



Ring of Scatterers Based Localization Using Single Base Station

Zengshan Tian, Yueyue Shu^(✉), Yong Li, Mu Zhou, and Ze Li

Chongqing University of Posts and Telecommunications, Chongqing 400065, China
{tiansz,zhoumu}@cqupt.edu.cn, shuyueyue6@163.com,
ly94ong@163.com, lizecqpt@yahoo.com

Abstract. In this paper, a new Mobile Station (MS) Localization approach based on Ring of Scatterers (ROS) is proposed in response to the Non-Line-of-Sight (NLOS) environments. By exploiting the geometrical relations among the MS, scatterers, and the single base station, we present a Geometric Characteristics Based (GCB) localization algorithm with ROS model which provides conditional information for accurate location estimation of MS and scatterers. Simulation results illustrate the superior performance of proposed algorithm in typical NLOS environments.

Keywords: Mobile Station Localization · Non-Line-of-Sight
Single Base Station · Ring of Scatterers

1 Introduction

The Federal Communication Commission (FCC) released an act in 1996 which required cellular service providers to estimate subscribers locations for Enhanced-911 (E-911) demand [1]. And this act inspired significant research enthusiasm on the modern cellular mobile communication system which is utilized as a new positioning mechanism in people's daily travel, transportation navigation, and national information security [2]. By using measurements such as Time of Arrival (TOA) [3], Angle of Arrival (AOA) [4], or their joint adoption [5], the Mobile Station (MS)s position can be estimated with accurately collected data. However, in the actual scenario, the measured data are dramatically deteriorated due to Non-Line-of-Sight (NLOS) effect. And the absence of a direct Line-of-Sight (LOS) path between MS and Base Station (BS) also makes traditional LOS-based positioning methods unavailable. In order to alleviate the localization errors in the NLOS situation, a lot of work has been done, such as using the scattering model [6], or reconstructing the major positioning parameters [7]. Authors in [6] designed the single base station positioning method based on two layer scattering model. In [7], by analyzing the polynomial fit curve of TOA, authors reconstructed the TOA in LOS path to reduce the NLOS error. Most of related solutions above assume that either there is at least one BS with

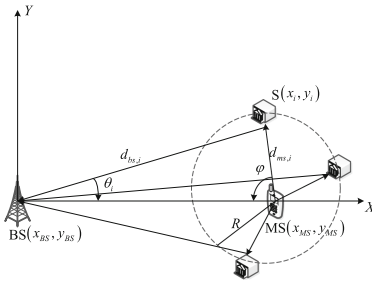


Fig. 1. ROS model.

LOS path or there are more than three BSs available. But the measurements are always associated with NLOS path from a single BS at a given MS location in many practical cases.

In this paper, a new localization approach is proposed based on the single BS to tackle the NLOS involved problem. Using the one-bounced Ring of Scatterers (ROS) model, the Geometric Characteristics Based (GCB) algorithm is designed without the LOS path and the feasible region is constricted for optimal location estimations corresponding to MS position and scatterers.

The remaining of this paper is organized as follows. In Sect. 2, the ROS model is presented. And the GCB algorithm is described in Sect. 3. Section 4 shows the associated simulations and numerical results. Finally, some concluding remarks are given in Sect. 5.

2 Scattering Model: ROS Model with a Single Base Station

In the ROS model, the signal undergoes a single reflection from the mobile station to the base station through the scatterers which are uniformly distributed on a MS centered ring with the radius, R . The angle φ between MS to BS and scatterers is uniformly distributed within $[0, 2\pi]$ as shown in Fig. 1. In this model, the height of antennas are relatively high. Thus, there is no scatterer near the base station. At the same time, there is no LOS path, and only one base station is used for positioning here.

In Fig. 1, the BS is assumed to locate at the origin with coordinate $(0, 0)$ and the line from BS to MS is the x axis. The signal are reflected by m scatterers from MS and BS. And the TOA and AOA measurements at BS corresponding to i -th scatterer are denoted by τ_i and θ_i respectively. The associated propagation distance in (1), L_i , includes the distance $d_{ms,i}$ between the MS location, (x_{MS}, y_{MS}) , and the i -th scatterer location, (x_i, y_i) , and the distance between BS location, (x_{BS}, y_{BS}) , and (x_i, y_i) .

$$L_i = c\tau_i = d_{ms,i} + d_{bs,i}, i = 1, \dots, m \tag{1}$$

where m is the number of scatterers on the ring. $c = 3 \times 10^8$ m/s. Meanwhile, the coordinate of i -th scatterer (x_i, y_i) , also can be expressed as

$$\begin{cases} x_i = d_{bs,i} \times \cos \theta_i \\ y_i = d_{bs,i} \times \sin \theta_i. \end{cases} \quad (2)$$

Then, (1) is equal to

$$\sqrt{(x_{MS} - d_{bs,i} \cdot \cos \theta_i)^2 + (y_{MS} - d_{bs,i} \cdot \sin \theta_i)^2} + d_{bs,i} = c\tau_i, i = 1, \dots, m. \quad (3)$$

Equation (3) contains m equations and $m + 2$ unknown parameters which involve $d_{bs,i}$ ($i = 1, \dots, m$) and (x_{MS}, y_{MS}) .

