Performance Analysis of Frequency Division Multiplex Complementary Coded CDMA Systems

Siyue Sun[1(✉)], Kun Wang[2], Guang Liang[1], Feng Tian[1], and Zaiyang Jiang[1]

[1] Shanghai Engineering Center for Micro-satellites, Shanghai, China
sunsiyue@hit.edu.cn
[2] Huawei Technologies Co., Ltd., Shanghai, China

Abstract. As a kind of two-dimensional spreading codes, complementary codes (CCs) are able to provide ideal correlation properties, which is an attractive feature for CDMA systems to eliminate multiple access interference (MAI) and multi-path interference (MPI). Based on modeling the frequency selective fading channel for a kind of frequency division multiplex (FDM) CC-CDMA system, this paper provides a detailed performance analysis of such kind of CC-CDMA systems and proves the limits of channel conditions for the interference-free character of such systems, which is verified by the simulated results at the end of this paper.

Keywords: Complementary codes · CDMA · Correlation properties
Multiple access interference · Multi-path interference

1 Introduction

As one of the most popular multiple access techniques in the past 50 years, Code Division Multiple Access (CDMA) provides higher frequency efficiency and security with lower radiation for wireless communications. However, due to its interference-limited problem, CDMA has lost competitiveness compared with Orthogonal Frequency Division Multiplexing Access (OFDMA) in ground cellular systems, although it is still the preferred multiple access technique in satellite communications.

It has been widely approved that all existing CDMA systems are interference-limited, particularly in the presence of multiple access interference (MAI) and multi-path interference (MPI) due to the undesirable properties of the spreading sequences. In order to improve the performance of CDMA technique, [1–3] proposed a new kind of spreading sequences — complementary codes (CCs) which

This work was partly supported by National Nature Science Foundation Program of China (No. 61601295), Shanghai Sailing Program (16YF1411000) and CAS Innovation Fund (No. CXJJ-16S033).

C. Li and S. Mao (Eds.): WiCON 2017, LNICST 230, pp. 319–327, 2018.
https://doi.org/10.1007/978-3-319-90802-1_28

are able to realize both idea auto- and cross- correlation properties relying on the two-dimensional code structure and the definition of complementary correlation.

Owing to such desired properties, the complementary coded CDMA (CC-CDMA) system has become one of the research hotspots, especially during discussing the candidates for 4G cellular systems. In [4], we have present a survey on the history of CCs and a deeply studies on the correlation properties of CCs with realistic communication environment was presented in [6]. However, the two-dimensional code structure and definition of complementary correlation make it hard to implement a CC-CDMA architecture. In [5], we have divided the existing CC-CDMA solutions into two categories according to the kinds of independent sub-channels and compared them in terms of resist of MAI and MPI, implementation complexity and spread and spectrum efficiency. In the last decade, CC-CDMA systems have been widely studied and reported by a large number of works [8–12] which showed their superior performance compared with the traditional CDMA systems. However, the desired interference-free features of CC-CDMA relies mainly on the same fading pattern on each sub-channel, which is almost impossible in realistic communication environment.

In this paper, based on modeling the frequency selective fading channel for a kind of frequency division multiplex (FDM) CC-CDMA systems, a detailed performance analysis of such kind of CC-CDMA systems is provided and the paper will prove the limits of channel conditions for the interference-free character of such systems, which will be verified by the simulated results at the end of this paper. The analysis and simulated work in this paper will provide theoretical bases and new ideas for future research work.

2 Definitions

2.1 Definitions of Complementary Codes

A family of CCs is denoted as $\mathcal{C}(K, M, N)$ which contents K CCs each with M element sequences. A CC can be represented by a $M \times N$ matrix $\mathbf{C}^{(k)}$, which is unfold as

$$\mathbf{C}^{(k)} = \begin{bmatrix} \mathbf{c}_0^{(k)} \\ \mathbf{c}_1^{(k)} \\ \vdots \\ \mathbf{c}_{M-1}^{(k)} \end{bmatrix} = \begin{bmatrix} c_{0,0}^{(k)} & c_{0,1}^{(k)} & \cdots & c_{0,N-1}^{(k)} \\ c_{1,0}^{(k)} & c_{1,1}^{(k)} & \cdots & c_{1,N-1}^{(k)} \\ \vdots & \vdots & \ddots & \vdots \\ c_{M-1,0}^{(k)} & c_{M-1,1}^{(k)} & \cdots & c_{M-1,N-1}^{(k)} \end{bmatrix},$$

where $c_{m,n}^{(k)} \in \{1, -1\}$, $k \in \{0, 1, \cdots, K-1\}$, $n \in \{0, 1, \cdots, N-1\}$, and $m \in \{0, 1, \cdots, M-1\}$. M is called flock size and N is the code length. In a CC-CDMA system, MN is the "congregated length" of a CC, and it determines the corresponding processing gain.

