Radar Burst Control Based on Constrained Ordinal Optimization Under Guidance Quality Constraints

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Abstract. Radar burst control has come into use in order to improve the survivability of combat aircraft and ensure operational effectiveness in the increasingly harsh electronic warfare environment. The critical factor in radar burst control is the radar burst timing. In this paper, a novel method is proposed to determine the optimal timing based on constrained ordinal optimization. Taking the combat effectiveness of air-to-air missile as the constraint condition, the constrained ordinal optimization method is applied to the radar burst detection of hybrid control. The optimal burst timing can be selected quickly and efficiently while making the combat effectiveness maximized. Simulation results indicate that the proposed method can significantly improve the searching efficiency of the optimal radar burst timing.

Keywords: Radar optimal burst timing · Hybrid control
Constrained ordinal optimization · Operational effectiveness

1 Introduction

In order to improve the combat effectiveness and survivability of a combat aircraft in the increasingly harsh electronic warfare environment, [1] proposed a radar burst control technology based on airborne multi-sensor coordination. In the area of the airborne multi-sensor co-tracking and radiation control, a significant research effort has been made. [2] proposed a joint detection filter, a measurement of measuring uncertainty with multiple sensors and single target tracking in the presence of clutter. In [3], a distributed multi-sensor collaborative management method was proposed. [4–6] focused on the sensor tracking, radiation control algorithm, and tracking accuracy. However, to improve the overall combat effectiveness, the sensor needs to coordinate with other units of command and control, communication and weapons. In the literature [7], a coordinated tracking method of airborne multi-sensor system based on radiation control was discussed. The main idea of this method is to apply a real-time control into radar switch machine in order to generate the radar intermittently radiate electromagnetic waves. However, the radar burst detection problem is a complex optimization problem which involves various random factors, computational time

constrain, and an NP-hard type problem, and it cannot be expressed as an explicitly definite optimization function. As such it is difficult to utilize the commonly used optimization algorithms to the problem. As well-known, the Ordinal Optimization theory can greatly reduce the computational complexity while ensuring a reasonably good solution at a sufficiently high probability. Compared with the Ordinal Optimization method, the selection set of the constrained ordinal optimization (COO) [8] is smaller, and the computation time is greatly reduced.

In this paper, a hybrid control method is proposed for the radar burst detection control. This method can effectively reduce the computational complexity caused by calculating the residual norm in each measurement update. At the same time, the multi-sensor information fusion tracking performance is linked with the combat effectiveness. The effect of the radar burst detection technology on the combat effectiveness is studied by using the air-to-air missile target intercept probability as the combat effectiveness evaluation method. All the feasibility models are selected by using the target probability of air-to-air missiles as the constraint condition, and then the ordinal optimization method is used in the candidate solution set to determine the optimal timing of the radar burst under the given combat effectiveness constraints.

This reminder of the paper is organized as follows: Sects. 2 and 3 are about the principle of radar burst detection and its various control modes. Section 4 introduces the method of determining the optimal burst timing of the radar under the guidance of quality constraint. In Sect. 5 the determination of the optimal burst time of radar based on COO method is discussed. The simulation results are discussed in Sect. 6 and the performance of the proposed approach is evaluated. Finally, in Sect. 7 the main contribution of this paper and the relevant future work are highlighted.

2 Radar Burst Detection

The so-called radar burst detection is a detection method when performing a probing task during commission operations, the radar no longer continuously sends electromagnetic signals, but uses the periodic (or aperiodic) switch machine model. When a probe signal is turned on, the frequency and the waveform of the transmitted signal are also pseudo-random agility in the pulse. This makes it look like a radar were working at different frequencies, and therefore, enables the system to have a low detectability so to avoid being detected by other systems. Essentially it is a way of radar radiation control.

Figure 1 shows the working principle of the fire control radar burst detection. Its essence is to moderate the normal radar working waveform by adding a control signal to it. The burst time interval of the radar depends on the time interval of the control signal T_c. The number of burst pulses N is determined by the pulse width of the burst signal τ_c and the pulse width of the normal operating waveform τ, i.e., $N = \tau_c/\tau$. The radar burst control signal is based on tracking accuracy or control system requirements to determine the size of T_c and τ_c by the radar computer.

