Optimal Smart Prepayment for Mobile Access Service via Stackelberg Game

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Abstract. In this paper we propose a smart prepayment for mobile access network Service Provider (SP) to charge End-Users (EUs). Prepayment is a desirable charging approach, since it helps the SP to reduce its loss in bad-debt and capital devaluation. Meanwhile, Quality of Service (QoS) is a major concern from the EUs' perspective, especially when they have heavy traffic demands and suffer from network congestion due to limited access bandwidths. Our proposed prepayment thus aims at improving both the SP's economic reward and the EUs' QoS. To analyze the benefit from the proposed prepayment scheme, we model the interaction between the SP and the EUs as a Stackelberg game, which is based on the rationale that improved QoS will be an incentive for the EUs to prepay. In this game model, the SP plays as a leader and determines its prepayment policy to optimize its reward, and the EU plays as a game follower and determines its prepaid amount as a response to the SP's policy. The equilibrium of this game model strongly depends on the EUs' traffic load level, which we quantify and analyze in depth. Our results show that both of the SP and the EUs can benefit from the equilibrium of the game model, implying that the proposed prepayment scheme will yield a desirable win-win outcome.

Keywords: Smart pricing · Mobile network service · Optimization Stackelberg game

1 Introduction

Facing a rapid growth in wired/wireless access networks, many mobile access Service Providers (SPs) are expected to provide access to a large number of End-Users (EUs) with heterogeneous traffic demands. The SPs, however, usually have limited bandwidths in access links (e.g., cellular networks), which results in a dilemma between obtaining higher profits through providing services to more EUs and suffering from the consequent network congestions due to excessive EUs' traffic demands. One of the most effective approaches to tackle this dilemma is

C. Li and S. Mao (Eds.): WiCON 2017, LNICST 230, pp. 34–50, 2018.
https://doi.org/10.1007/978-3-319-90802-1_4

to increase the bandwidths of access links, e.g., through updating their network infrastructures. However, the SPs usually hesitate to do so because of huge capital investments and more importantly, their uncertainties in cost-recovery, e.g., due to failure to collect the entire service fees from EUs (i.e., the so-called "bad-debts"). A mechanism to motivate the SPs to increase the access bandwidths is prepayment, i.e., the SPs are allowed to collect (or partially collect) the service fee at the beginning of a service cycle. Then the SPs can increase the EUs' access bandwidths based on the received prepayment. For example, this can be logically implemented through the SPs' traffic-shaping, i.e., the SPs increase the bandwidth-limit on the EU's traffic according to the EU's prepaid amount. Hence, the EU is motivated to choose the option of prepayment for a better QoS-performance. Prepayment will generally benefit the SPs economically from the following two aspects: (i) the prepayment can save the SPs' loss due to capital devaluation. Specifically, the capital usually devalues over time (for example, a current value v is only worth of $v\alpha$ at the end of a service cycle, where α denotes the discount factor), and hence obtaining the service fee at the beginning of a service cycle means a greater profit compared to obtaining it at the end of the cycle; (ii) the prepaying can reduce the SPs' loss due to the bad-debt, i.e. the SP fails to collect the entire service fee from the EU at the end of the service cycle. Therefore, prepayments have been widely accepted and used by SPs [8–10]. Different from the SP's interest, Quality of Service (QoS) is a key concern of EUs, especially when they have heavy traffic loads and hence suffer from network congestions due to limited access bandwidths. For instance, most of the popular applications on mobile terminals, such as multimedia streaming, online gaming and group communications are usually QoS-sensitive and suffer from insufficient access bandwidths (in particular, as reported in [11], prepayment will be well suitable to charge realtime applications like multimedia streaming, online gaming and etc). As described earlier, choosing the option of prepayment can motivate the SPs to increase its access bandwidth, and thus can effectively alleviate the network congestions and improve the EUs' QoS-experience.

Considering different interests of SP and EUs, a smart prepayment scheme can be envisioned to encourage the collaborations between the SP and the EUs. Put it simply, the SP commits to using a portion of the EUs' prepayment to increase its access bandwidth (or equivalently to increase the EUs' bandwidth-limit in traffic shaping), and as a response the EUs are encouraged to choose the option of prepayment for better QoS. However, the questions about this prepayment scheme are (i) how much of the EUs' prepayment the SP will spend in upgrading its access bandwidth, (ii) as a response, how much the EUs will choose to prepay. More importantly, (iii) how much will the SP and EUs benefit from this scheme, and do they have a joint incentive to adopt this scheme? These questions motivate our work. To analyze the prepayment scheme and answer the above questions, we model the strategic interaction between the SP and the EUs as a noncooperative game. Intuitively, the interests of the SP and the EUs are conflicting with each other, i.e., the SP expects to obtain a greater prepayment from the EU but spending less in increasing its bandwidth. In comparison, the EU expects to receive a better QoS but spending less prepayment. Hence, the game naturally fits our need for modeling. In particular, based on the rationale that *the improved QoS from this prepayment scheme will be an incentive for EUs to prepay*, we adopt the Stackelberg game, a leader-follower dynamic game, to model the specific SP-EUs interaction [19] (another reason for us to choose the Stackelberg game is that the access service providers usually possess dominant positions in practical markets compared to individual EU). The equilibrium of this Stackelberg game, i.e., the ultimate state that either SP or EUs will not change their decision unilaterally, represents the optimal prepayment scheme and hence facilitates our quantitative analysis on the benefits from this scheme. We analyze the equilibrium of the proposed Stackelberg game via backward induction. We find that the equilibrium strongly depends on the EUs' traffic load, and hence we differentiate three traffic load regions to quantify the equilibrium (we also show the uniqueness of the equilibrium for each region). Based on the obtained equilibrium, we quantify the SP and EUs' rewards from the prepayment scheme. Our results show that both the SP and the EUs will benefit significantly from the game equilibrium, which implies a win-win outcome from the prepayment. The proposed prepayment scheme can effectively motivate the cooperation among different users and operators in different paradigms in future 5G cellular networks [14–16].

