Analysis of Conditional Connectivity Based on Two Lanes for VANETs

Bin Pan[1,2,4] and Hao Wu[1,3,4](✉)

[1] State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University, Beijing 100044, China
{binpan, hwu}@bjtu.edu.cn
[2] School of Electronic and Information Engineering, Beijing Jiaotong University, Beijing 100044, China
[3] Beijing Key Laboratory of Intelligent Traffic Data Safety and Privacy Protection Technology, Beijing Jiaotong University, Beijing 100044, China
[4] Beijing Engineering Research Center of High-Speed Railway Broadband Mobile Communications, Beijing Jiaotong University, Beijing 100044, China

Abstract. Conditional connectivity is important for the design of the upper layer communication protocol and the network deployment in different scenarios. This paper analyzes the performance of conditional connectivity with the network topology change by establishing the model of inter-vehicle communication. The paper aims at the two-lane highway scenario for vehicular ad hoc networks (VANETs), considering the factors of communication range, vehicle flow characteristics and moving speed. The main conditional connectivity performance index is the conditional connectivity probability. Based on the simulation, the correctness of the theoretical analysis is verified. We also make an explanation of the simulation results.

Keywords: VANETs · Conditional connectivity performance
Two lanes · User level

1 Introduction

As a realization form of distributed wireless network, mobile ad hoc networks (MANETs) have received extensive attention from numerous institutes and researchers around the world in recent years. MANETs don't rely on the support of fixed infrastructure, consisting of mobile communication nodes which can form the networks quickly and have the ability of storing and computing information. MANETs can complete the transmission of information through the multi-hop communication between nodes. As a special kind of MANETs, vehicular ad hoc networks (VANETs) are formed with self-organizing vehicle nodes. Vehicles in VANETs can perform the multi-hop communication by vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I), completing information transmission and data distribution [1]. VANETs can ensure traffic safe and improve transportation efficiency, and therefore it has aroused people's wide attention in the field of transportation.

Compared with the traditional MANETs, the communication vehicles in VANETs can provide enough electric energy. However, its trajectory is limited by the geometry

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of the road. Hence, it has certain predictability and regularity. Unlike traditional ad hoc networks, the high speed mobility of the vehicle nodes can cause the network topology to change dynamically, making the inter-vehicle communication links prone to interrupt and reducing the effective coverage and transmission performance. In recent years, a large number of scholars have carried out research on the connectivity of VANETs [2–4]. Based on the assumption that nodes obey the uniform distribution, the connectivity performance of vehicular networks is analyzed by [5]. However, the mobility of nodes is not considered. Literature [6] has verified the basic characteristics connectivity performance in VANETs through a lot of simulations and how the change of the relative position between the vehicle and the road can affect the connectivity performance has been discussed. In [7], a new type of mobility model is established, and the connection probability was derived based on this model. The author discussed how the mobility of vehicle nodes will affect the performance of the connectivity. In [8], a vehicle flow mobility model is established which is more close to the actual highway scenario. The paper analyzed the statistical characteristics of multiple connectivity performance indexes, considering the effects of different system parameters. In this system model, the vehicle arrival rate depends on the speed of vehicles. Therefore, this hypothesis does not apply to general scenarios for VANETs. These papers mentioned above have not considered the influence of the network topology change on the stability of data transmission. And most of the papers have analyzed the impact of vehicle mobility on the whole network connectivity performance from the system level. Literature [9] analyzes the influence of the vehicle movement characteristics and the network topology change on the conditional connectivity performance based on the model of freeway scene from the user level. However, its scene is restricted to the single-lane highway, which can't adapt to the more complex reality scene, therefore it has certain limitations.

In order to meet the need of continuous data service for a long time, this paper establishes a model in the two-lane highway scenario based on literature [9]. The influence of the system parameters such as the vehicle flow arrival rate, the speed of the vehicle, the communication range and the data transmission time on the conditional connectivity performance is considered. The main performance index is conditional connectivity probability. First, we derive the statistical distribution characteristics of the initial inter-vehicle distance in the two-lane scenario. The initial inter-vehicle distance represents the distance between the two vehicles when they enter the highway entrance. Then we calculate the distribution characteristics of the relative speed of the vehicles in the two-lane scenario. Finally, we obtain the analytical expression for the conditional connectivity probability in the two-lane scenario. The conclusions of this paper can provide theoretical guidance for the design and deployment of VANETs in the two-lane highway scenario.

