A Method for Analysing and Improving the Multi-user Detection Algorithm of SCMA

Shuai Han(✉), Yiteng Huang, and Bin Wang

Communications Research Center, Harbin Institute of Technology, Harbin, China
hanshuai@hit.edu.cn

Abstract. Sparse code multiple access (SCMA) is a novel kind of non-orthogonal multiple access technology which combines the ideas of CDMA and OFDMA. The modulation scheme based on the mapping of the codebook can be regarded as a kind of spread spectrum coding technology and the encoding gain helps to improve spectrum utilization, system capacity and transfer rate. At the receiver, the message passing algorithm (MPA) whick is employed to detect multi-user signals can reduce the computational complexity because of the sparse structure. In this paper, we use the extrinsic information transfer chart (EXIT chart) to analyze MPA detection performance under different SNR conditions and propose serial MPA algorithm based on fairness which can improve the convergence performance of the algorithm and reduce the computational complexity. The theoretical derivation and simulation results demonstrate that serial MPA can improve the algorithm convergence performance and MPA detection are not apply to low SNR scenario.

Keywords: SCMA · EXIT chart
Serial MPA algorithm based on fairness · Multi-user detection
MPA convergence

1 Introduction

In the future 5G networks, the data flow rate of the system is greatly improved and occupies a wider range of bandwidth compared with 4G [1]. However, the reality is that a variety of standard communication systems mix together and the continuous spectrum resources are scant, so only limited discrete spectrum resources are available. Obviously, OFDM technology can not meet the needs of 5G mass connectivity. Therefore, 5G network sets higher demands upon the air interface techniques, in which sparse code multiple access (SCMA) is one of the standard candidates of air interface technology for 5G mobile communications [2].

B. Wang—This paper is sponsored by the funds of Science and Technology on Communication Networks Laboratory (No. KX162600029), the Huawei Innovation Research Program and the National Natural Science Foundation of China (No. 91438205).

C. Li and S. Mao (Eds.): WiCON 2017, LNICST 230, pp. 103–115, 2018.
https://doi.org/10.1007/978-3-319-90802-1_9

SCMA combines the idea of CDMA and OFDMA [3] to achieve non-orthogonal multiple access in the frequency domain. It modulates the signal onto the multi-dimensional OFDMA subcarrier. The codebook mapping can be considered as a spread-spectrum coding technique, which can improve the spectrum utilization and improve the system performance. SCMA uses multi-dimensional codebook instead of QAM modulation and LDS spread [4] to provide the shaping gain and spread gain. In the SCMA system, the user's information is modulated on multiple carriers for diversity transmission, and its codebook mapping is various, so that the receiver can demodulate user information based on different carriers and different codebook to reduce the information collision. Message passing algorithm (MPA) with affordable complexity can be adopted to achieve near ML performance [5] in SCMA detection module. Weighted Message Passing Algorithm for SCMA is proposed in reference [6]. Bayesteh studied the blind detection algorithm for SCMA uplink [7].

However, there are few works have been done to study the MPA algorithm performance in SCMA detection system under different conditions. What's more, the convergence performance of the original MPA algorithm need to be improved in engineering implementation.

In this paper, we use the extrinsic information transfer chart (EXIT chart) to analyze MPA detection performance under different channel conditions [8]. It's convenient to find out whether the MPA algorithm can converge to exact solutions and the approximate number of iteration required for convergence through the simulation diagram. Besides, we propose serial MPA based on fairness to improve the convergence performance of original multi-user detection algorithm. Simulation result shows that the algorithm that we propose reduces the number of iterations significantly, improves the convergence performance of the algorithm, and reduces the computational complexity of the system. It will be introduced in Sect. 4 separately.

The remainder of this paper is organized as follows. In Sect. 2 we introduce the basic principle and system model of SCMA. Section 3 introduces extrinsic information transfer (EXIT) chart to analyse the MPA algorithm performance. the improvement detection algorithm is presented in Sect. 3B. Then, the original MPA algorithm performance is analysed through simulations in Sect. 4. Moreover, we propose serial detection algorithm based on fairness to improve the convergence performance of the algorithm and reduce the computational complexity of the system in this section. Finally, the conclusion is given in Sect. 5.