Related Work: Models of charging have been longstanding concerns for network operators and service providers [1]. Related work can be roughly categorized according to the subjects investigated, which include *whom to charge* [2,3], *what to charge* [4,5], and *how to charge* [6,7]. However, an issue less explored before is *when to charge*. Regarding this issue, prepaying and post-paying are two important candidates in practice. Compared to the prepayment, the post-payment means that the SP is only allowed to collect the service fee at the end of the service cycle. While the post-paying has been widely adopted at present, the prepaying has its own advantages and hence attracts lots of interests (please refer to [8] for a survey of different models of prepaid mobile services and [9] for general customers and markets). In fact, with a rapid growth in 3G commercial communication networks, the prepaid access service has been widely adopted by many access SPs [10] and has been considered to be well suitable for realtime multimedia applications like video streaming, online-gaming and etc. From customers' point of view, it is reported that the prepayment is especially preferred by customers with tight budgets and eager to control their expenses closely, e.g., the reported demand for prepaid billing is 90–95% at initial sign-up in fast growing regions such as South America, Asia and Africa [11]. While, in developed regions, prepayment is also attractive to customers without good credit history, teenagers, and early-adopters of new services. Recent work [12,13] investigated the implementation issue of the prepayment for mobile network operators. Specifically, [12] focused on the prepaid voice-service and analyzed the optimal frequency for credit checking and updating with the objective to achieve a good balance between minimizing the checking cost and the bad-debt. [13] investigated how each user allocated its prepaid credit to executed sessions.

2 Design of Smart Prepayment Scheme

2.1 Proposed Prepayment Scheme and Its Game Model

We specify the proposed prepayment scheme as follows. Suppose that the SP adopts traffic-shaping mechanism to manage the EUs' traffic in access network. The SP is allowed to collect the service fee (in partial) from EUs at the beginning of the service cycle. The SP designs its prepayment policy to encourage the EUs' prepayment. Specifically, the SP's policy is characterized by a parameter β, which specifies the portion of the EU's prepaid amount used to increase the EU's bandwidth-limit in traffic-shaping. Given the SP's policy β, the EU determines how much to prepay, i.e., its prepaid amount p out of the original service-fee T. With the prepayment scheme, the SP controls β to trade off between its benefit from obtaining the EUs' prepayment and its expense on increasing the bandwidth of access link, and the EUs controls p to trade off between its improved QoS-performance from increased link bandwidth and its economic loss due to prepaying.

The objectives of the SP and EUs are conflicting with each other, i.e., the SP expects to obtain a greater prepayment from the EU but spending less in increasing its bandwidth-limit. In comparison, the EU expects to receive a better QoS but prepay less. Hence, it is important to investigate what will be the equilibrium for the prepayment scheme where both the SP will not change its policy β and the EU will not change its prepaid amount p unilaterally. To investigate this equilibrium, we consider a simple yet illustrative model as follows. The strategic interaction between the SP and EUs is modeled as a Stackelberg game, where the SP plays as the game leader and determines its policy β in advance. The EUs play as the game followers and determine their prepaid amount p as a response to the SP's prepayment policy β. The Stackelberg game captures the practical market where the SP usually possesses a dominant position in comparison with individual EU. Notice that as an initial step to analyze the prepayment scheme in this work, we first choose to model a representative EU, which represents a class of EUs with homogeneous traffic characteristic (thus, it suffices for all EU to use the same prepayment p and for the SP to use the same policy β). This choice helps us focus on analyzing the strategic interaction between the SP and the EU and quantifying their benefits from the prepayment scheme. Nevertheless, in practice multiple classes of EUs with heterogeneous characteristic (e.g., traffic demands) need to be investigated, which is an important extension for our future work. We illustrate the decision-making of the SP and the EUs in the next two subsections.

2.2 Modeling of the Decision-Making of the SP

The SP controls its prepayment policy β to optimize its own reward by solving the following problem

$$\texttt{(SP-P)}: \quad \max_{0 \leq \beta \leq 1} V(\beta, p) = p(1-\beta)(1-\alpha) + \theta p \alpha, \tag{1}$$

where the discount factor α (ranging from 0 to 1) measures the rate at which the capital devalues over time. $\alpha = 1$ means no devaluation, and $\alpha = 0$ means a complete devaluation. The risk factor θ measures the probability that the SP fails to get the service fee from the EU, which results in a bad-debt. The EUs' prepaid amount is denoted by p. We explain the two parts of the SP's reward function $V(\beta, p)$ in details as follows.