The remainder of the paper is organized as follows. The system model in two-lane highway scenario is briefly described in Sect. 2. In Sect. 3, the conditional connectivity performance is analyzed by the theoretical derivation. In Sect. 4, we verify the correctness of the theoretical analysis by the system simulation and make an explanation for the results. Section 5 summarizes the whole paper and gives a conclusion.

2 System Model

In this paper, a two-lane highway scenario is modeled based on Poisson Point Process. The main model assumptions are as follows.

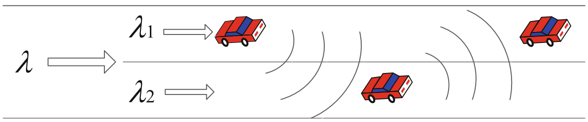

Fig. 1. Two-lane highway communication model

- Each vehicle node in the network is assumed to have the same ability to communicate with others. It is the same as the communication radius of each node, which is called R. When the distance between the two vehicles is less than R, the communication link can be considered connected.
- As shown in Fig. 1, the arrival of vehicles in the two-lane highway entrance follows a Poisson process with λ (vehicles per second). After the arrival at the highway, each vehicle will choose a lane to enter, based on the wishes of the driver. According to the drivers' choices, we assume that the arrival rate in the fast lane is λ_1 and the arrival rate in the slow lane is λ_2. Hence, we can get the relation $\lambda = \lambda_1 + \lambda_2$. When the vehicle enters the fast or the slow lane, it will choose a constant speed v_i, which is independent uniformly distributed. If the vehicle enters the fast lane, the distribution interval of speed is $[v_{min1}, v_{max1}]$. If the vehicle enters the slow lane, the interval is $[v_{min2}, v_{max2}]$. In order to facilitate the calculation, assume that the minimum speed of the fast lane v_{min1} is equal to the maximum speed of the slow lane v_{max2} and the total arrival rate of two lane entrances is a constant 1veh/sec. Therefore, the probability density function (pdf) of the speed v_i is given as:

$$f_{v_i}(x) = \begin{cases} \frac{1}{v_{max1} - v_{min1}} & v_{min1} \leq x \leq v_{max1} \\ \frac{1}{v_{max2} - v_{min2}} & v_{max2} \leq x \leq v_{min2} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

- Random variable T_i represents the arrival time interval of vehicles. According to the stochastic process theory, the arrival time interval T_i is independent identically distributed (i.i.d) with exponential distribution with parameter λ. Therefore, the pdf of T_i can be expressed as:

$$f_{T_i}(y) = \begin{cases} \lambda e^{-\lambda y} & y \geq 0 \\ 0 & y < 0 \end{cases} \tag{2}$$

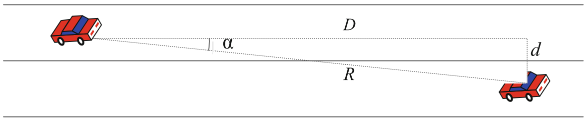

Fig. 2. Different-lane communication model

- As shown in Fig. 2, we discuss the critical situation of communication interruption when the two vehicles run in different lanes. The distance d between the adjacent lanes is very small compared with the communication radius R. That is α is approximately equal to zero. Therefore, this model can be simplified as the communication model that vehicles run in the same lane.
- Assume that the mobility of each vehicle is independent and is not affected by other vehicles. There can be the overtaking phenomenon without lane-changing to simplify the theoretical analysis.

According to [9], at the initial observation time $t = 0$, the vehicle i arrives at the highway entrance. Then it will choose a lane to enter according to the driver and choose a constant speed v_i in the speed range of the lane. After a period of time T_i, another vehicle j arrives at the entrance of the highway. It will also choose a lane to enter according to the driver and choose a constant speed v_j in the speed range of the lane. Assuming the vehicles i and j begin to transmit data, the communication distance between them at present is about $d_i = v_iT_i$. d_i is defined the initial inter-vehicle distance. Assuming the whole transmission time is T_t, if the link between i and j stay stable during the whole transmission, the packet can be received successfully at the time $t = T_i + T_t$. At this time, the distance between vehicles is defined d_t.