In order for the radar to find target more quickly and accurately in the use of burst detection mode, we can let the radar work under the guidance of other airborne sensors. This requires the radar to work with other sensors, and among them, is the multi-sensor coordination radar burst detection tracking, and its process is shown in Fig. 2.
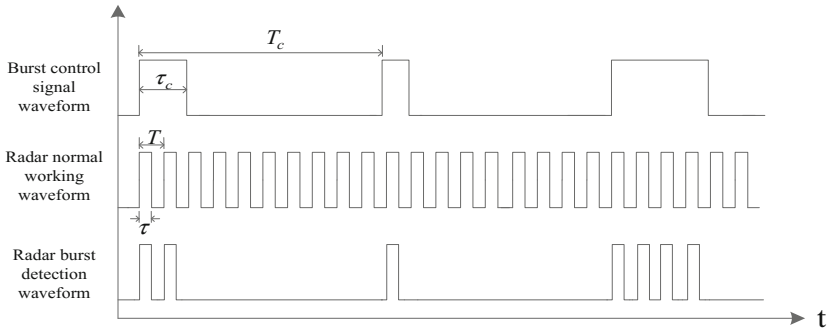
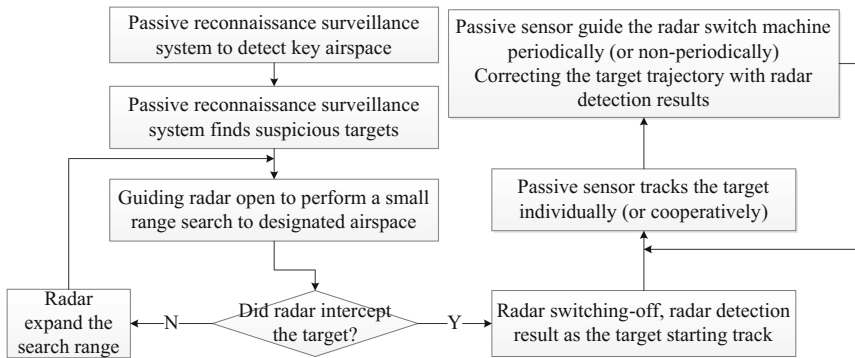**Fig. 1.** Radar burst detection working principle diagram

Fig. 2. Radar burst detection process under multi-sensor coordination

3 Radar Burst Detection Control Method

To ensure the quality of the target tracking, the radar start-up time and the length of each radiation must be strictly controlled. The method of determining the size of τ_c can be found in [9]. In terms of the radar burst control technology, there are three main ways to control the radar burst time as discussed below.

3.1 Equal Interval Control

The equal interval control method uses a fixed radar burst control signal interval T_c in order for the radar to perform periodic burst detection. This control method is simple and easy to implement. However, the off-line calculation process of the optimal radar burst interval with different tracking accuracies is time-consuming, and the time interval cannot be adjusted in real-time during the tracking process.

3.2   Real-Time Control

The basic idea of the real-time burst control is: according to a real-time tracking quality assessment of the target being tracked by the sensor system, we can determine if the radar burst is to be used to detect the target. This method has a great flexibility.

In [7], a radar real-time burst control method based on target residual norm was discussed, which uses the comparison results of the target filter residual norm and a given threshold to control the radar switch in real-time, and accordingly makes the radar intermittently radiate electromagnetic waves. Consider the target residual norm as

$$d(k) = v_k^T S_k^{-1} v_k \tag{1}$$

where v_k is the filter residual (innovation), S_k is innovation covariance, and $d(k)$ obeys the chi-square distribution with a degree of freedom m (m is the observed dimension). We have

$$E[d(k)] = m$$
$$\sigma[d(k)] = \sqrt{2m} \tag{2}$$

Hence, the problem of whether the radar is turned on by the filter residual control can be considered a hypothetical testing problem to test whether $d(k)$ is in a confidence interval centered at m.

When

$$m - k\sigma_{d(k)} < d(k) < m + k\sigma_{d(k)} \tag{3}$$

holds, the radar is to be turned off, and no external radiation; otherwise, the radar is on to detect. In Eq. (3) k is the coefficient that determines the length of the confidence interval, called burst control factor, and its size is selected according to the actual situation.

3.3   Hybrid Control

Each of the two methods discussed above has its own advantages. In practical applications, these two control methods can be combined to use, that is, with a fixed interval cycle T_c use the target residual norm to determine whether to turn the radar on. In this way, we can not only avoid the problem of poor flexibility due to burst detection at fixed cycles, but also reduce the computational complexity of calculating the residual norm for each measurement update. This method can have a good balance between flexibility and computational complexity compared with the other two control methods. This paper adopts this method.