Part (i): the first part $p(1-\beta)(1-\alpha)$ in (1) denotes the *additional* reward the SP can obtain compared to the post-payment. It can be explained as follows. Recall that T denotes the original service fee the EU has to pay. With the prepayment, the SP obtains $p(1-\beta)+(T-p)\alpha$, which includes $p(1-\beta)$ at the beginning of the service and $(T-p)\alpha$ at the end. Notice that the SP spends $p\beta$ in increasing the EU's bandwidth limit according to its prepayment policy, hence $p\beta$ is deduced from the SP's reward. By contrast, with the post-payment, at the end of the service cycle, the SP can obtain $p(1-\beta)\alpha + (T-p)\alpha = (T-p\beta)\alpha$. Notice that, to make a fair comparison, we also assume that the SP spends $p\beta\alpha$ in increasing its bandwidth with the post-payment, which however happens at the end of the service cycle. Table 1 lists the difference between the post-payment and the prepayment. The difference between the sum of the first row for post-payment and the sum of the second row for prepayment gives the first part in (1), i.e., $p(1-\beta)(1-\alpha)$.

Part (ii): the second part $\theta p\alpha$ in (1) denotes the SP's gain from reducing the loss due to bad-debt. Specifically, with prepayment, the SP has a risk of losing $\theta(T-p)\alpha$ due to the bad-debt at the end of the service cycle. In comparison, with postpayment, the SP faces a risk of losing $\theta T\alpha$ at the end. The difference between them yields $\theta p\alpha$.

Table 1. Difference between the post-payment and prepayment

	Beginning of the service cycle	End of the service cycle
Post-payment	0	$(T-p\beta)\alpha$
Prepayment	$p(1-\beta)$	$(T-p)\alpha$

In summary, the SP's reward function (1) trades off between the economic gain the SP obtains from prepayment and its additional cost in increasing the bandwidth-limit for EUs. The SP's reward function (1) measures the *additional* gain from the prepayment compared to that from the conventional postpayment, i.e., a positive reward means that the SP gains more from the prepayment and will choose the option of prepaying self-incentively. This corresponds to a key motivation for our work, i.e., investigating whether both the SP and EU can benefit from the proposed prepayment (compared to the postpayment).

2.3 Modeling of Decision-Making of the Presentative EU

The presentative EU plays as a follower and determines its prepaid amount p (as a response to the SP's policy β) by solving the following problem.

$$\texttt{(EU-P)}: \max_{0 \leq p \leq T} U(\beta, p) = \omega \frac{p\beta}{\mu} \frac{1}{(C-x)(\frac{p\beta}{\mu} + C - x)} - p(1-\alpha), \tag{2}$$

where C denotes the bandwidth-limit without prepaying, and x denotes the EU's traffic load. μ denotes the marginal cost for the SP to increase bandwidth-limit.

As shown in the EU's reward function $U(\beta, p)$, we measure the EU's reward as the difference between its improved delay performance and its economic loss. Specifically, with prepayment, the EU's bandwidth-limit is increased and hence its average traffic delay is improved by $\frac{p\beta}{\mu} \frac{1}{(C-x)(\frac{p\beta}{\mu}+C-x)}$. For simplicity, we use the M/M/1 queue model to quantify the EU's average delay under a given traffic load. Similar models quantifying user's dissatisfaction due to average traffic delay also appeared in [17, 18] regarding to the optimization of network operators. Here, ω denotes the weighting factor that maps the EU's improved delay-performance into its perceptive reward (which is comparable with its economic loss). Without loss of generality, we set $\omega = 1$ in the rest of this paper. Meanwhile, $p(1-\alpha)$ denotes the EU's economic loss due to prepaying, which is the difference between the EU's payment $T\alpha$ with post-payment and its payment $p + (T-p)\alpha$ with prepayment. In summary, $U(\beta, p)$ represents that the EU aims to achieve a good balance between receiving an improved QoS-performance and reducing its economic loss.

Functions (1) and (2) show the conflicting objectives of SP and EUs. The equilibrium of the above interaction between the SP and EU represents the stable state the SP and EUs will reach, and no one will deviate from this equilibrium unilaterally. We derive this equilibrium in the next section.

3 Analysis of Equilibrium

3.1 Backward Induction and Equivalent Formulation

In the above formulated Stackelberg game, the SP possesses the dominant position and hence has the so-called first-move advantage [19], i.e., the SP can optimize its policy β by expecting the EU's response (i.e., the prepaid amount p). Thus, we adopt the backward induction to derive this equilibrium.

We first derive the EU's best choice of prepaid amount given the SP's prepayment policy β. Since in problem `(EU-P)`, the EU's reward function is strictly concave with respect to p, the first order condition for optimality is applicable. By setting $\frac{\partial U(\beta,p)}{\partial p} = 0$, we obtain the EU's optimal prepaid amount p as a response to the SP's policy β as follows

$$p(\beta) = \min\Big\{\sqrt{\frac{\mu}{\beta}}\sqrt{\frac{1}{1-\alpha}} - \frac{\mu}{\beta}(C-x), T\Big\}. \tag{3}$$

The above result shows that to guarantee $p(\beta) \geq 0$, it is required that $0 \leq \sqrt{\frac{\mu}{\beta}} \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$. It means that to encourage a positive prepayment from the EU, the SP has to commit a minimum investment, i.e., β should be above a certain threshold which depends on the EU's traffic load. By substituting (3) into the EU's objective function, we obtain the EU's reward as a function of the SP's policy β as follows

$$U^*(\beta) = U(\beta, p(\beta)) = \max\big\{\frac{1}{C-x}\big(1-\sqrt{\frac{\mu}{\beta}}\sqrt{1-\alpha}(C-x)\big)^2, U(\beta, T)\big\}. \quad (4)$$

Remark 1: Equation (4) shows that, with a favorable SP's prepayment policy (i.e., condition $\sqrt{\frac{\mu}{\beta}}\sqrt{1-\alpha}(C-x) \leq 1$ is met to prevent a negative EU's prepaid amount), the EU always receives a nonnegative reward by choosing its prepaid amount according to (3), and hence the EU is encouraged to choose the option of prepaying.