2 System Model

2.1 System Model

The structure of SCMA transmitter is shown in Fig. 1. The data streams from multiusers is processed by FEC encoder module first and then interleaving the encoded data to prevent burst errors. The processed data streams are sent to SCMA encoder and directly mapped to several orthogonal subcarriers according to the pre-designed codebooks. Finally, the whole data streams output from the

SCMA encoder are transmitted through the channel. The transmission signal is superimposed on the data of other users carried by the same subcarrier and introduces noise in the transmission process. Then the transmission signal is sent to receiver. The SCMA receiver structure is shown in Fig. 2. The SCMA decoder detects the user data streams which are interferenced by channel noise and other user data streams according to codebook and carrier allocation information we have known. After that, decode convolution code and restore data bits with hard decisions.

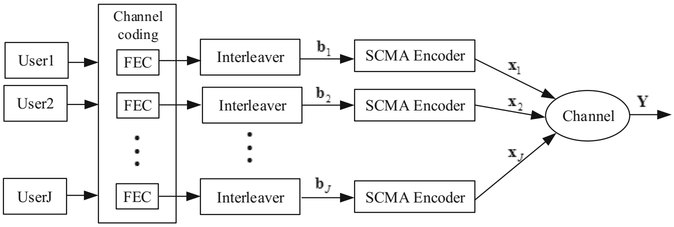

Fig. 1. SCMA transmitter structure

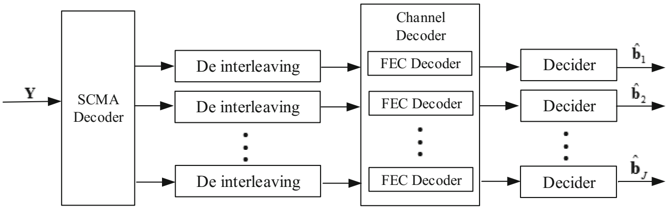

Fig. 2. SCMA receiver structure

SCMA Encoder. The SCMA encoder maps the interleaved binary bits to a multi-layer constellation symbol and then scatters the constellation point symbols on different subcarriers. In this paper, $\mathbf{B}$ and $\mathbf{C}$ denote sets of binary and complex numbers respectively. The map from binary bits to K-dimensional complex codebook can be expressed as $f : \mathbf{B}^{\log_2 M} \to \mathbf{C}^K$, where M is the number of constellation points, and K is the number of orthogonal subcarriers. The SCMA system allocates subcarriers for the user according to the allocation matrix $\mathbf{F}$ The row index k and the colomn index i of the matrix represent subcarrier k and user i respectively. If the element $(\mathbf{F})_{kj}$ in the matrix is 1, that means the subcarrier k is assigned to the user j. The matrix $\mathbf{F}$ below express a system which allocates 4 subcarriers to 6 users and each user occupies 2 subcarriers.

$$\mathbf{F} = \begin{bmatrix} 1\,1\,1\,0\,0\,0 \\ 1\,0\,0\,1\,1\,0 \\ 0\,1\,0\,1\,0\,1 \\ 0\,0\,1\,0\,1\,1 \end{bmatrix} \quad (1)$$

Signal Transmission Model. The signals of the J users are multiplexed to K orthogonal subcarriers. At the receiver, the attenuation signal of different users after channel transmission are superimposed on the subcarriers Therefore, the received symbol can be expressed as

$$\mathbf{y} = \sum_{j=1}^{J} \text{diag}(\mathbf{h}_j)\mathbf{x}_j + \mathbf{n}, \tag{2}$$

where $\mathbf{h}_j = (h_{1j}, h_{2j}, \ldots, h_{Kj})^T$ is the channel parameters vector of user j, indicating signal attenuation during transmission, $\mathbf{x}_j = (x_{1j}, x_{2j}, \ldots, x_{Kj})^T$ is SCMA codebook of the user j, and $\mathbf{n}$ is white noise, complying with Gaussian distribution in the complex domain.

SCMA Multiuser Detection. The optimal detection can be achieved by the maximum a posteriori probability (MAP) algorithm. The codeword matrix of each user is expressed as $\mathbf{X}_j = (x_1, x_2, \ldots, x_J)^T$ The estimated value of $\mathbf{X}$ is expressed as

$$\hat{\mathbf{X}} = \arg\max_{\mathbf{X} \in M^J} p(\mathbf{X}|\mathbf{y}) \tag{3}$$

Considering the complexity of detection, we transform the equation above into a marginal probability distribution problem and get the following equation

$$\hat{\mathbf{x}}_j = \arg\max_{a \in M} \sum_{\substack{\mathbf{X} \in M^J \\ \mathbf{X}_j = a}} P(\mathbf{X}|\mathbf{y}) \tag{4}$$

In the downlink, the number of system users is huge, and the amount of computation is so large that it is impossible to implement for a receiver. Therefore, multi-user detection based on factor graph is used to reduce the receiver computation complexity.