4   Determining the Optimal Radar Burst Timing Under Guidance Quality Constraint

For the equal interval control, real-time control and hybrid control discussed above, it is necessary to determine the burst control signal interval T_c or the burst control factor k in advance to establish a pre-defined database, and then select appropriate T_c or

k according to the specific problem in the actual commission delivery process. For the problem of air-to-air missile combat guidance under multi-sensor coordination, the theoretical analysis shows that the combat effectiveness of air-to-air missile is positively correlated with radar burst detection times. For this reason, with the goal of maximizing the air-to-air missile operational effectiveness and the radar burst detection times after missile launch, we establish an optimization problem in respect to T_c or k by solving different target values under different conditions, and build a database to be used for actual operations.

When the multi-sensor co-tracking a target, the timing of radar burst detection will directly affect the tracking quality. And corresponding to the specific air-to-air missile guidance process, this timing directly affects the guidance accuracy, and hence affects seeker interception probability of the missile handover phase between midcourse guidance and terminal guidance, and ultimately affects the operational effectiveness of an air-to-air missile. This paper uses the missile intercept target probability P_c to reflect the air-to-air missile operational effectiveness. To unify the different combat situations and radar radiation durations of a missile guidance process under a certain air-to-air missile launch distance, we define the radar burst boot rate as

$$\alpha = \frac{\delta n_e}{t_g} \tag{4}$$

to reflect the radar burst boot frequency of a combat process guided by an air-to-air missile under multi-fighter multi-sensor coordination, where n_e is the radar burst detection times during the guidance, the duration of each detection is δ; and t_g indicates the missile guidance duration, from the missile launch to the handover phase between the midcourse guidance and the terminal guidance.

For the combat problem guided by an air-to-air missile under multi-fighter multi-sensor cooperative, it is more convenient to reduce the airborne radar burst detection times by as much as possible while maintaining the operational effectiveness of air-to-air missiles. Therefore, the following constraint optimization problem can be established:

For equal interval control,

$$\begin{cases} \min \alpha = f_1(T_c) \\ s.t. \ P_c = g_1(T_c) \geq p \end{cases} \tag{5}$$

For real-time control,

$$\begin{cases} \min \alpha = f_2(k) \\ s.t. \ P_c = g_2(k) \geq p \end{cases} \tag{6}$$

For hybrid control,

$$\begin{cases} \min \alpha = f_3(T_c, k) \\ s.t. \ P_c = g_3(T_c, k) \geq p \end{cases} \tag{7}$$

Note that for the above optimization problems, the objective function $f(\bullet)$ and the constraint condition $g(\bullet)$ cannot be expressed by an explicit function. We therefore use air combat simulations to generate the relevant output results as the function output values. As such, the above optimization problem is based on experimental optimization.

5 Determination of the Optimal Radar Burst Timing Based on COO Method

We explore the advantages of using the Ordinal Optimization theory to solve the complex simulation optimization problems we are facing.

5.1 Ordinal Optimization Theory

The Ordinal Optimization (OO) theory consists of two basic ideas: sorting comparison and target softening. In a simple way, the ordinal optimization method divides the problem optimization process into two phases: establishing a rough model in the first stage based on known information and the understanding of the characteristics for selecting a better performance solution from a large search space, and make the number of solutions to be considered within a manageable scale. And then in the second stage the real model is to be used to evaluate the performance of the solutions selected in the first stage.

5.2 Constrained Ordinal Optimization

The traditional ordinal optimization only considers an unconstrained optimization problem of a single objective. To address this limitation, a Constrained Ordinal Optimization has been proposed by Zhao [8] to solve the problem of simulation optimization with constraints.

The basic idea of the Constrained Ordinal Optimization is: First, the feasibility model is used to scan and obtain a feasible solution, and then apply OO in the estimated feasible solution set. The COO requires a smaller set of selection than the direct application of OO without the feasibility model. At the same time, the size of the selection set also depends on the accuracy of the feasibility model. The step of determining the radar burst timing method based on COO is as follows (shown in Fig. 3):

Step 1. Establish the air-to-air missile guidance combat simulation model under multi-fighter multi-sensor cooperation. On this basis, a feasibility analysis model is created based on the constraint condition function $g(\bullet)$, and the objective function of the accurate calculation model is built up based on the objective function $f(\bullet)$. At the same time, this objective function is used to determine the rough calculation model;

Step 2. Sample from the effective range of T_c or k values evenly at a regular interval to form a decision solution set Θ;

Step 3. Use the feasibility model, randomly extract N feasible solutions (i.e., the solution of $P_c \geq p$) from Θ according to the principle of equal opportunity to form a feasible solution set Φ;

Step 4. Use the blind selection rule and the rough calculation model to calculate the feasible solution set Φ.

