Capacity Analysis of Secondary User System in Cognitive MIMO Networks Based on NOMA

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Abstract. Nowadays, 5G puts forward a number of candidate multiple access technologies, among which the non-orthogonal multiple access (NOMA) is attracting more and more attention. Combining cognitive MIMO with NOMA is of great significance to improve the capacity for future mobile communication. Cognitive system includes two kinds of users, which are secondary users (SUs) and primary users (PUs), and the underlay spectrum sharing paradigm needs to consider the interference of the SUs system to the PUs system is below the predetermined threshold. And therefore, in order to reduce interference and improve capacity, we precode firstly at the transmitter. Then SUs were clustered according to the merits of the channel quality and performed power allocation for each cluster. During this process, the mean of the channel matrixs' trace is used as the dynamic reception weight to enhance the system capacity. Meanwhile, taking the SUs' quality of service (QoS) and the requirement of successive interference cancelation (SIC) into account. The objective function is NP-hard problem, we need to transform it into system capacity for sub-cluster, and finally using Lagrange function, nonconvex KKT conditions and mathematical induction (MI) to solve the optimal power allocation coefficient, which is between zero and one. The simulation shows that this proposed scheme can improve the capacity obviously compared with the average power allocation.

Keywords: Non-orthogonal multiple access · Cognitive MIMO
Underlay paradigm · Power allocation · Lagrange function · KKT conditions

1 Introduction

The combination of cognitive radio technology and MIMO technology is called cognitive MIMO network, which can greatly improve mobile communication capacity because of MIMO spatial parallel transmission advantage. Facing with the upcoming 5G era, spectrum scarcity is still a serious problem. MIMO Cognitive, which have advantages of both the flexibility of cognitive and the spatial transmission of MIMO, may enhance the spectrum utilization and improve the system capacity, and therefore, it has a wide range of prospect for future mobile communications [1]. Looking back 1G to 4G, are using orthogonal multiple access technology, in 5G era, non-orthogonal

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multiple access technology is increasingly concerned about the industry, which perhaps not only further enhance the spectral efficiency, but also is an effective approach to approaching the capacity domain of multi-user channels [2]. It can be seen from the literature [3] that the underlay scheme is widely used due to its simplicity and promising prospect. In underlay paradigm, the PUs and SUs exist and transmit messages at the same time, they occupy the same frequency to accomplish their own communication, but the SUs to the PUs' harmful interference may not affect the normal communication of PUs, otherwise SUs cannot work properly [4].

NOMA is a new type of multiple access technology that introduces interference at the transmitter and uses the SIC to eliminate the interference at the receiver [5]. Since this technology was put forward so far, more and more researchers study NOMA with MIMO, and their main purpose is to improve the system capacity, but there are little people study NOMA cognitive MIMO, but this method not only improve the system capacity greatly, but also frequency efficiency. In [6], the optimal solution is obtained by solving the maximum value of the quadratic equation, and it is obvious that the method is simple and can get the global optimal solution. However, this method can only solve only two users situation, so this literature does not have universality. The reference [7] only studies two users which meet the requirements of many users, and the document does not pre-code the transmission signal, so that the interference control cannot be carried out well. The reference [8] studied the downlink NOMA multi-user beamforming system, which divides multiple users into clusters with only two users in each cluster, there are only two users in each cluster, and the number of transmitting antenna of clusters and the base station are the same, otherwise, it is not suitable to meet the requirements of this method.

According to the literature [8], the number of sub-user clusters is redesigned, and the number of clusters are determined according to the number of effective antennas of the secondary subscriber base station. That is, if the SBS has two effective transmitting antennas, the secondary users are divided into two cluster. It is known from the literature [9] that the influence of the power allocation factor on the user with poor channel quality is much greater than that of the user with good channel quality. Therefore, when the power allocation is performed, the users with poor channel status are assigned more power, the allocation of less power. Compared to the traditional water-filling, this power allocation is more conducive to system performance.

Where $(\bullet)^H$ represents the conjugate transpose of the matrix or vector, and $tr(\bullet)$ represents matrix trace.

2 System Model

Considering a multi-user downlink communication network, the cognitive system adopts the underlay spectrum sharing mode. At this time, it is necessary to consider the interference constraint of the secondary user system to the primary user system. The secondary users receive the signals from the secondary base station (SBS), and the signals include the useful signals and the harmful signals, the PUs receive the interference signal from the SBS as well. The system model shown in Fig. 1. Assuming that SBS have N_t antennas, and there are N SUs in this system. In order to facilitate the analysis, SUs are divided into N_t clusters, and there are K users in each cluster, so $\sum_{n=1}^{N_t} K = N$.

When performing power, water-filling is a common method to choose, but water filling power allocation algorithm must know the channel information, so the receiver needs current feedback channel information, which introduces some delay and additional overhead, and especially when the channel changes more slowly, the water-filling method is also easier to achieve, but when the channel conditions change faster, the method is difficult to achieve. In addition, the water-filling method needs to deal with the channel information, thus introducing a greater complexity. In this paper, for the sake of analysis, it is assumed that the power of each antenna accounted for the same proportion of the total power.