3 Performance Analysis

3.1 Initial Inter-vehicle Distance

After the vehicle i arriving at the entrance of the highway, the vehicle chooses a lane and continues moving with a speed of v_i. After a time interval T_i, another vehicle j arrives at the entrance. At the moment, two vehicles are d_i apart, which can be expressed as:

$$d_i = v_iT_i \tag{3}$$

v_i and T_i are independent random variables, therefore their joint probability density can be expressed as:

$$f_{v_i,T_i}(x,y) = f_{v_i}(x)f_{T_i}(y) \tag{4}$$

By combining (1) (2) with (3), the pdf of initial inter-vehicle distance d_i can be calculated as:

$$f_{d_i}(z) = \int_{-\infty}^{+\infty} \frac{1}{|z|} f_{v_i}(\frac{z}{y}) f_{T_i}(y)dy = \begin{cases} \frac{\lambda}{v_{max1} - v_{min1}} \int_{z/v_{max1}}^{z/v_{min1}} \frac{1}{y} e^{-\lambda y} dy \\ \frac{\lambda}{v_{max2} - v_{min2}} \int_{z/v_{max2}}^{z/v_{min2}} \frac{1}{y} e^{-\lambda y} dy \end{cases}$$
$$= \begin{cases} \frac{\lambda}{v_{max1} - v_{min1}} \int_{\lambda z/v_{max1}}^{\lambda z/v_{min1}} t^{-1} e^{-t} dt & \text{the front vehicle in fast lane} \\ \frac{\lambda}{v_{max2} - v_{min2}} \int_{\lambda z/v_{max2}}^{\lambda z/v_{min2}} t^{-1} e^{-t} dt & \text{the front vehicle in slow lane} \end{cases} \quad (5)$$

By analyzing (5), we can notice that the pdf $f_{di}(z)$ of initial inter-vehicle distance d_i is related to the Exponential Integral $E_1(z)$ [10]. The special function $E_1(z)$ can be expressed as:

$$E_1(z) = \int_z^{\infty} e^{-t} t^{-1} dt \quad (6)$$

Hence, we can get the pdf $f_{di}(z)$ of initial inter-vehicle distance d_i through calculating (5) with $E_1(z)$.

$$f_{d_i}(z) = \begin{cases} \frac{\lambda}{v_{max1} - v_{min1}} \left[E_1\left(\frac{\lambda z}{v_{max1}}\right) - E_1\left(\frac{\lambda z}{v_{min1}}\right) \right] & \text{the front vehicle in fast lane} \\ \frac{\lambda}{v_{max2} - v_{min2}} \left[E_1\left(\frac{\lambda z}{v_{max2}}\right) - E_1\left(\frac{\lambda z}{v_{min2}}\right) \right] & \text{the front vehicle in slow lane} \end{cases} \quad (7)$$

According to (7), the cumulative distributed function (CDF) $F_{di}(x)$ of d_i can be given as:

$$F_{d_i}(x) = Pr\{di \le x\} = \int_0^x f_{d_i}(z)dz \quad (8)$$

Therefore, we can get the probability that the initial inter-vehicle distance is not more than the communication range R according to (8), which can be expressed as:

$$\Pr\{di \le R\} = \int_0^R f_{d_i}(z)dz \quad (9)$$

3.2 Conditional Connectivity Probability

Conditional connectivity probability is defined as the probability that two vehicles are constantly connected during the whole data transmission time T_t, given that the vehicles begin to communicate when they arrive at the highway entrance. Therefore, the conditional connectivity probability can be considered as the probability that the vehicles are always within the communication range of each other until the transmission is finished successfully.