Step 5. Order the feasible solution set Φ by rough calculation solutions to estimate the type of ordered performance curve, and constitute a sub-decision set S_f according to the top s_f feasible solutions from the sorted solutions. In accordance of the relevant theory in the literature [10], the size of s_f is determined by

$$s_f = e^z k^\rho g^\gamma + \eta \tag{8}$$

where z, ρ, γ and η can be found from the regression table in [10], g is the number of elements in the sufficient subset G, and k is the number of elements in the intersection of the sub-decision set S_f and the sufficient subset G.

Step 6. Simulate the s_f feasible schemes selected from the sub-decision set S_f, and sort out the results.

Step 7. Analyze the previously sorted k_s feasible solutions of the exact simulation results, the values of T_c and k corresponding to the radar burst optimal timing are determined. Verify whether they are satisfied the constraint conditions.
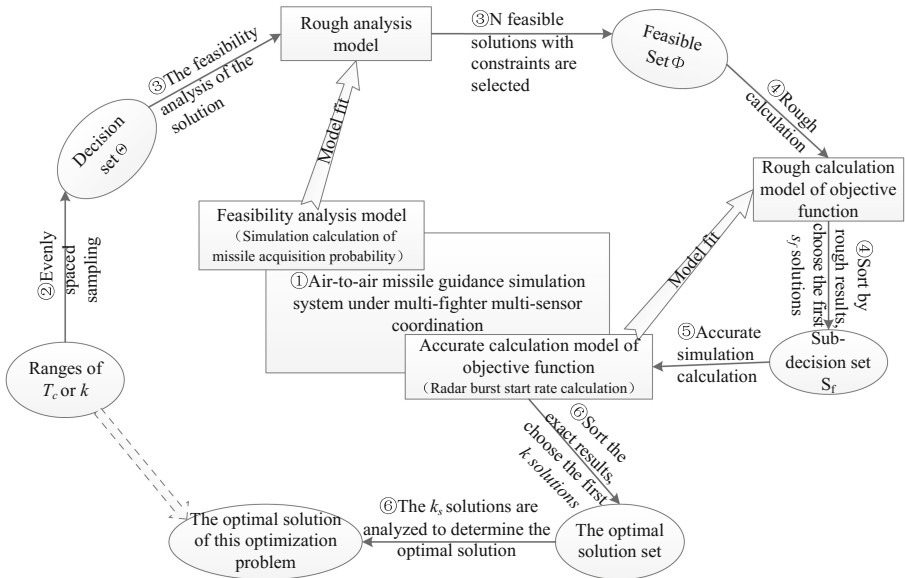

Fig. 3. The process of determining optimal radar burst timing based on the COO method

In the above process, the establishment of an appropriate and reliable simulation model is the first step to solve the optimization problem, and it is the most important step. Below are the relevant simulation models to be adopted:

(1) Air combat simulation model based on multi-sensor cooperative
    The model includes the basic combat aircraft model, air-to-air missile model, sensor model, weapon fire control model, and detailed sensor management model and tracking filter model.

(2) Feasibility analysis model
    The feasibility analysis model is based on the constraint function $g(\bullet)$. For the optimization problem of the optimal radar timing, the solution of the feasibility analysis model is the air-to-air missile target intercept probability calculation model. For air-to-air missile target intercept probability, the most commonly used method is Monte Carlo simulation.

(3) Accurate calculation model of objective function
    The exact calculation model is a detailed calculation model of the radar burst rate based on the objective function $f(\bullet)$. According to the definition of radar burst detection rate, it is necessary to record the number of radar bursts detected during the period from the missile launch to the beginning of handing over the midcourse to the terminal guidance. This process is carried out in combat simulation guided by air-to-air missile under multi-fighter multi-sensor coordination. Due to the randomness of the sensor measurement process, the statistical results obtained by the single simulation have a significant uncertainty. Therefore, to make the statistical results more reliable, it is usually necessary to conduct Monte Carlo simulation several times repeatedly, then take the average of the results of the multiple experiments.