2.2 Definitions of the Complementary Correlation

The correlation properties of CCs are characterized by the complementary aperiodic correlation function which is calculated as the sum of the aperiodic correlation functions of all element sequences with the same delay τ, or

$$\rho\big(\mathbf{C}^{(k_1)}, \mathbf{C}^{(k_2)}; \tau\big) = \sum_{m=1}^{M} \phi\big(\mathbf{c}_m^{(k_1)}, \mathbf{c}_m^{(k_2)}; \tau\big) \tag{1}$$

where $\mathbf{C}^{(k_1)}, \mathbf{C}^{(k_2)} \in \mathcal{C}(K, M, N)$, $k_1, k_2 \in \{1, 2, \cdots, K\}$, and $\phi\big(\mathbf{c}_m^{(k_1)}, \mathbf{c}_m^{(k_2)}; \tau\big)$ is the aperiodic correlation function of $\mathbf{c}_m^{(k_1)}$ and $\mathbf{c}_m^{(k_2)}$. When $k_1 = k_2$, (1) is the complementary aperiodic auto-correlation function (ACF); otherwise, it gives the complementary aperiodic cross-correlation function (CCF).

A CC is said to be perfect if and only if its ACF is a delta function. A set of perfect CCs are considered to be perfect, if and only if the CCF of any two CCs is a zero function. The correlation properties of a perfect set of CCs, $\mathcal{C}(K, M, N)$, which are also called ideal correlation properties throughout this paper, can be expressed as

$$\rho(\mathbf{C}^{(k_1)}, \mathbf{C}^{(k_2)}; \tau) = \begin{cases} MN, & k_1 = k_2 \,\text{and}\, \tau = 0 \\ 0, & \text{elsewhere} \end{cases} \tag{2}$$

where $\forall k_1, k_2 \in \{1, \cdots, K\}$.

3 System Model and Performance Analysis

3.1 The Transmitter Mode

In this paper, a kind of FDM CC-CDMA system [7] will be studied. A family of CCs $\mathcal{C}(K, M, N)$ is employed as the spreading codes for a system with K users and let $\mathbf{C}^{(k)}$ be the spreading code for user k. The transmitter mode of an FDM CC-CDMA system is shown in Fig. 1.

The original data is serial/parallel conversed into $b^{(k,1)}, b^{(k,2)}, \cdots, b^{(k,N)}$. Then the N streams of data are spread by the same CC and they are summed after delayed different number of chips latency.

The spreading wave of mth element code of user k is

$$C_m^{(k)}(t) = \sum_{n=1}^{N} c_{m,n}^{(k)} q(t - nT_c + T_c) \tag{3}$$

where, $q(t) = \frac{1}{\sqrt{MNT_c}}$, $0 \le t < T_c$, T_c is the chip duration. Let $b^{(k,\eta)}(i)$ be the ith BPSK symbol of ηth stream of data after S/P operation, $\eta \in \{1, 2, \cdots, N\}$, $i \in \{0, 1, \cdots, B-1\}$ and B is the length of each stream of data burst. Then the spread signal after delayed $\eta - 1$ chips latency is

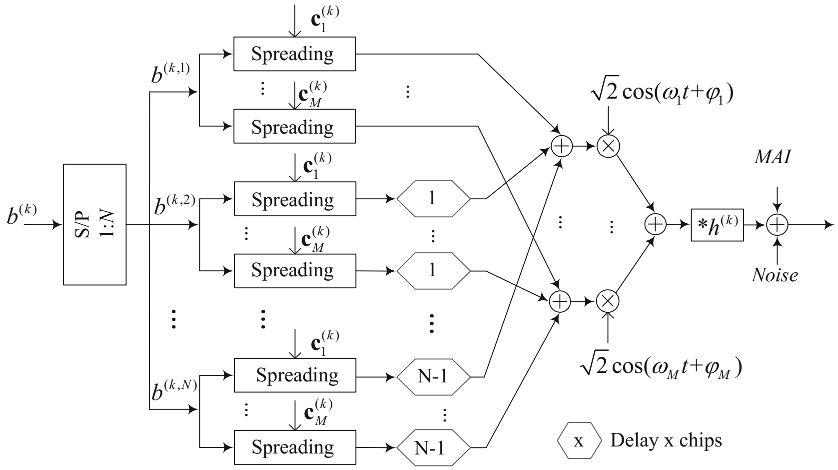

Fig. 1. The transmitter mode of parallel frequency-division-multiplexed CC-CDMA systems.