Next, knowing the EU's best response, we analyze the SP's best prepayment policy. Suppose that the EU's original service fee T is relatively large. Then, using (3), the SP's optimization problem `(SP-P)` becomes

$$\max_{0\leq\beta\leq 1} \big(\sqrt{\frac{\mu}{\beta}}\sqrt{\frac{1}{1-\alpha}} - \frac{\mu}{\beta}(C-x)\big)\big((1-\beta)(1-\alpha)+\theta\alpha\big). \quad (5)$$

Let β^* denote the SP's optimal prepayment policy for the above problem. Then, the EU's optimal prepaid amount can be given by $p^* = p(\beta^*)$ according to (3). Hence, the profile (β^*, p^*) serves as the equilibrium for our formulated Stackelberg game. However, despite its simple form, the above problem (5) is nonconvex in general with respect to β. To tackle this difficulty and explore the hidden structural property, we first make a change of variables as follows: $z = \sqrt{\frac{\mu}{\beta}}$, or equivalently $\beta = \frac{\mu}{z^2}$. Notice that $z \geq \sqrt{\mu}$ since $0 \leq \beta \leq 1$.

Based on this change of variables, the EU's best choice (3) can be re-expressed as

$$p(z) = z\big(\sqrt{\frac{1}{1-\alpha}} - z(C-x)\big). \quad (6)$$

Meanwhile, the SP's optimization problem `(SP-P)` can be equivalently transformed into the following problem (here the symbol prime denotes the "Equivalence")

$$\texttt{(SP-P}'\texttt{)}: \max_{z\geq 0} \widetilde{V}(z) = F(z)G(z)$$
$$\text{subject to: } \sqrt{\mu} \leq z \leq \sqrt{\frac{1}{1-\alpha}}\frac{1}{C-x}, \quad (7)$$

where

$$F(z) = z\big(\sqrt{\frac{1}{1-\alpha}} - (C-x)z\big), \quad (8)$$

$$G(z) = \big(1-\alpha+\theta\alpha-(1-\alpha)\frac{\mu}{z^2}\big). \quad (9)$$

However, directly solving the above problem (SP-P$'$) is yet straightforward because it is still nonconvex with respect to z. Fortunately, we can exploit the hidden structural property in problem (SP-P$'$), which facilitates our following solution procedures. As we will illustrate soon, the hidden property strongly depends on the EU's traffic load, which implies that *the EU's traffic load significantly influences the interaction between SP and EU as well as the resulting equilibrium-prepaying.* This influence can be intuitively interpreted as follows. With a heavy traffic load, the EU has a strong desire to improve its delay performance (according to the formula measuring the average traffic delay, a slight increase of the bandwidth limit will reduce the delay significantly under a heavy traffic load). Thus, the EU is willing to prepay more. In consequence, the SP sets a small β to maximize its reward. In contrast, with a light traffic load, the EU already has a good delay performance and hence has a weak desire to improve its delay performance. Thus, the EU will prepay less and the SP has to set a greater β (even close to 1).

In the next three subsections, we will illustrate the above intuition by quantifying three different traffic load regions (namely, the heavy, medium and light regions) and investigating the equilibrium for each region, respectively.

3.2 Equilibrium Under the Heavy Traffic Load Region

We first consider the heavy traffic load region. Suppose that $\sqrt{\mu} \leq \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$ holds, which is equivalent to that the traffic load x meets $C \geq x \geq C - \frac{1}{2}\sqrt{\frac{1}{1-\alpha}}\frac{1}{\sqrt{\mu}}$. We thus call it the heavy traffic load region. Our key findings in this region are as follows.

Key Findings in the Heavy Region: Each traffic load x in the heavy load region corresponds to a unique equilibrium, where the SP's policy $1 > \beta^* > 0$, and the SU's prepaid amount $p^* > 0$. Correspondingly, both SP and EU receive positive rewards at the equilibrium, i.e., they benefit from the this prepayment simultaneously.

As will be explained in Sect. 4, Fig. 1 shows the profile of equilibrium under different traffic loads via a numerical example. Figure 2 further shows the corresponding rewards at the equilibrium. Specifically, the above findings along with the numerical results match our intuition that the EU is strongly motivated to prepay under a heavy traffic load to improve its delay-performance and both the SP and EU will benefit from the prepayment significantly.

We explain these findings as follows. To solve problem (SP-P$'$) in the heavy traffic region, we first divide the feasible range of the decision variable z into two subranges, namely *Subrange 1:* $\sqrt{\mu} \leq z \leq \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$ and *Subrange 2:* $\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}} \leq z \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$. Next, we quantify the value of z which can maximize $\widetilde{V}(z)$ for each subrange as follows.

(1) Subrange 1: $\sqrt{\mu} \leq z \leq \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$

It can be checked that both $F(z)$ and $G(z)$ are increasing in z. Hence, $\widetilde{V}(z)$ is increasing. As a result, in *Subrange 1*, the value of z that maximizes the objective function $\widetilde{V}(z)$ of problem (SP-P′) can be directly given as $z^* = \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$.