The initial inter-vehicle distance is d_i, when the vehicle i and j begin to transmit data. After the complete data transmission, the vehicle communication link still keeps connected, and at the moment two vehicles are d_t apart. That is

$$d_t = \left|d_i + (v_i - v_j)T_t\right| = |d_i + \Delta v T_t| \tag{10}$$

Hence, the conditional connectivity probability P_{con} is the probability that i and j stay connected during the whole data transmission, given that i and j are within the communication range of each other at the beginning of transmission.

$$P_{con} = \Pr\{d_t \le R | d_i \le R\} = \frac{\Pr\{d_t \le R, d_i \le R\}}{\Pr\{d_i \le R\}} \tag{11}$$

It can be found that P_{con} depends on initial inter-vehicle distance d_i, relative speed Δv, data transmission time T_t and communication range R.

As for relative speed Δv, we can get the derivation results with reference to [9].

- When the two vehicles are all in the fast lane, the pdf of Δv is

$$f_{\Delta v_1}(u) = \begin{cases} \frac{u + v_{max1} - v_{min1}}{(v_{max1} - v_{min1})^2} & v_{min1} - v_{max1} \le u \le 0 \\ \frac{-u + v_{max1} - v_{min1}}{(v_{max1} - v_{min1})^2} & 0 \le u \le v_{max1} - v_{min1} \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

- When the two vehicles are all in the slow lane, the pdf of Δv is

$$f_{\Delta v_2}(u) = \begin{cases} \frac{u + v_{max2} - v_{min2}}{(v_{max2} - v_{min2})^2} & v_{min2} - v_{max2} \le u \le 0 \\ \frac{-u + v_{max2} - v_{min2}}{(v_{max2} - v_{min2})^2} & 0 \le u \le v_{max2} - v_{min2} \\ 0 & \text{otherwise} \end{cases} \tag{13}$$

- When i is in the fast lane and j is in the slow lane, the pdf of Δv is

$$f_{\Delta v_3}(u) = \begin{cases} \frac{-v_{min2} - u + v_{max1}}{(v_{max1} - v_{min1})(v_{max2} - v_{min2})} & 0 \le v_{max1} - v_{max2} \le u \le v_{max1} - v_{min2} \\ \frac{u - v_{min1} + v_{max2}}{(v_{max1} - v_{min1})(v_{max2} - v_{min2})} & 0 \le v_{min1} - v_{max2} \le u \le v_{min1} - v_{min2} \\ 0 & \text{otherwise} \end{cases} \tag{14}$$

- When i is in the slow lane and j is in the fast lane, the pdf of Δv is

$$f_{\Delta v_4}(u) = \begin{cases} \frac{u - v_{min2} + v_{max1}}{(v_{max1} - v_{min1})(v_{max2} - v_{min2})} & v_{min2} - v_{max1} \le u \le v_{min2} - v_{min1} \le 0 \\ \frac{-u - v_{min1} + v_{max2}}{(v_{max1} - v_{min1})(v_{max2} - v_{min2})} & v_{max2} - v_{max1} \le u \le v_{max2} - v_{min1} \le 0 \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

Due to the randomness of two vehicles' speed, the vehicle i and j may be far away from each other or may be close to each other. We discuss the two cases respectively. We assume that $|\Delta v_{max} T_t| \le R$ to simplify the analysis, where the $|\Delta v_{max} T_t|$ represents the upper limit to the variation of the inter-vehicle distance.

When i and j are far away from each other, the conditional connectivity probability can be expressed as the joint probability of $\{d_t \le R\}$, $\{d_i \le R\}$ and $\{v_i \ge v_j\}$.

$$\begin{aligned} p_1 &= \Pr\{d_t \le R, d_i \le R, v_i \le v_j\} \\ &= \Pr\{0 \le d_i \le R - \Delta v T_t, \Delta v \ge 0\} \\ &= \frac{\lambda_1^2}{\lambda^2} \int_0^{v_{max1} - v_{min1}} \int_0^{R - \Delta v T_t} f_{d_i}(z) f_{\Delta v_1}(u) dz du + \frac{\lambda_2^2}{\lambda^2} \int_0^{v_{max2} - v_{min2}} \int_0^{R - \Delta v T_t} f_{d_i}(z) f_{\Delta v_2}(u) dz du \\ &\quad + \frac{\lambda_1 \lambda_2}{\lambda^2} \int_{v_{max1} - v_{max2}}^{v_{max1} - v_{min2}} \int_0^{R - \Delta v T_t} f_{d_i}(z) f_{\Delta v_3}(u) dz du + \frac{\lambda_1 \lambda_2}{\lambda^2} \int_{v_{min1} - v_{max2}}^{v_{min1} - v_{min2}} \int_0^{R - \Delta v T_t} f_{d_i}(z) f_{\Delta v_3}(u) dz du \end{aligned} \quad (16)$$