$$s_m^{(k,\eta)}(t) = \sqrt{p_t}\sum_{i=0}^{B-1} b^{(k,\eta)}(i)C_m^{(k)}\left[t - iT_b - (\eta-1)T_c\right] \tag{4}$$

where, p_t is the transmitting power, $T_b = NT_c$ is the bit duration.

As shown in Fig. 1, combine the above N streams of data, we get

$$s_m^{(k)}(t) = \sum_{\eta=1}^{N} s_m^{(k,\eta)}(t) \tag{5}$$

Finally, the M combined signal will be modulated on M sub-carriers, as

$$s^{(k)}(t) = \sum_{m=1}^{M} s_m^{(k)}(t)\sqrt{2}\cos(\omega_m t + \varphi_m) \tag{6}$$

3.2 The Receiver Mode

In this paper, a tap-delay-line model is used to describe the multi-path channel and we get

$$\begin{aligned} r(t) &= \sum_{k=1}^{K} \mathit{\Gamma}_k h^{(k)}(t) * s^{(k)}(t) \\ &= \sqrt{2}\sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{m=1}^{M} \mathit{\Gamma}_k h_l^{(k)} s_m^{(k)}(t-\tau_{k,l})\cos\left[\omega_m(t-\tau_{k,l}) + \widehat{\varphi}_m\right] \end{aligned} \tag{7}$$

where $\mathit{\Gamma}_k$ is channel coefficient of user k, $h_l^{(k)}$ and $\tau_{k,l}$ denote the channel coefficient and the time delay of lth channel.

Assuming ideal carrier-synchronization, the received signal on ω_ith sub-carrier is coherent demodulated in the receiver, as

$$\begin{aligned} r_i(t) &= \left\{ r(t) \cdot \sqrt{2}\cos(\omega_i t + \widehat{\varphi}_i) \right\}_{LPF} \\ &= \sum_{l=1}^{L}\sum_{k=1}^{K}\sum_{m=1}^{M} \Gamma_k h_l^{(k)} s_m^{(k)}(t-\tau_{k,l})\cos(\omega_i \tau_{k,l}) \end{aligned} \tag{8}$$

where $\{\cdot\}_{LPF}$ means low pass filtering. Therefore, the equivalent baseband multi-path channel model is

$$h_m^{(k)}(t) = \sum_{l=1}^{L} h_l^{(k)} \cos(\omega_m \tau_l)\delta(t-\tau_{k,l}) = \sum_{l=1}^{L} h_{l,m}^{(k)} \delta(t-\tau_{k,l}) \tag{9}$$

This paper considers an asynchronous multi-path channel with each sub-band suffering flat fading. Based on such channel condition, the carrier-demodulated signal from sub-carrier m can be written as

$$r_m^{(g)}(t) = \sum_{k=1}^{K} \Gamma_k h_m^{(k)} s_m^{(k)}(t-\tau_k) + n_m(t) \tag{10}$$

where $n_m(t)$ is the complex additive white Gaussian noise with the power spectrum density N_0 which is independent over M sub-carriers. The receiver mode of such CC-CDMA system is shown in Fig. 2.