(2) Subrange 2: $\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}} \leq z \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$

$F(z)$ is decreasing in z while $G(z)$ is increasing in z. By checking $\frac{d\widetilde{V}(z)}{dz}$, we obtain

$$H(z) = \frac{d\widetilde{V}(z)}{dz} = \sqrt{\frac{1}{1-\alpha}}(1-\alpha+\theta\alpha)+\sqrt{1-\alpha}\frac{\mu}{z^2}-2z(C-x)(1-\alpha+\theta\alpha). \quad (10)$$

Notice that $H(z)$ is strictly decreasing in z, which implies that $\widetilde{V}(z)$ is strictly concave with respect to z. Therefore, in *Subrange 2*, there will be a unique value of z that maximizes $\widetilde{V}(z)$. Nevertheless, quantifying this unique z in a closed-form expression is challenging. Fortunately, we can identify the following result regarding to $H(z)$.

Proposition 1: In *Subrange 2* (i.e., $\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}} \leq z \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$), there exists a unique value of z, which meets the condition that $H(z) = 0$.

Proof: We first show the following two important properties.
Property (i): by substituting $z = \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$ into $H(z)$ (10), we obtain the following result

$$H(\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}) = \sqrt{\frac{1}{1-\alpha}}\big((1-\alpha)\mu(1-\alpha)(C-x)^2 - ((1-\alpha)+\theta\alpha)\big).$$

Since $\sqrt{\mu} \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$, i.e. $\mu(1-\alpha)(C-x)^2 \leq 1$, there exists $H(\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}) \leq 0$.
Property (ii): by substituting $z = \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$ into $H(z)$ (10), we obtain the following result

$$H(\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}) = 4\mu(1-\alpha)^{\frac{3}{2}}(C-x)^2 \geq 0.$$

The above *property (i)* and *property (ii)*, along with the property that $H(z)$ is strictly decreasing, indicate that within *Subrange 2*, i.e. $\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}} \leq z \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$, there exists a unique value of z satisfying the condition $H(z) = 0$. Hence, we finish the proof. ■

(3) Summary of the Results in Subrange 1 and Subrange 2

Notice that the condition $H(z) = 0$ suffices to be the optimality condition for problem $(\texttt{SP-P}')$ under the heavy load region. Our analysis of *Subrange 1* and *Subrange 2* shows that, in the heavy load region, there exists a unique optimal solution for $(\texttt{SP-P}')$, and moreover this optimal solution resides within the range of $\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}} \leq z \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$. Let z^* denote this optimal solution. By exploiting the property that $H(z)$ is strictly decreasing, we can use the bisection algorithm to find z^* within the range of $\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}} \leq z \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$, thus solving problem $(\texttt{SP-P}')$. In particular, by using the bisection algorithm, we can determine this z^* within $\log_2 \frac{\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}}{\epsilon}$ rounds of iteration, where ϵ denotes the tolerance level of numerical error. After that, at the equilibrium, the SP's prepayment policy is given by

$$\beta^* = \frac{\mu}{(z^*)^2}, \tag{11}$$

and according to (6) the EU's prepaid amount is given by

$$p^* = z^*\big(\sqrt{\frac{1}{1-\alpha}} - z^*(C-x)\big). \tag{12}$$

3.3 Equilibrium Under the Medium Traffic Load Region

Next, we consider the medium traffic load region. Specifically, suppose that $\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}} \leq \sqrt{\mu} \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$, which is equivalent to that the traffic load x meets the condition $C - \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{\sqrt{\mu}}} \geq x \geq C - \sqrt{\frac{1}{1-\alpha}\frac{1}{\sqrt{\mu}}}$. We thus call it the medium traffic load region. Our findings in this region are as follows.

Key Findings in the Medium Region: Each load x the medium region corresponds to a unique equilibrium. In particular, there exists a special traffic threshold Γ in the medium region. When the EU traffic load x is below Γ, then the SP's policy $\beta^* = 1$ (i.e., the SP has to spend entire the EU's prepayment in increasing the bandwidth-limit to attract the EU), and the EU's prepaid amount $p^* > 0$. Correspondingly, both the SP and EU obtain positive rewards, i.e., they benefit from the prepayment scheme (notice that even with $\beta^* = 1$, the SP still obtains a positive reward from saving its loss due to bad-debt). However, these rewards decrease as the EU traffic load x decreases.

As to be explained in Sect. 4, Fig. 1 shows the profile of equilibrium via a numerical example, and Fig. 2 shows the corresponding rewards, which verify the above findings. Compared to the heavy region, the EU becomes less motivated to prepay as its traffic load decreases in the medium region. As a result, the SP has to spend more in increasing its bandwidth to encourage the EU to prepay. Moreover, when the EU's traffic load is below the threshold Γ, the SP has to spend the entire prepayment in increasing its bandwidth. All these match our intuitions well.