3 The Proposed GCB Algorithm

3.1 MS Location Model

From ROS model in Fig. 1, we have

$$c\tau_1 - d_{bs,1} = c\tau_{j+1} - d_{bs,j+1}, j = 1, \dots, m - 1. \quad (4)$$

Thus, combining (3) and (4), we can convert the underdetermined equations in (3) into the overdetermined ones as if $m > 3$ as shown in (5).

$$\begin{cases} \sqrt{(x_{MS} - d_{bs,i} \cdot \cos \theta_i)^2 + (y_{MS} - d_{bs,i} \cdot \sin \theta_i)^2} + d_{bs,i} = c\tau_i, i = 1, \dots, m \\ c\tau_1 - d_{bs,1} = c\tau_{j+1} - d_{bs,j+1}, j = 1, \dots, m - 1. \end{cases} \quad (5)$$

Here (5) provides $2m - 1$ independent equations with $m + 2$ unknown parameters. Since the AOA and TOA measurements are generally with errors due to multipath effect, we utilize NLOS propagation by considering the errors into range estimation, thus the localization towards target can be obtained by minimizing the Nonlinear Least Squares Problem (NL-LS) in (6).

$$F(\mathbf{x}) = \frac{1}{2} \sum_{p=1}^{2m-1} f_p^2(\mathbf{x}) = \frac{1}{2} (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m, \xi_1, \xi_2, \dots, \xi_{m-1}) (\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m, \xi_1, \xi_2, \dots, \xi_{m-1})^T \quad (6)$$

where $\mathbf{x} = (x_{MS}, y_{MS}, d_{bs,i})^T$, $\chi = (d_{bs,1}, d_{bs,j+1})^T$. The $\varepsilon_i(\mathbf{x})$ and $\xi_j(\chi)$ are derived by moving the right hand side of the two equations in (5) to the left and making the equation equal to zero.

In our system, the minimum range $L_{\min} = c\tau_{\min}$ implies that the MS is inside a BS centered circle with radius L_{\min} . And the maximum and minimum of AOA measurements constrain MS to a fan-shaped area with angle α as shown by dash lines in Fig. 2. Thus feasible region of MS can be expressed as

$$\begin{cases} \sqrt{(x_{MS} - x_{BS})^2 + (y_{MS} - y_{BS})^2} \leq c\tau_{\min} \\ \alpha_1(x_{MS}, y_{MS}) \leq \alpha \\ \alpha_2(x_{MS}, y_{MS}) \leq \alpha \end{cases} \quad (7)$$

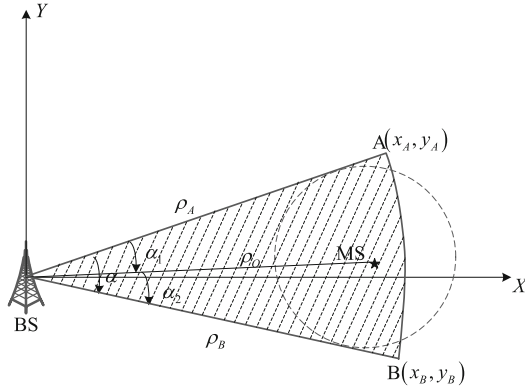


Fig. 2. Feasible region of MS.

and $d_{bs,i} \leq c\tau_{\max}$, where α denotes the maximal angle formed by vectors $\vec{\rho}_A$ and $\vec{\rho}_B$ with equal length L_{\min} . $\alpha_1(x_{MS}, y_{MS})$ and $\alpha_2(x_{MS}, y_{MS})$ are the functions of x_{MS} and y_{MS} corresponding to α_1 and α_2 respectively.

Thus, the above localization problem is transformed into a nonlinear constrained least squares problem.