When i and j are close to each other, the conditional connectivity probability can be expressed as the joint probability of $\{d_t \le R\}$, $\{d_i \le R\}$ and $\{v_i \le v_j\}$.

$$\begin{aligned} p_2 &= \Pr\{d_t \le R, d_i \le R, v_i \le v_j\} \\ &= \Pr\{|d_i + \Delta v T_t| \le R, d_i \le R, \Delta v \le 0\} \\ &= \frac{\lambda_1^2}{\lambda^2} \int_{v_{min1} - v_{max1}}^{0} \int_0^{R} f_{d_i}(z) f_{\Delta v_1}(u) dz du + \frac{\lambda_2^2}{\lambda^2} \int_{v_{min2} - v_{max2}}^{0} \int_0^{R} f_{d_i}(z) f_{\Delta v_2}(u) dz du \\ &\quad + \frac{\lambda_1 \lambda_2}{\lambda^2} \int_{v_{min2} - v_{max1}}^{v_{min2} - v_{min1}} \int_0^{R} f_{d_i}(z) f_{\Delta v_4}(u) dz du + \frac{\lambda_1 \lambda_2}{\lambda^2} \int_{v_{max2} - v_{max1}}^{v_{max2} - v_{min1}} \int_0^{R} f_{d_i}(z) f_{\Delta v_4}(u) dz du \end{aligned} \quad (17)$$

According to the law of total probability and combining (16) with (17), we can get the joint probability of $\{d_t \le R\}$ and $\{d_i \le R\}$.

$$\begin{aligned} &\Pr\{d_t \le R, d_i \le R\} \\ &= \Pr\{d_t \le R, d_i \le R, v_i \ge v_j\} + \Pr\{d_t \le R, d_i \le R, v_i < v_j\} \\ &= p_1 + p_2 \end{aligned} \quad (18)$$

Hence, substituting (9) and (18) to (11), we can get the analytical result of P_{con}.

4 Simulation Results

In this section, we will give the analytical and simulation results of the initial inter-vehicle distance and the conditional connectivity performance for different system parameters, such as the communication range and the vehicle flow arrival rate in each lane. System simulation parameters are set as shown in Table 1. In this paper, we adopt the Monte Carlo simulation method. For different simulation parameters, 10^5 trials are generated.

Table 1. Simulation parameters.

Parameters	Values
v_{min1}, v_{max2}	25 m/s
v_{max1}	35 m/s
v_{min2}	15 m/s
T_t	5 s
λ_1	0, 0.2, 0.5, 0.8, 1 veh/sec
λ	1 veh/sec

In Fig. 3, we can acquire the analytical and simulation results of $F_{di}(x)$ with different vehicle flow arrival rates λ_1, $\lambda_2(\lambda_2 = \lambda - \lambda_1)$ in two lanes. As shown in the figure, the value of the cumulative distribution function $F_{di}(x)$ will increase with the increase of the initial inter-vehicle distance d_i. In addition, with the increase of the vehicle flow arrival rate λ_1 in the fast lane, the value of the cumulative distribution function $F_{di}(x)$ will be decreased. For example, the probability that d_i is not more than 50 meters is about 0.9 when the value of λ_1 is 0.2. However, that probability can decrease to 0.83 when the λ_1 increased to 0.8. This is because the higher proportion of vehicles in the fast lane, the higher mobility of the whole vehicles. Hence, the initial inter-vehicle distance gets larger and the connectivity performance becomes worse. We can also notice that when the value of λ_1 increased, the change of the value of $F_{di}(x)$ is not large, because the total vehicle flow arrival rate λ in two lanes is constant. When the value of λ_1 is 0, the result is the same as the $F_{di}(x)$ in [9] (when $\lambda = 1$, $v_{max} = 25$ m/s, $v_{min} = 15$ m/s).

