_2[i-1] \tag{31}$$

may be mapped to the outer position of the constellation, and the probability of correct demodulation will be higher. Assuming Eq. (31) is correctly demodulated, then $m_1[i]$ and $m_2[i-1]$ can be known, and they can help to demodulate $m_R[2i]$, which causes the sample value being successfully solved to the correct information because of the high confidence of $m_1[i]$ and $m_2[i-1]$. In this way, BP algorithm can enhance the overall decoding system.

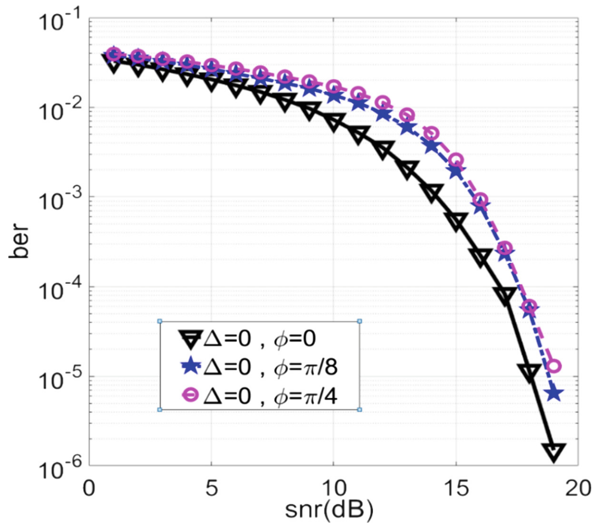


Fig. 10. BER performance of 16-QAM PNC with half-symbol BP algorithm

5.4 Simulation on 16-QAM PNC with Half-Symbol BP Algorithm

By generating half-symbol asynchrony BP algorithm, the performance of the 16-QAM PNC system at different phase offsets is shown in Fig. 10. In the simulation here, the SNR progressively advances from 0 to 20 in steps of 1. Under the same condition and different SNR, node 1 and node 2 simultaneously transmit 100 data packets in turn, each containing 1,000 16-QAM symbols.

In Fig. 10, Δ is a half-symbol asynchronous parameter. $\Delta = 0$ means half-aligned, and $0 \leq \Delta < 0.5$. ϕ means the phase offset φ. As can be seen, the BER at $\varphi = \pi/8$ is 6 dB higher than $\varphi = 0$ when SNR is 18 dB, and the performance between $\varphi = \pi/4$ and $\varphi = \pi/8$ is not much difference. Compared with Fig. 5, the proposed decoding method solves the problem of phase offset, and when the SNR is large ($snr = 18\,\text{dB}$), the BER performance is improved by 34 dBs ($\varphi = \pi/4$) and 30.7 dB ($\varphi = \pi/8$), which means the result can meet the normal communication needs. So, it can be seen that this algorithm has very good performance.

6 Conclusion

In this paper, we study the phase asynchrony issue in 16-QAM modulated PNC system, including the establishment and mapping of 16-QAM PNC, the general performance of system, system performance combined with channel coding. In the situation that the traditional idea cannot solve this problem, this paper proposes a half-symbol asynchronous BP algorithm to solve the problem caused by the phase offset by constructing asynchronously half symbols using the belief propagation algorithm.

The essence of PNC is to achieve the relay at the source of two data packets to XOR operation. In this paper, the mapping rule of 16-QAM relay has been successfully constructed, and the bitwise XOR at 16-QAM relay has been implemented. This method is general and can be extended to higher order modulations (such as 64-QAM, 1024-QAM, etc.). Moreover, for the half-symbol asynchronous BP algorithm in this paper, it has been successfully achieved on 16-QAM and the performance is greatly improved. It can be seen that this algorithm still has space for further expansion.

Future Work. There are some further discussions can be done in the future. For example, the way to combine half-symbol asynchronous BP algorithm with the existing means of communication technology (such as channel coding, OFDM, etc.) is still need for further study. For the research in this article, it can be continued to simulate the actual situation (such as using the USRP to do the actual simulation) to get more accurate results.

References

1. Liew, S.C., Lu, L., Zhang, S.: A primer on physical-layer network coding. Synth. Lect. Commun. Netw. **8**(1), 1–218 (2015)
2. Zhao, M.F., Zhou, Y.J., Yuan, Q., Yang, Y.X.: Research survey on physical layer network coding. J. Comput. Appl. **31**(8), 2015–2128 (2011)

3. Zhang, S., Liew, S.C., Lam, P.P.: Physical-layer network coding. In: International Conference on Mobile Computing and Networking, MOBICOM 2006, pp. 358–365 (2006)
4. Chen, L.: The research of physical layer network coding strategy based on various modulation techniques. Beijing Institute of Technology (Doctoral dissertation) (2015)
5. Liew, S.C., Zhang, S., Lu, L.: Physical-layer network coding: tutorial, survey, and beyond. Phys. Commun. **6**, 4–42 (2011)
6. Pan, H., Lu, L., Liew, S.C.: Network-coded multiple access with high-order modulations. In: IEEE Global Communications Conference, pp. 1–7 (2016)
7. Du, J., Yang, L., Yuan, J., Zhou, L., He, X.: Bit mapping design for LDPC coded BICM schemes with multi-edge type exit chart. IEEE Commun. Lett. **21**(4), 722–725 (2017)
8. Lu, L., Wang, T., Liew, S.C., Zhang, S.: Implementation of physical-layer network coding. Elsevier Phys. Commun. **6**(1), 74–87 (2013)
9. Goldsmith, A.: Wireless Communications. Cambridge University Press, Cambridge (2005)
10. Leon-Garcia, A., Widjaja, I.: Communication Networks. Tsinghua University Press, Beijing (2000)
11. Arikan, E.: Channel polarization: a method for constructing capacity-achieving codes for symmetric binary-input memoryless channels. IEEE Trans. Inform. Theory **55**(7), 3051–3073 (2009)
12. Lu, L., Liew, S.C.: Asynchronous physical-layer network coding. IEEE Trans. Wirel. Commun. **11**(2), 819–831 (2012)