We further explain these findings as follows. Notice that for the same reason as we describe in the previous subsection, there exists $H(\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}) \leq 0$. By substituting $\sqrt{\mu}$ into $H(z)$, we obtain that

$$H(\sqrt{\mu}) = 2(C-x)((1-\alpha)+\theta\alpha)(S-\sqrt{\mu}),$$

where

$$S = \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}} + \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}\frac{1-\alpha}{(1-\alpha)+\theta\alpha}}.$$

Notice that $\frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}} \leq S \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$ always holds. In particular, the value of $H(\sqrt{\mu})$ is of interest since it helps to divide the medium traffic load region into two subregions with different characteristics, i.e., (i) the subregion that yields the SP's prepayment policy $\beta = 1$, and (ii) the subregion that yields the SP's prepayment policy $\beta < 1$. The details are illustrated as follows. By setting $H(\sqrt{\mu}) = 0$, we determine a threshold $\varGamma$ with respect to the traffic load as

$$\varGamma = C - \frac{1}{\sqrt{\mu}}(\frac{1}{2}\sqrt{\frac{1}{1-\alpha}} + \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1-\alpha}{1-\alpha+\theta\alpha}}). \tag{13}$$

Using the threshold $\varGamma$, we thus consider two different subregions for the traffic load, namely, *Subregion (i):* $C - \sqrt{\frac{1}{1-\alpha}}\frac{1}{\sqrt{\mu}} \leq x \leq \varGamma$ and *Subregion (ii):* $\varGamma \leq x \leq C - \frac{1}{2}\sqrt{\frac{1}{1-\alpha}}\frac{1}{\sqrt{\mu}}$. The details are as follows.

Subregion (i): the traffic load subregion of $C - \sqrt{\frac{1}{1-\alpha}}\frac{1}{\sqrt{\mu}} \leq x \leq \varGamma$.

When $C - \sqrt{\frac{1}{1-\alpha}}\frac{1}{\sqrt{\mu}} \leq x \leq \varGamma$, then $H(\sqrt{\mu}) \leq 0$. Since $H(\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}) \leq 0$ and $H(z)$ is strictly decreasing in z, there always exists $H(z) \leq 0$ within the range of $\sqrt{\mu} \leq z \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$. Therefore, the optimal solution for problem (`SP-P`$'$) can be directly given by $z^* = \sqrt{\mu}$ and $\beta^* = 1$. This result means that at the equilibrium, the SP has to use the entire EU's prepayment to increase its bandwidth. Correspondingly, the EU's prepaid amount can be determined as $p^* = \sqrt{\mu}\big(\sqrt{\frac{1}{1-\alpha}} - \sqrt{\mu}(C-x)\big)$ according to (3). Notice that even using $\beta^* = 1$, the SP still gets a nonnegative reward equal to $\theta\alpha\sqrt{\mu}\big(\sqrt{\frac{1}{1-\alpha}} - \sqrt{\mu}(C-x)\big)$, which stems from saving its loss in bad-debt.

Subregion (ii): the traffic load subregion of $\varGamma \leq x \leq C - \frac{1}{2}\sqrt{\frac{1}{1-\alpha}}\frac{1}{\sqrt{\mu}}$.

When $\varGamma \leq x \leq C - \frac{1}{2}\sqrt{\frac{1}{1-\alpha}}\frac{1}{\sqrt{\mu}}$, then $H(\sqrt{\mu}) \geq 0$. Since $H(\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}) \leq 0$ and $H(\sqrt{\mu}) \geq 0$, further along with the property that $H(z)$ is strictly decreasing in z, there exists a unique value of z within the range of $\sqrt{\mu} \leq z \leq \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}$ such that $H(z) = 0$, which corresponds to the optimality condition for problem (`SP-P`$'$). Again, let z^* denote the optimal value of z for problem (`SP-P`$'$). We can use the bisection algorithm to determine z^* such that $H(z^*) = 0$ is reached (within $\log_2 \frac{\sqrt{\frac{1}{1-\alpha}}\frac{1}{C-x}-\sqrt{\mu}}{\epsilon}$ rounds of iteration). Therefore, the equilibrium can be given by $\beta^* = \frac{\mu}{(z^*)^2}$ and $p^* = z^*\big(\sqrt{\frac{1}{1-\alpha}} - z^*(c-x)\big)$.

In summary, in the region of the medium traffic load, by further differentiating two subregions (which are separated by the threshold Γ), we successfully quantify the equilibrium (β^*, p^*) for the proposed game.

Remark 2: The traffic threshold Γ, which separates the medium traffic load region, is an important indicator that determines *whether the SP will spend the entire EU's prepaid amount in increasing its bandwidth-limit or not.* Intuitively, with a traffic load below Γ, the EU is less motivated to prepay for an improved QoS performance. As a result, the SP has to spend more to attract the EU and thus gain less.

Remark 3: The traffic load threshold Γ given in (13) has the following three properties. Property (i): Γ is increasing in the risk factor θ; Property (ii): Γ is increasing in the marginal bandwidth-cost μ; Property (iii): Γ is decreasing in the discount factor α.

Property (i) is a direct result from (13). Intuitively, when the risk factor θ is larger, the SP can save more from reducing the bad-debt according to (1). Thus, the SP tends to increase its β, i.e. using a large portion of EU's prepaid amount to increase its access bandwidth.

Property (ii) is also a direct result from (13). It can be interpreted as follows. When the marginal cost μ is larger, the EU's improved delay performance tends to decrease according to (2) and (4). As a result, the SP has to increase its β to compensate for the EU's loss more significantly such that the EU is still motivated to prepay.

Property (iii) can be proved by showing that $\frac{\partial \Gamma}{\partial \alpha} \leq 0$. First, let $\Omega = \sqrt{\frac{1}{1-\alpha}}(1 + \frac{1-\alpha}{1-\alpha+\theta\alpha})$. We thus have $\Gamma = C - \frac{1}{\sqrt{\mu}}\frac{1}{2}\Omega$. Specifically, there exists

$$\frac{\partial \Omega}{\partial \alpha} = \sqrt{\frac{1}{1-\alpha}}\{\frac{(1-\alpha)(1-\theta)}{(1-\alpha+\theta\alpha)^2} + \frac{1}{2}\frac{1}{1-\alpha+\theta\alpha}\frac{\theta\alpha}{1-\alpha}\} \geq 0.$$

Since Ω is increasing in the discount factor α, Γ is decreasing in α. Intuitively, when the discount factor α is greater, the EU's economic loss decreases, and thus EU will receive a greater reward according to (2) and (4). As a result, the SP can reduce its β accordingly, meaning that the SP spends less in increasing its access bandwidth.