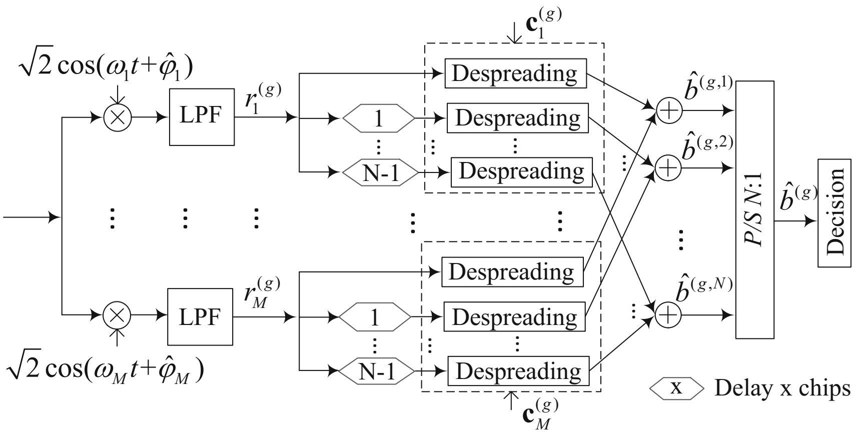

Fig. 2. The receiver mode of parallel frequency-division-multiplexed CC-CDMA systems.

Step 1. De-spreading (assuming ideal carrier-, bit- and chip-synchronous)

$$
\begin{aligned}
y_m^{(g,\eta')}(j) &= \int_0^{NT_c} r_m^{(g)}\left[t + jT_b + (\eta' - 1)T_c + \tau_g\right] C_m^{(g)}(t)dt \\
&= \int_0^{NT_c} \sum_{k=1}^{K}\sum_{\eta=1}^{N} \Gamma_k h_m^{(k)} s_m^{(k,\eta)}\left[t + jT_b + (\eta' - 1)T_c + \tau_g - \tau_k\right] C_m^{(g)}(t)dt + v_m \\
&= \sum_{k=1}^{K}\sum_{\eta=1}^{N}\sum_{i=0}^{B-1} \int_0^{NT_c} C_m^{(k)}\left[t + (j-i)T_b + (\eta' - \eta)T_c + \tau_g - \tau_k\right] C_m^{(g)}(t)dt \\
&\times \sqrt{p_t}\Gamma_k h_m^{(k)} b^{(k,\eta)}(i) + v_m \\
&= \frac{1}{M}\sqrt{p_t}\Gamma_g h_m^{(g)} b^{(g,\eta')}(j) + I_m^{(g)} + I_m^{(K)} + v_m
\end{aligned} \tag{11}
$$

where $v_m = \int_0^{NT_c} n_m(t)C_m^{(g)}(t)dt$ is the noise, $I_m^{(g)}$ is the interference from η'th stream of signal of user g on mth subcarrier, as

$$
\begin{aligned}
I_m^{(g)} =& \sqrt{p_t}\Gamma_g \sum_{\eta=1,\eta\neq\eta'}^{N} h_m^{(g)} b^{(g,\eta)}(j) \frac{1}{MN}\phi(\mathbf{c}_m^{(g)}, \mathbf{c}_m^{(g)}; \delta_\eta) \\
&+ \sqrt{p_t}\Gamma_g \sum_{\eta=1,\eta\neq\eta'}^{N} h_m^{(g)} b^{(g,\eta)}\left[j + \mathbf{sgn}(\delta_\eta)\right] \frac{1}{MN}\phi(\mathbf{c}_m^{(g)}, \mathbf{c}_m^{(g)}; N - \delta_\eta)
\end{aligned} \tag{12}
$$

where $\delta_\eta = \eta' - \eta$, $\mathbf{sgn}(x)$ equals to 1 when $x \geq 0$, and equals -1 when $x < 0$.

$I_m^{(K)}$ denotes the interference from other users on mth subcarrier, as

$$
I_m^{(K)} = \sum_{k=1,k\neq g}^{K}\sum_{\eta=1}^{N} \sqrt{p_t}\Gamma_k h_m^{(k)} \frac{1}{MN}\left\{\alpha_m^{(k,\eta)} b^{(k,\eta)}(j) + \beta_m^{(k,\eta)} b^{(k,\eta)}\left[j + \mathbf{sgn}(\delta_{k,\eta})\right]\right\}
$$

where $\delta_{k,\eta} = (\tau_g - \tau_k + \eta' - \eta)/T_c$.