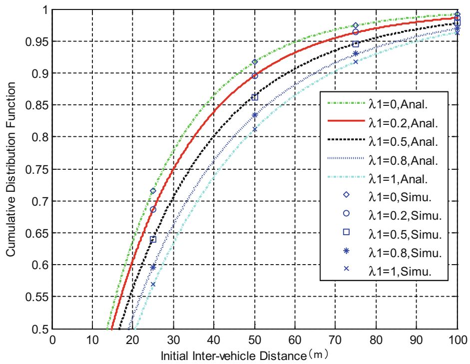

Fig. 3. CDF of initial inter-vehicle distance with various λ_1

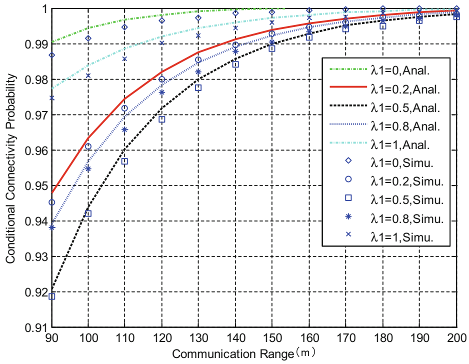

Fig. 4. Conditional connectivity probability with various λ_1

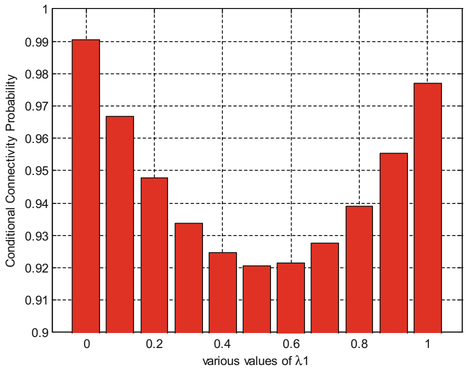

Fig. 5. Conditional connectivity probability with various λ_1 when R is 90 m

In Fig. 4, we can acquire the performance of conditional connectivity probability P_{con} in different communication range R and vehicle flow arrival rate λ_1 and λ_2. According to the curve in the figure, we can find that P_{con} increases with the increase of R. That is, the improvement of the node communication ability is helpful to achieve the link connected constantly. P_{con} can't keep increasing or decreasing at all time with the increasing of λ_1, but performs decreasing previously and increasing later as shown in Fig. 5. This is because when the proportions of the vehicles in the fast and slow lane are the same, the mean value of the speed difference among the vehicles is the largest, which will result in the worst conditional connectivity performance. In addition, we consider the cases that the proportions of vehicles in the fast and slow lane are different. P_{con} with $\lambda_1 = 0.1$ is higher slightly than the P_{con} with $\lambda_1 = 0.9$. This is because while in these two cases the mean values of the speed difference among the vehicles are the same, the low-speed vehicles with $\lambda_1 = 0.1$ are more than $\lambda_1 = 0.9$. The communication link between the vehicles with low speed is more stable, therefore the conditional connectivity performance is better when $\lambda_1 = 0.1$. When the value of λ_1 is 0, the result is the same as the P_{con} in [9] (when $\lambda = 1$, $v_{max} = 25$ m/s, $v_{min} = 15$ m/s).

5 Conclusion

According to the demand of the users' service in VANETs, this paper analyzes the influence of the distribution characteristics of the two-lane vehicle flow and the parameters of the communication system on the conditional connectivity performance. The main conditional connectivity performance index is the conditional connectivity probability. First, a system model in two-lane highway scenario is established. Then we derive the analytical expressions for the initial inter-vehicle distance and the conditional connectivity probability. Finally, we verify the theoretical results by software simulation and make an analysis of the conditional connectivity performance of the communication link. The results of this paper can be applied to design the upper layer communication protocol. For example, in the two-lane routing protocol, the conditional connectivity performance can be used as a routing criterion to determine whether the link can be selected as the next-hop communication link.

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