Due to the limited space, we will skip the details about the analysis for the light traffic load region in which the traffic load x satisfies $0 \leq x \leq C - \sqrt{\frac{1}{1-\alpha}}\frac{1}{\sqrt{\mu}}$. Notice that the methodology to analyze the light traffic load region is similar to that for the heave and medium load regions.

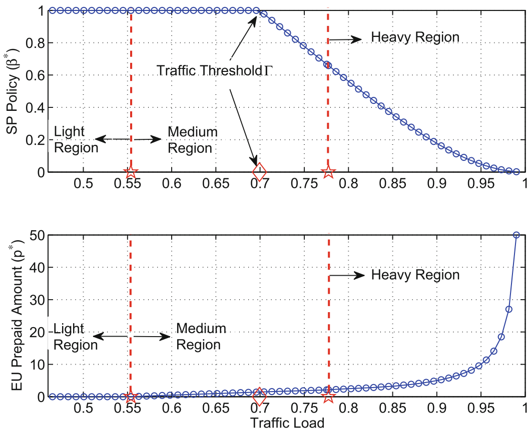

Fig. 1. Examples of equilibrium profile (β^*, p^*) under different traffic loads. Top subfigure: SP's prepayment policy β^* at the equilibrium. Bottom subfigure: EU's prepaid amount p^* at the equilibrium. We set $C = 10, \mu = 1, \alpha = 0.95, \theta = 0.1$.

4 Numerical Results

4.1 Equilibrium and the Corresponding Rewards

Figure 1 shows the equilibrium (β^*, p^*) under different traffic loads. We normalize both the traffic load x and the load threshold Γ by the link bandwidth C (i.e., the horizontal axis denotes the value of $\frac{x}{C}$). In Fig. 1, different traffic load regions are separated by the dash lines. Notice that in the light region we always have the SP's prepayment policy $\beta^* = 1$ and the EU's prepaid amount $p^* = 0$ at the equilibrium.

The results shown in Fig. 1 verify the equilibrium analyzed in Sect. 3. Specifically, when the EU's traffic load is in the medium region and below the traffic threshold Γ (which is denoted by the diamond), the EU has a relatively weak desire to improve its delay performance, and hence its prepaid amount p^* is relatively small. Correspondingly, the SP sets its prepayment policy $\beta^* = 1$ to encourage the EU's prepaying. In comparison, when the EU's traffic load is above the traffic threshold Γ, the EU's desire to improve its delay performance increases, and its prepaid amount p^* increases. Correspondingly, the SP can adopt a smaller β^*. Furthermore, when the EU's traffic load falls into in the heavy traffic-load region, the EU has a strong desire to improve its delay performance, and hence its prepaid amount p^* increases dramatically. Correspondingly, the SP further reduces its β^*.

Figure 2 further shows the rewards obtained by the SP and the EU at the equilibrium. Specifically, in the medium traffic load region, both the rewards of the SP and the EU are relatively low due to the EU's weak desire to improve its delay. In comparison, when the EU's traffic load is heavy, both the SP and the EU obtain significant rewards from the prepayments since the EU has a strong desire

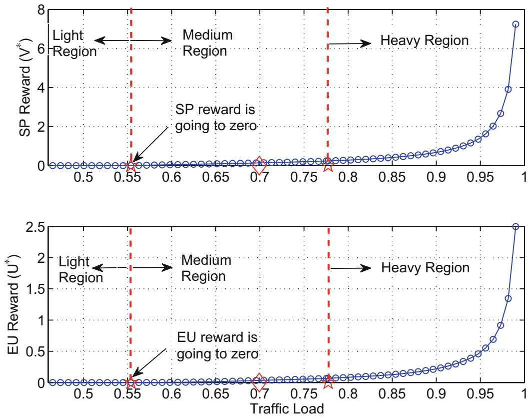

Fig. 2. Examples of equilibrium rewards under different traffic loads. Top subfigure: SP's reward at the equilibrium. Bottom subfigure: the EU's reward at the equilibrium. We set $C = 10, \mu = 1, \alpha = 0.95, \theta = 0.1$.

to improve its delay and hence a greater motivation for prepaying. In particular, the positive EU's reward and the positive SP's reward at the equilibrium (in the medium and heavy traffic regions) shown in Fig. 2 indicate that both the EU and SP can positively gain from prepaying in comparison with conventional post-payment scheme, i.e., thus achieving a desirable win-win result. Notice that as stated in Sect. 2, our definitions for the SP's reward and the EU's reward can be considered as measures of relative gains achieved by the prepayment schemes in comparison with the post-payment scheme. Therefore, the positive EU's reward and SP's reward at the equilibrium indicate the superior performance of the prepayment scheme.