2.2 Classical Detection Algorithm

In the factor graph every circle represents a user (variable node) and every block represents a tone (function nodes), the factor graph represents system includes 6 users and 4 carriers is shown in Fig. 4. There are loops in the factor graph, which causing none of the line can be calculated first. The message passing algorithm (MPA) is applied for multi-user detection. The MPA algorithm decomposes the complex multi-user signal detection process into a number of relatively simple iterative steps, which are iteratively updated on the basis of probability information.

Each user and each subcarrier can be represented as a variable node (VN) and a function node (FN) respectively. Message transfer using a parallel mechanism, in each iteration process, all the function nodes and variable nodes are all parallel processing and delivery the messages (Fig. 3).

MPA iteratively updates the probability associated with the edges in the factor graph by passing the extrinsic information between VNs and FNs.

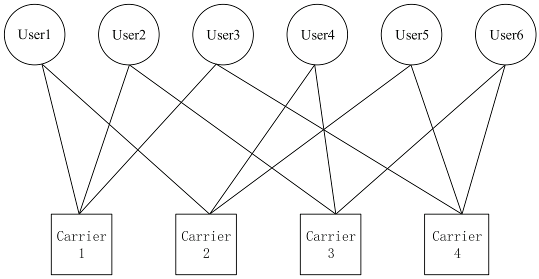

Fig. 3. SCMA system factor graph

First, initialize all variable nodes. Then, update the function node and variable node. The received signal for every VN is combined with the extrinsic information passed by the FN through the remainder of the edges connected to the FN. The passed message from FN to VN can be expressed as

$$\mu_{f_k \to V_m}(x_m) = \sum_{\sim\{x_m\}} \left\{ f_k(\{V_n = x_n\}_{V_n \in N(f_k)}) \prod_{V_n \in N(f_k)\backslash\{V_m\}} \mu_{V_n \to f_k}(x_n) \right\} \quad (5)$$

The prior information for every VN is combined with the extrinsic information passed by FN through the remainder of the edges connected to the VN. The passed message from VN to FN can be expressed as

$$\mu_{V_n \to f_l}(x_n) = \prod_{f_k \in N(V_n)\backslash\{f_l\}} \{\mu_{f_k \to V_n}(x_n)\} \quad (6)$$

Finally, after a few iterations, the probability message converges to the reliable value and the marginal probability of the variable can be expressed as

$$g_{V_n}(x_n) = \mu_{f_k \to V_n}(x_n) \times \mu_{V_n \to f_k}(x_n) \quad (7)$$

3 MPA Algorithm Performance Analysis and Enhance

3.1 Analyze the Performance of MPA Algorithm with Extrinsic Information Transfer (EXIT) Chart

The message passed between the VNs and FNs increases under the iterative process, and its own information is fed back as a priori information of the next iteration, so the independence of the message and the improvement in decoding performance is affected. Therefore, it is important to research the convergence of MPA algorithm.

From the system point of view, the iterative decoder can be regarded as a dynamic system. As an efficient tool for analyzing the convergence of iterative decoding system, EXIT chart analysing the convergence based on mutual information. The SCMA receiver consists of the function node decoder (FND) and the variable node decoder (VND) [9], and the EXIT chart technology can be used to analyze the convergence of detection algorithm [10].

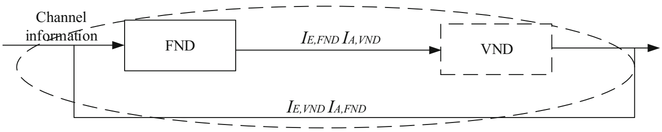

Fig. 4. SCMA detector structure

SCMA detector structure is shown in Fig. 4. In the multi-user detection process of the SCMA system, the prior information of a component detector is obtained from the extrinsic information of the other component detector, and the extrinsic information of the detector is independent of each other.