(4) Rough calculation model
    Due to the limitation of the computational capacity and the evaluation time, it is not feasible to use the simulation model to evaluate each feasible solution in real time. According to the order comparison characteristics of the ordinal optimization theory, we only need to establish the rough model of the evaluation function, and make a quick assessment of each option. Therefore, we proposes to use a partial sample value obtained by the simulation model to fit the precise model and using the fitted model as a rough model for a rapid assessment.
    We use an n-2n-2 three-layer error back propagation (BP) network model shown in Fig. 4 to fit the simulation model. The network contains 2n hidden nodes (n is the number of the network input nodes), and the activation function of each hidden node is a sigmoid function. The node on the output layer adopts a linear activation function. The network parameters will be determined throughout training with the partial data obtained by the simulation evaluation model.
    Compared with the rough calculation using the simulation model, the BP network after fitting can not only reduce the time of rough calculation, but also ensure the validity of the calculated data.
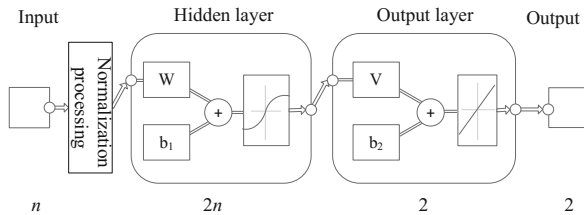
Fig. 4. BP network structure for rough calculation

6 Simulation Experiment

In our simulation, the burst period of an airborne radar was assumed to be controlled within 0.1 s–10.0 s. A proper radar burst period was determined based on the requirements that the target intercept probability would not less than 85% and the burst rate would take the minimum when the air-to-air missile has been launched within 60 km–80 km.

Sampling the radar burst cycle by 0.1 s step length, missile launch distance by 1 km step gives a total of $100 \times 21 = 2100$ combination groups. It is not advisable to do scheme optimization by traversal search, since it is very time-consuming using Monte Carlo approach to determine the missile intercept probability. Our simulation experiments have shown that: when taking 1 s as the simulation step, completing a missile intercept probability calculation usually requires 70.59 s of CPU time, even without considering the time of scenario edit and simulation framework start-up. The simulation time would be about 132 s when the radar burst period was optimized by 0.2 s step. To complete more than 2000 sets of simulation calculation, it would take about 70 h. Therefore, the COO method proposed in this paper was used to optimize:

(1) Rough calculation and evaluation model. From the above-mentioned 2100 combinations, 50 (10×5) groups were extracted by $T_c = 1, 2, \ldots, 10$ s and D = 60, 65, …, 80 km, respectively. Calculating the corresponding missile intercept probabilities and radar burst boot rates by step 1 s, and training a BP network with the resultant data. The inputs of the BP network were the radar burst period and the missile launch distance, and the output was the missile intercept probability and the radar burst boot rate. The process took about 1 h.

(2) Using the trained BP network to roughly calculate the 2100 combinations quickly. The process was completed in a minute.

(3) According to the rough calculation results, selecting any combination of the missile intercept probability that satisfies $P_c > 83\%$ (considering the possible impact of the model fitting error on the result, so to relax the constraints). Then the selected combinations were merged together according to the radar burst period, and sorted by the average radar burst rate from small to large.

(4) Selecting the five radar burst periods of the forward order to do accurate calculation by simulation step 1 s. At this time, a total of $5 \times 21 = 105$ groups of experiments were conducted, consuming about 4 h.

(5) Comparing and analyzing the accurate simulation results, the radar burst period T_c = 3.6 s corresponding to the minimum average radar burst boot rate and satisfying the target intercept probability not less than 85% when the missile launch distance is within 60 km–80 km. Table 1 shows the target intercept probability corresponding to the partial launch distance at T_c = 3.6 s, all of which meet the requirements.

From the steps described above, it is evident that the COO method has greatly reduced the workload in determining the radar optimal burst timing. Making the duration of calculations reduced from more than 70 h of CPU time to 5 h. Hence, the ordinal optimization method has a great advantage in solving the problem of simulation optimization.

Table 1. A part of the target capture probability corresponding to the launch distance when T_c = 3.6 s

Transmitting distance/km	60	64	68	72	76	80
Target capture probability	88.2%	87.5%	87.0%	86.4%	85.9%	85.3%

7 Conclusion

This paper presents a method to determine the radar optimal burst timing based on Constrained Ordinal Optimization, and has applied it to air-to-air missile guidance process. Simulation results indicate that using the Constrained Ordinal Optimization method to determine the optimal radar burst timing can greatly improve the searching probability under the conduction that the combat effectiveness is maximized, and therefore effectively control the radar radiation and improve the stability and viability of combat aircraft. In the future, we can use parallel computing technology to further improve the searching efficiency.

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