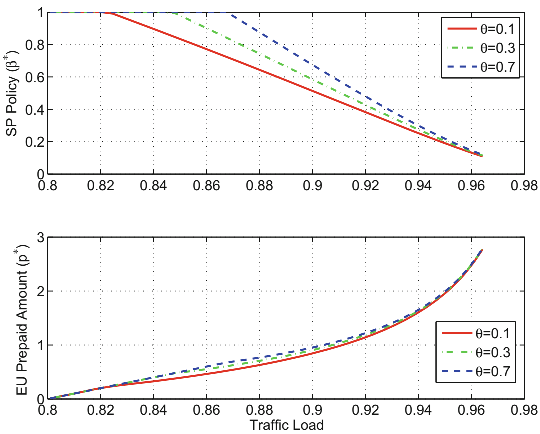

Fig. 3. Impact of the risk factor θ on the equilibrium profile. We set $\alpha = 0.75, \mu = 1$. Top subfigure: SP's prepayment policy β^*; Bottom subfigure: EU's prepaid amount p^*.

Figures 3 and 4 show the impact of the risk factor θ on the SP's prepayment policy and EU's prepaid amount at the equilibrium as well as their rewards.

The top subfigure in Fig. 3 shows that when the risk factor θ increases, the SP also increases its β^*. This result is consistent with the intuition well. Since as θ increases, the SP tries to save more by reducing its bad-debt according to its reward (5). Thus, the SP is willing to spend a greater part of the EU's prepaid amount in increasing the link capacity. Analytically, this result can also be illustrated as follows. To guarantee $H(z) = 0$, it requires (after some manipulations):

$$\sqrt{\frac{1}{1-\alpha}} = 2z(C-x) - \frac{\sqrt{1-\alpha}}{1-\alpha+\theta\alpha}\frac{\mu}{z^2}. \tag{14}$$

The right hand side (RHS) of (14) is increasing in both θ and z. Therefore, to meet condition (14), the optimal value of z decreases as θ increases. Thus, the SP's $\beta = \frac{\mu}{z^2}$ increases accordingly. Notice that the top subfigure in Fig. 3 also verifies that the traffic threshold Γ (i.e., the threshold beyond which the SP's policy $\beta^* < 1$) is increasing in θ, as stated in Property (i) Remark 3.

The bottom subfigure in Fig. 3 shows that as the risk factor θ increases, the EU also increases its prepaid amount p^*. This result matches the intuition that as the risk factor θ increases, the SP is willing to spend a greater portion of the EU's prepayment in increasing its link capacity, and the EU thus finds it is profitable to prepay more. Also analytically, this result can be illustrated as follows. Based on our analysis in Sect. 3, the optimal value of z can only reside in the interval of $[\max(\sqrt{\mu}, \frac{1}{2}\sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}}), \sqrt{\frac{1}{1-\alpha}\frac{1}{C-x}})]$. Thus, according to the EU's best choice function (6), the EU's prepaid amount $p(z)$ is decreasing in z. Since z decreases as θ increases, the EU's prepaid amount p increases accordingly.

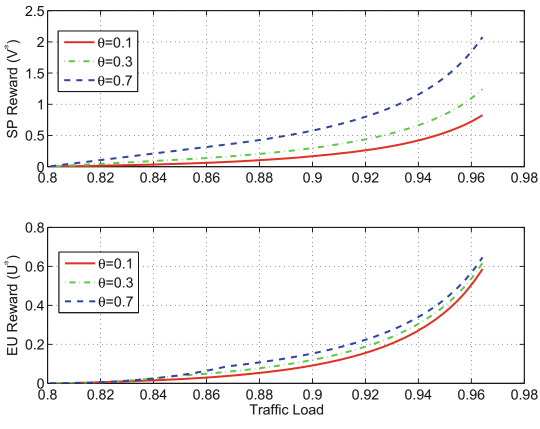

Fig. 4. Impact of the risk factor θ on the equilibrium rewards. We set $\alpha = 0.75, \mu = 1$. Top subfigure: SP's reward at the equilibrium; Bottom subfigure: EU's reward at the equilibrium.

Meanwhile, the results of Fig. 4 show that both the SP and the EU can benefit more from the prepayment scheme when the risk factor θ increases. Intuitively, as the risk factor increases, the prepayment scheme can save the SP's loss more due to bad-debt. Thus, both the SP and EUs benefit more. Specifically, the bottom subfigure in Fig. 4 shows as the risk factor θ increases, the EU's reward at the equilibrium increases. This result can be illustrated analytically as follows. According to (4), the EU's reward at equilibrium can be given by $U^*(z) = \frac{1}{C-x}(1 - z\sqrt{1-\alpha}(C-x))^2$ (for an easy presentation we stick with the use of $U(.)$ as the EU's reward function, which is a function of z instead of β). Since the SP's optimal decision z decreases as θ increases, the EU's utility increases consequently. Meanwhile, the top subfigure in Fig. 4 shows that the SP's reward at the equilibrium also increases as the risk factor θ increases.

5 Conclusion and Future Work

In this paper, we propose a smart prepayment scheme for the SP of access networks to improve its economic reward and better EUs' QoS-performance. Our analysis based on the model of Stackelberg game shows that both the SP and EUs can benefit from the optimal prepayment scheme (i.e., the equilibrium of the proposed game), thus yielding a win-win outcome. We also find that the EU's traffic load influences the optimal prepayment scheme, which we quantify and analyze by considering three different traffic load regions.

Acknowledgement. This work was supported in part by the National Natural Science Foundation of China under Grant 61572440 and Grant 61379122, in part by the Zhejiang Provincial Natural Science Foundation of China under Grant LR17F010002 and Grant LR16F010003, and in part by the Young Talent Cultivation Project of Zhejiang Association for Science and Technology (2016YCGC011).

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