The relationship between the discrete input and output signals in the Gaussian white noise channel is shown in the following equation,

$$a = b + n \tag{8}$$

where b is the input signal, n is a Gauss noise with a mean value of 0 and an variance of δ^2. z is the output signal of the channel (only consider additive noise). The conditional probability function under the Gaussian noise channel can be expressed as

$$p(z|\mathbf{b} = b) = \frac{e^{-\frac{(z-b)^2}{2\delta^2}}}{\sqrt{2\pi}\delta_n} \tag{9}$$

The symbol b represents the random data information with a discrete value of 1 or 0. Thus, the log likelihood ratio of the transmitted data can be expressed as

$$Z = \ln \frac{p(z|b = 0)}{p(z|b = 1)} \tag{10}$$

We represent the prior information and the output information as mutual information. $I(X, Y)$ represents the mutual information between the variables X and Y. Variable X represents the signal sent by the SCMA system, and Y is the logarithmic likelihood of variable X. $I(X, Y)$ can be expressed as

$$I(X, Y) = H(X) - H(X|Y) \tag{11}$$

where the source entropy $H(X)$ represents the uncertainty of X, and $H(X|Y)$ represents the uncertainty of X under the condition that variable Y have been known. According to the derivation of probability relation, $I(X, Y)$ can be expressed as

$$I(X; Y) = \sum_{x,y} p(x, y) \log(\frac{p(y|x)}{p(y)}) \tag{12}$$

The value of variable X is 0 or 1, and Y is a continuous random variable, so $p(y)$ can be expressed as

$$p(y) = 1/2[p(y|x=0) + p(y|x=1)] \tag{13}$$

The mutual information between the transmitted data and the priori information of the component detector can be expressed as

$$\begin{aligned} I_A(X;Y) =& \frac{1}{2} \sum_{x=0,1} \int_{-\infty}^{+\infty} p_A(y|X=x) \\ & \log_2\left(\frac{2 \times p_A(y|X=x)}{p_A(y|X=0) + p_A(y|X=1)}\right) dy \end{aligned} \tag{14}$$

where factor p represents probability distribution.

Under the condition of BPSK modulation, p follows the Gaussian distribution, however, for the case of multi-modulation, there is no general conclusion. In the SCMA system, the constellation mapping based on the codebook design is various because of the change of the codebook, so it is complicated to obtain the probability distribution by the formula expression. In this case, the probability distribution is obtained by the way of Monte Carlo simulation and histogram statistics approximately.

Similar to the expression of $I_A(X;Y)$, $I_E(X;Y)$ can be expressed as

$$\begin{aligned} I_E(X;Y) =& \frac{1}{2} \sum_{x=0,1} \int_{-\infty}^{+\infty} p_E(y|X=x) \\ & \log_2\left(\frac{2 \times p_E(y|X=x)}{p_E(y|X=0) + p_E(y|X=1)}\right) dy \end{aligned} \tag{15}$$

Then the EXIT chart of the SCMA detector can be simulated.

3.2 Serial Message Pass Algorithm Based on Fairness

Message Update Based on Codeword Probability. Each iteration of the parallel MPA algorithm, all FNs update the message in parallel at the same time first, and then all VNs update the message simultaneously. The multi-user detection algorithm based on a parallel strategy whose updated message is passed to the corresponding variable node or function node at the beginning of the next iteration can't deliver the updated message immediately.

In addition, the algorithm requires large-capacity registers to store intermediate variables for parallel processing. Serial MPA algorithm based on fairness improves the convergence performance and save hardware resources.

According to the previous introduction to the MPA algorithm, the probability of outputting the codeword after t iterations can be expressed as

$$g^t(v_i) = \mu^t_{V_i \to f_j}(x_i) \times \mu^t_{f_j \to V_i}(x_i) \tag{16}$$

We can use $Q_{V_i}^t(x_i)$ and $\mu_{f_j \to V_i}^t(x_i)$ to calculate $\mu_{V_i \to f_j}^t(x_i)$ based on the above formula.

$$\mu_{V_i \to f_j}^t(x_i) = \frac{g^t(x_i)}{\mu_{f_j \to V_i}^t(x_i)} \tag{17}$$

The variable node update can be integrated into the function node update to calculate the marginal probability of the variable and save part of the intermediate variable storage space.