$$
\begin{cases}
\delta_{k,\eta} > 0 : \alpha_m^{(k,\eta)} = \phi(\mathbf{c}_m^{(g)}, \mathbf{c}_m^{(k)}; \delta_{k,\eta}),\ \beta_m^{(k,\eta)} = \phi(\mathbf{c}_m^{(k)}, \mathbf{c}_m^{(g)}, N - \delta_{k,\eta}) \\
\delta_{k,\eta} < 0 : \alpha_m^{(k,\eta)} = \phi(\mathbf{c}_m^{(k)}, \mathbf{c}_m^{(g)}; -\delta_{k,\eta}),\ \beta_m^{(k,\eta)} = \phi(\mathbf{c}_m^{(g)}, \mathbf{c}_m^{(k)}, N + \delta_{k,\eta}) \\
\delta_{k,\eta} = 0 : \alpha_m^{(k,\eta)} = \phi(\mathbf{c}_m^{(g)}, \mathbf{c}_m^{(k)}; 0),\ \beta_m^{(k,\eta)} = 0
\end{cases} \tag{13}
$$

Step 2. Combination

$$
\begin{aligned}
\widehat{b}^{(g)}(j) &= \sum_{m=1}^{M} y_m^{(g,\eta')}(j) \\
&= \frac{\sqrt{p_t}}{M}\Gamma_g \sum_{m=1}^{M} h_m^{(g)} b^{(g,\eta')}(j) + \sum_{m=1}^{M} I_m^{(g)} + \sum_{m=1}^{M} I_m^{(K)} + \sum_{m=1}^{M} v_m \\
&= \frac{\sqrt{p_t}}{M}\Gamma_g \sum_{m=1}^{M} h_m^{(g)} b^{(g,\eta')}(j) + I^{(g)} + I^{(K)} + \omega
\end{aligned} \tag{14}
$$

where $I^{(g)}$, $I^{(K)}$ and ω are interference and noise respectively.

Step 3. Despreading and combination with different chip delays according to step 1 and 2, and then after serial/parallel conversion, we get the decision signal.

4 Analysis on Interference-Elimination Performance

Simplify the interference in (11), we get

$$\begin{aligned} I^{(g)} =& \frac{\sqrt{p_t}\Gamma_g}{MN}\sum_{m=1}^{M}\sum_{\eta=1,\eta\neq\eta'}^{N} h_m^{(g)} b^{(g,\eta)}(j)\phi(\mathbf{c}_m^{(g)},\mathbf{c}_m^{(g)};\delta_\eta) \\ &+ \frac{\sqrt{p_t}\Gamma_g}{MN}\sum_{m=1}^{M}\sum_{\eta=1,\eta\neq\eta'}^{N} h_m^{(g)} b^{(g,\eta)}\big[j+\mathbf{sgn}(\delta_\eta)\big]\phi(\mathbf{c}_m^{(g)},\mathbf{c}_m^{(g)};N-\delta_\eta) \\ =& \frac{\sqrt{p_t}\Gamma_g}{MN}\sum_{\eta=1,\eta\neq\eta'}^{N}\Big\{b^{(g,\eta)}(j)\Psi_1 + b^{(g,\eta)}\big[j+\mathbf{sgn}(\delta_\eta)\big]\Psi_2\Big\} \end{aligned}$$

where $\Psi_1 = \sum_{m=1}^{M} h_m^{(g)}\phi(\mathbf{c}_m^{(g)},\mathbf{c}_m^{(g)};\delta_\eta)$, $\Psi_2 = \sum_{m=1}^{M} h_m^{(g)}\phi(\mathbf{c}_m^{(g)},\mathbf{c}_m^{(g)};N-\delta_\eta)$.

$$\begin{aligned} I^{(K)} &= \frac{\sqrt{p_t}}{MN}\sum_{m=1}^{M}\sum_{k=1,k\neq g}^{K}\sum_{\eta=1}^{N}\Gamma_k h_m^{(k)}\Big\{\alpha_m^{(k,\eta)} b^{(k,\eta)}(j) + \beta_m^{(k,\eta)} b^{(k,\eta)}\big[j+\mathbf{sgn}(\delta_{k,\eta})\big]\Big\} \\ &= \frac{\sqrt{p_t}}{MN}\sum_{k=1,k\neq g}^{K}\sum_{\eta=1}^{N}\Gamma_k\Big\{b^{(k,\eta)}(j)\Psi_3 + b^{(k,\eta)}\big[j+\mathbf{sgn}(\delta_{k,\eta})\big]\Psi_4\Big\} \end{aligned} \tag{15}$$

where $\Psi_3 = \sum_{m=1}^{M} h_m^{(k)}\alpha_m^{(k,\eta)}$, $\Psi_4 = \sum_{m=1}^{M} h_m^{(k)}\beta_m^{(k,\eta)}$.