3.2 The LM-BFGS Algorithm

We adopt Levenberg-Marquard (LM) method to solve the nonlinear least squares problem. Specifically, the increment of variants $\Delta\mathbf{x}$ is formed as follows.

$$\Delta\mathbf{x}_k = -(\mathbf{H}_k + \mu_k\mathbf{I})^{-1}\mathbf{A}_k^T\mathbf{b}_k, \tag{8}$$

where $\Delta\mathbf{x}_k = (\Delta x, \Delta y, \Delta L_i) = \mathbf{x} - \mathbf{x}_k$, $\mathbf{H}_k = \mathbf{A}_k^T\mathbf{A}_k$ is the approximate Hessian matrix, k is the number of iterations, \mathbf{I} is the identity matrix, μ_k is a scalar. \mathbf{A}_k is Jacobean matrix and $\mathbf{b}_k = [f_1(\mathbf{x}_k) f_2(\mathbf{x}_k) \cdots f_{2m-1}(\mathbf{x}_k)]^T$. As a consequence, successive location estimations are updated according to the recursion

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \lambda_k\Delta\mathbf{x}_k, \tag{9}$$

where λ_k is learning rate which is updated according to the Armijo criterion [8].

However, the LM algorithm obtains the approximate Hessian matrix only by $\mathbf{H}_k = \mathbf{A}_k^T\mathbf{A}_k$ which ignores the term with second order, $\mathbf{M}(\mathbf{x}) = \sum_p^{2m-1} f_p(\mathbf{x})\nabla^2 f_p(\mathbf{x})$, and thus causes a greater error with large residuals.

Based on quasi-Newton updating [9] we conduct the Broyden-Fletcher-Goldfarb-Shanno (BFGS) to update the approximate value of the second order according to [10]. Thus the the approximate Hessian matrix can be rewritten as $\tilde{\mathbf{H}}_k = \mathbf{A}_k^T\mathbf{A}_k + \mathbf{B}_k$, where $\mathbf{B}_k \approx \mathbf{M}_k$. The use of BFGS generally has better performance for full Hessian approximations in nonlinear programming objective function [11]. Based on LM-BFGS algorithm, the final estimate position $(\hat{x}_{MS}, \hat{y}_{MS})$ is obtained by minimizing the cost function.

4 Simulations and Numerical Results

Simulations are performed to validate the efficiency of the proposed GCB localization algorithm. Specifically, 1000 independent simulations are conducted. The BS and MS are located at (0,0) and (1000,0). The TOA and AOA measurements are added by Gaussian noises with standard deviations σ_{TOA} and σ_{AOA} respectively. And we use the Root Mean Square Error (RMSE) $RMSE = \frac{1}{1000} \sum_{n=1}^{1000} \sqrt{(\hat{x}_{MS}(n) - x_{MS})^2 + (\hat{y}_{MS}(n) - y_{MS})^2}$ to evaluate the positioning accuracy.

Firstly, we discuss the variations of positioning accuracy with respect to the number of scatterers and the radius of ROS. We set $\sigma_{TOA} = 5$ m and $\sigma_{AOA} = 2^\circ$. From Fig. 3, we can observe that the positioning performance is improved with increased number of scatterers, and the improvement is insignificant when the number of scatterers is greater than 12. Besides, the RMSE decreases as the radius of ROS increases.

In the second experiment, we vary the standard deviations of TOA and AOA to evaluate the performance behavior of our method with $m = 12$ and $R = 100$ m.

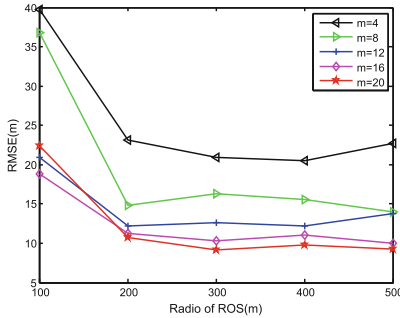


Fig. 3. RMSE with respect to the different number of scatterers and the different radius of ROS.

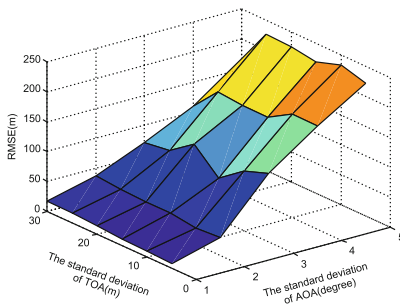


Fig. 4. RMSE with respect to the different standard deviations of TOA and AOA.

As illustrated in Fig. 4, the AOA error has a significant effect on the position error. TOA error also has certain influence on the positioning accuracy which is not obvious when the AOA error is large.

5 Conclusion

This paper presents a new MS positioning approach based on single BS in NLOS environment. The proposed GCB algorithm fully utilizes the geometrical characteristics of ROS to make localization problem resolvable with a simple LS method. Specifically, the positioning error will decrease as the radius of ROS increases which is different from the usual case. However this feature is beneficial for macrocell based positioning.

Acknowledgment. This work was supported by the National Natural Science Foundation of China (61301126, 61471077), Program for Changjiang Scholars and Innovative Research Team in University (IRT1299), Special Fund of Chongqing Key Laboratory (CSTC).

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