In serial MPA message delivery, as long as the function node information $\mu_{f_j \to V_i}^t(x_i)$ is updated, the probability information $g^{t-1}(x_i)$ of the corresponding user output codeword is updated immediately. $g^{t-1}(x_i)^{old}$ and $g^{t-1}(x_i)^{new}$ represent the probability information that before and after message update respectively. In addition, $g^{t-1}(x_i)^{new}$ is more accurate obviously. Then variable node update can be expressed as

$$\mu_{V_i \to f_j}^t(x_i) = \frac{g^{t-1}(x_i)^{new}}{\mu_{f_j \to V_i}^{t-1}(x_i)} \tag{18}$$

Based on the above discussion and the Eq. 15, the function node update of the serial MPA multiuser detection algorithm can be expressed as

$$\begin{aligned}\mu_{f_i \to V_i}^t(x_i) = \sum_{\sim v_i} \Bigg\{ & \frac{1}{\sqrt{2\pi}\delta} \exp\left(\frac{-1}{N_{0,n}} \|y_j - \sum_{x \in \xi_j} h_{j,x} v_{j,x}\|^2\right) \\ & \frac{Q^{t-1}(x_k)^{new}}{\mu_{f_j \to V_k}^{t-1}(x_i)} \prod_{l \in \xi_j / \{i,k\}} \frac{Q^{t-1}(x_k)^{old}}{\mu_{f_j \to V_l}^{t-1}(x_l)} \Bigg\}\end{aligned} \tag{19}$$

where ξ_j represents the set of users associated with the kth resource block, i and j represent the user in the set.

Serial Message Update Based on Fairness. The reliability of the function node update $\mu_{f_j \to V_i}^t(x_i)$ determines the accuracy of the codeword probability $g^t(x_i)$ that the detector calculates.

If the message $\mu_{V_i \to f_l}(v_i)$ passed from the VN to the FN is calculated based on the message $\mu_{f_k \to V_i}(v_i)$ that has been updated in this iteration we consider the message $\mu_{V_i \to f_l}(v_i)$ on the branch of the factor graph is credible and define it as credible branch. The function node update message $\mu_{f_j \to V_i}^t(v_i)$ is more accurate if the FN connects with more numbers of credible branches.

Because the serial MPA algorithm updates the message asynchronously, the further back function node update message is the more reliable. Serial message update based on fairness ensures uniform distribution of credible branches associated with each function node so that users' message in the system can be decoded fairly. The algorithm is expressed as Algorithm 1.

Algorithm 1. Serial message pass algorithm based on fairness

Require: y, H, δ^2, t_{max}
intialization:
for i=1,2,$\cdots$,J and i=1,2,$\cdots$,I **do**
  $g\left(x_i\right) = 1/M, \mu^0_{f_j \to V_i}\left(x_i\right) = 1/M$
end for
message update:
while $t \le t_{max}$ **do**
  for $j = 1, 2, \cdots, J$ **do**
    $M^j_{all}(x_i) = \prod\limits_{i \in \xi_j} \frac{g^{t-1}(x_i)}{\mu^{t-1}_{f_j \to V_i}(x_i)}$
    for $i = 1, 2, \cdots, I$ **do**
      if $i \in \xi_j$ **then**
        $M^j_{temp}(x_i) = \frac{g^{t-1}(x_i)}{\mu^{t-1}_{f_j \to V_i}(x_i)}$
        $\mu^t_{f_j \to V_i}(x_i) = \sum\limits_{\sim v_i} \left\{ \frac{1}{\sqrt{2\pi}\delta} \exp(-\frac{1}{2\delta^2} \left\| y_j - \sum\limits_{v \in \xi_j} h_{j,v} x_{j,v} \right\|^2) \frac{M^j_{all}(x_i)}{M^j_{temp}(x_i)} \right\}$
        $g^{t-1}(x_i) = M^j_{temp}(v_i) \mu^t_{f_j \to V_i}(x_i)$
        $M^j_{all}(v_i) = \prod\limits_{i \in \xi_j} \frac{g^{t-1}(x_i)}{\mu^{t-1}_{f_j \to V_i}(x_i)}$
      end if
    end for
  end for
  t=t+1
end while
for i= $1, 2, \cdots$,I **do**
  $g(x_i) = \prod\limits_{j \in \xi_i} \mu^{t_{max}}_{f_j \to V_i}(x_i)$
end for
Ensure: $g\left(x_i\right)$

4 Simulation Results and Analysis

4.1 Extrinsic Information Transfer (EXIT) Chart Simulation

We simulate the two component detector EXIT curve, and choose one to reversal the vertical and horizontal axis. Then, put the two simulation curve into a chart to analyse.