Substitutes (13) into Ψ_3 and Ψ_4. Comparing $\Psi_1 \sim \Psi_4$ and (2), it is proved that when the channel is flat fading, i.e., $h_1^{(k)} = h_2^{(k)} = \cdots = h_M^{(k)}$, $k \in \{1,2,\cdots,K\}$, $\Psi_1 = \Psi_2 = \Psi_3 = \Psi_4 = 0$. Therefore, the interference $I^{(g)} = I^{(K)} = 0$ and we get

$$\widehat{b}^{(g)}(i) = \sqrt{p_t}\Gamma_g h^{(g)} b^{(g)}(i) + \sum_{m=1}^{M} v_m \tag{16}$$

Since $\{v_m\}_{m=1}^{M}$ are M dependent Gaussian random variables, their sum is also a dependent Gaussian random variable. Let $p_t = \frac{E_b}{MNT_c}$, $\Gamma_g = 1$, we get

$$\gamma_b = |h_g|^2\frac{E_b}{N_0} \tag{17}$$

In Rayleigh channel, the probability density of γ_b is

$$p(\gamma_b) = \frac{1}{\overline{\gamma}_b}e^{-\gamma_b/\overline{\gamma}_b} \tag{18}$$

where $\overline{\gamma}_b = \frac{E_b}{N_0}$ is the average signal-noise ratio. We can get the corresponding bit error rate (BER) as

$$P_2(\gamma_b) = \int_0^\infty Q(\sqrt{2\gamma_b})p(\gamma_b)d\gamma_b = \frac{1}{2}\left[1 - \sqrt{\frac{E_b/N_0}{1 + E_b/N_0}}\right] \tag{19}$$

Then we compared the BER performance of CDMA systems with different spreading codes, as shown in Fig. 3. $NFR = 20lg(\Gamma_k/\Gamma_g)$ denotes the near-far effect.

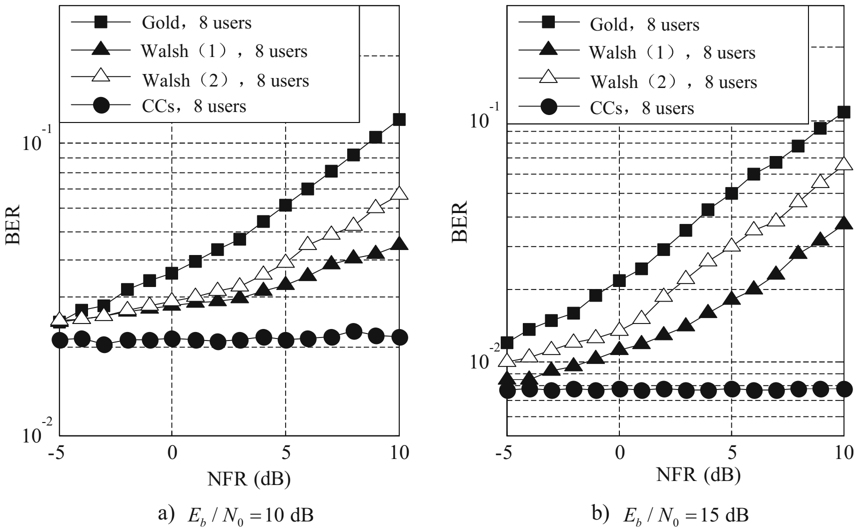

Fig. 3. Influence of near-far effect on BER of CDMA systems with three different codes.

As can be seen in Fig. 3, the CC-CDMA system performs better than that with traditional spreading codes, especially with serious near-far effect. However, when the channel is frequency selective fading, i.e., $h_1^{(k)}, h_2^{(k)}, \cdots, h_M^{(k)}$ are not equal, the equal gain combination defined in (2) can not be realized. Therefore, none of Ψ_1, Ψ_2, Ψ_3 and Ψ_4 equal to zero and CC-CDMA will suffer self-interference $I^{(g)}$ and MAI $I^{(K)}$ over the frequency selective fading channels.

5 Conclusions and Discussions

This paper has present a detailed analysis of an FDM CC-CDMA system and proves the limits of channel conditions for the interference-free character of such system. As can be seen from the analytical work, we get the conditions of the interference-free feature of CC-CDMA systems. As for the future work, we will study on new combining algorithms for such system to improve its performance over such frequency selective fading channels.

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