An EXIT chart example is shown in Fig. 5. The amount of mutual information transferred between the two component detectors varies with the direction of the arrow poles when the component detector iterative detection and decoding.

The space between the two curves is called the decoding space. The larger the space between the two curves, the less the number of iterations required for the algorithm convergence. The closer the intersection of the two curves get to one, the more accurate the iterative algorithm is.

The degrees of variable nodes in SCMA system factor graph represents the number of carriers that users multiplex. The three curves in Fig. 6 represent the

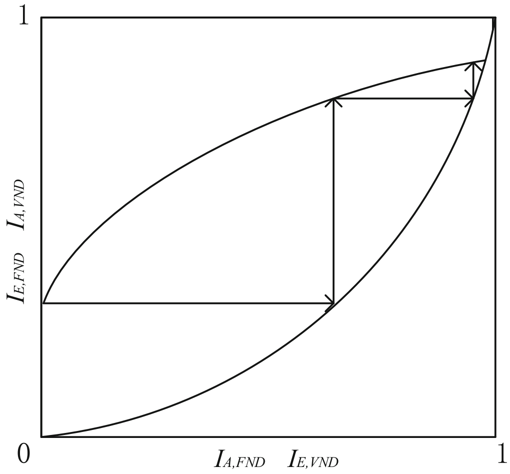

Fig. 5. EXIT chart example

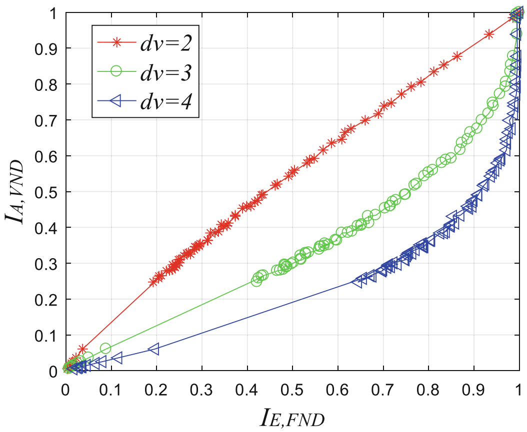

Fig. 6. The EXIT curve of the variable node

variable node EXIT curves when variable node degree is 2, 3 or 4 respectively. As the degree of variable node increases, the EXIT curve is closer to the horizontal axis and the decoding space is larger. Therefore, as the number of user multiplex resources increases, the iterative detection algorithm converges faster and gets better bit error rate performance. However, the system overload rate and the system capacity is reduce as user multiplex resources increases.

The EXIT chart of SCMA system under different SNR is shown as Fig. 7. It can be seen from the figure that the two component decoders can obtain new message through the iterations between the two component decoders, and the output information of the component decoders is more accurate. The iterations are stopped when the two component detector EXIT curves intersect.

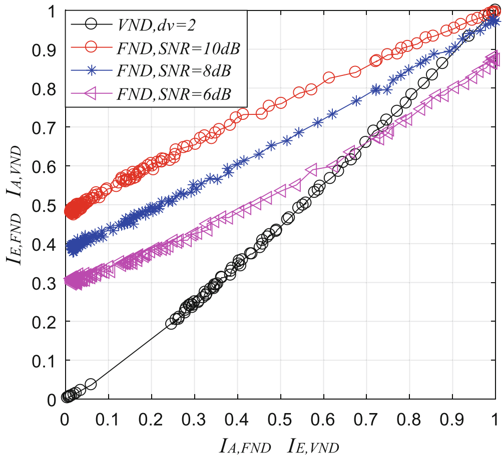

Fig. 7. The EXIT chart of SCMA system

Through the simulation results, we can find that the EXIT curves of the two component detectors have an intersection when using the MPA algorithm for multiuser detection in the SCMA system under different SNR. Therefore, we can use the EXIT chart to prove that MPA algorithm is convergent in the SCMA multi-user detection. The mutual information at the intersection of two EXIT curves in the simulation diagram increases as the SNR becomers higher, and the performance of the MPA algorithm is better correspondingly. The mutual information value is small at the intersection of two EXIT curves in the low SNR condition and therefore MPA algorithm can not converge to the exact value. The intersection coordinate of the EXIT curves in the chart is almost equal to one, the space between the two curves is wide and the decoding space is large in the condition that SNR is over 10 dB.

Therefore, MPA algorithm performances well in the high SNR condition and it isn't suitable for low SNR SCMA system.

4.2 Serial Message Pass Algorithm Based on Fairness

The SCMA system uses 16-point codebook mapping and then modulated signals are transmitted over Gaussian channels. Finally, BCJR algorithm is applied for decoding [11].

The convergence performance of the two algorithm that serial MPA based on fairness and original MPA algorithm are shown in Fig. 8. The serial MPA based on fainess converges only after 4 iterations, while the parallel MPA must iterate more than 10 times before it completely convergent.

Therefore, the algorithm that we propose reduces the number of iterations significantly, improves the convergence performance of the algorithm, and reduces the computational complexity of the system.

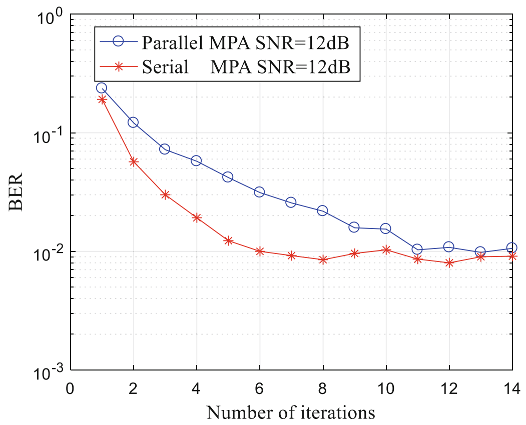

Fig. 8. The algorithm convergence performance comparison

5 Conclusion

In the paper, we use the extrinsic information transfer chart (EXIT chart) to analyze MPA detection performance. In the SCMA system, the constellation mapping based on the codebook design is various, so it is too complicated to obtain the probability distribution by mathematical formula derivation. Through the EXIT chart that we simulate, it is easy to analysis the MPA algorithm performance in SCMA detection system and come to the conclusion that MPA algorithm is not apply to the low SNR condition. Furthermore, we present serial MPA algorithm based on fairness to improve the convergence performance of original MPA algorithm and reduces the computational complexity of the detection system.

References

1. Agyapong, P.K., Iwamura, M., Staehle, D., Kiess, W., Benjebbour, A.: Design considerations for a 5G network architecture. IEEE Commun. Mag. **52**(11), 65–75 (2014)
2. Nikopour, H., Baligh, H.: Sparse code multiple access. In: IEEE 24th International Symposium on Personal Indoor and Mobile Radio Communications (PIMRC), pp. 332–336. IEEE, London, UK (2013)
3. Ren, B., Han, S., Meng, W., Li, C., Wu, X.: Enhanced turbo detection for SCMA based on information reliability. In: 2015 IEEE/CIC International Conference on Communications in China, Shenzhen, China, November 2015
4. Prasad, R., Ruggieri, M.: Technology Trends in Wireless Communications. Artech House, Norwood (2003)
5. Weingarten, H., Steinberg, Y., Shamai, S.: The capacity region of the Gaussian multiple-input multiple-output broadcast channel. IEEE Trans. Inf. Theory **52**(9), 3936–3964 (2006)

6. Wei, D., Han, Y., Zhang, S., Liu, L.: Weighted message passing algorithm for SCMA. In: 2015 International Conference on Wireless Communications and Signal Processing (WCSP), Nanjing, pp. 1–5 (2015)
7. Bayesteh, A., Yi, E., Nikopour, H., Baligh, H.: Blind detection of SCMA for uplink grant-free multiple-access. In: 11th International Symposium on Wireless Communications Systems (ISWCS), Barcelona, pp. 853–857 (2014)
8. Ashikhmin, A., Kramer, G., ten Brink, S.: Extrinsic information transfer functions: model and erasure channel properties. IEEE Trans. Inf. Theory **50**(11), 2657–2673 (2004)
9. Du, Y., Dong, B., Wang, X., Dang, G., Gao, P.: Multiuser detection scheme for SCMA systems based on serial strategy. J. Electron. Inf. Technol. **38**(8), 1–6 (2016)
10. Wang, X., Du, Y., Cheng, C., Gao, P., Dong, B.: Convergence property analysis for multiuser detection schemes in SCMA systems based on EXIT chart. J. Sig. Process. **32**(12), 1440–1445 (2016)
11. Tuchler, M., Singer, A.C.: Turbo equalization, an overview. IEEE Trans. Inf. Theory **57**(2), 920–952 (2011)