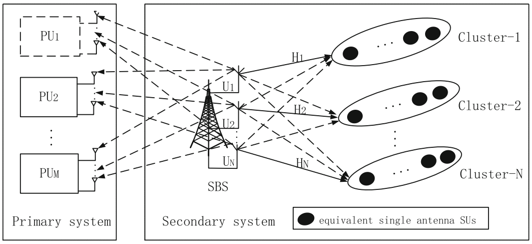

Fig. 1. System model

The signal sent by the sub user base station is expressed as

$$\mathbf{x} = [x_1, x_2, x_3, \cdots, x_{N_t}] \in C^{N_t \times 1}$$

The vector **x** represents the transmit information sequence from SBS

$$x_{n,k} = \sqrt{a_{n,k}{}^{P_t}\Big/{}_{N_t^{S_{n,k}}}}$$

Then, the signals were sent to the n-th cluster can be expressed as

$$x_n = \sum_{k=1}^{K} x_{n,k} = \sum_{k=1}^{K} \sqrt{a_{n,k}{}^{P_t}\Big/{}_{N_t^{S_{n,k}}}}$$

Where $x_{n,k}$ is the signals were sent to the k-th user of the n-th cluster, $a_{n,k}$ is the power allocation from the k-th user of the n-th cluster, and $s_{n,k}$ is the data information from the k-th user of the n-th cluster, and P_t represents the actual transmission power from SBS.

According to the system model, the SBS sends the information to the N users of N_t clusters. Meanwhile, the precoding is performed at the transmitter and the precoding matrix is $\mathbf{U} = \left\{ \mathbf{u}_1^{\mathbf{H}}, \mathbf{u}_2^{\mathbf{H}}, \mathbf{u}_3^{\mathbf{H}}, \cdots, \mathbf{u}_{N_t}^{\mathbf{H}} \right\} \in C^{N_t \times N_t}$. In this process, the primary user system will also receive interference from SBS. The received information at the the k-th user of the n-th cluster is

$$\begin{aligned} y_{n,k} &= \chi\left(\boldsymbol{h}_{n,k}\mathbf{U}\mathbf{x} + z_{n,k}\right) \\ &= \chi\boldsymbol{h}_{n,k}\mathbf{u}_n^T x_n + \chi\boldsymbol{h}_{n,k}\sum_{i=1,i\neq n}^{N}\mathbf{u}_i^T x_i + \chi z_{n,k} \\ &= \chi\boldsymbol{h}_{n,k}\mathbf{u}_n^T\sqrt{a_{n,k}\frac{P_t}{N_t}}s_{n,k} + \chi\boldsymbol{h}_{n,k}\mathbf{u}_n^T\sum_{j=1,j\neq k}^{K}\sqrt{a_{n,j}\frac{P_t}{N_t}}s_{n,j} + \chi\boldsymbol{h}_{n,k}\sum_{i=1,i\neq n}^{N}\mathbf{u}_i^T x_i + \chi z_{n,k} \end{aligned} \tag{1}$$

Because NOMA uses Successive interference cancellation (SIC) at the receiver, so concerning the nature of NOMA, the $y_{n,k}$ in (1) can be rewritten as:

$$y_{n,k} = \underbrace{\chi\boldsymbol{h}_{n,k}\mathbf{u}_n^T\sqrt{a_{n,k}\frac{P_t}{N_t}}s_{n,k}}_{\text{desired signals}} + \underbrace{\chi\boldsymbol{h}_{n,k}\mathbf{u}_n^T\sum_{j=1}^{k-1}\sqrt{a_{n,j}\frac{P_t}{N_t}}s_{n,j}}_{\text{intra-cluster interference}} + \underbrace{\chi\boldsymbol{h}_{n,k}\sum_{i=1,i\neq n}^{N}\mathbf{u}_i^T x_i}_{\text{inter-cluster interference}} + \underbrace{\chi z_{n,k}}_{\text{channel noise}} \tag{2}$$

Where χ the mean of the channel matrixs' trace is used as the dynamic reception weight, that is $\chi = \frac{1}{K}\sum_{i=1}^{K} tr(h_{n,i}h_{n,i}^H)$, which means the weight of the received signal is in association with the mean of the channel trace. When channel station is better, the received signal is large, but when the channel station is relatively poor, the intensity of the received signal is relatively small. $z_{n,k}$ is the Gauss distribution noise signal whose mean value is zero and variance is σ^2.

From the literature [8], we can see that the influence of power factor on users with poor channel quality is far greater than that of users with good channel quality, and therefore, considering the influence of power factor on the channel quality and made the overall performance of the system is further improved, we assign less power to the users with better channel quality, more power allocation with poor channel quality. Suppose in the n-th cluster, channel gains are sorted as $h_{n,1} > h_{n,2} > h_{n,3} > \cdots > h_{n,K}$, and the corresponding power allocation factor is $a_{n,1} < a_{n,2} < a_{n,3} < \cdots < a_{n,K}$.

We can get the signal-to-interference-and-noise-rate (SINR) from the formulation (2), which is

$$\begin{aligned}\mathrm{SINR}_{n,k} &= \frac{\left|\chi \boldsymbol{h}_{n,k}\mathbf{u}_n^T\right|^2 a_{n,k}\frac{P_t}{N_t}}{\left|\chi \boldsymbol{h}_{n,k}\mathbf{u}_n^T\right|^2 \sum_{j=1}^{k-1} a_{n,j}\frac{P_t}{N_t} + \sum_{i=1,i\neq n}^{N}\sum_{j=1}^{K}\left|\chi \boldsymbol{h}_{n,k}\mathbf{u}_i^T\right|^2 a_{i,j}\frac{P_t}{N_t} + \chi z_{n,k}} \\ &= \frac{\chi\left|\boldsymbol{h}_{n,k}\mathbf{u}_n^T\right|^2 a_{n,k}}{\chi\left|\boldsymbol{h}_{n,k}\mathbf{u}_n^T\right|^2 \sum_{j=1}^{k-1} a_{n,j} + \chi\sum_{i=1,i\neq n}^{N}\left|\boldsymbol{h}_{n,k}\mathbf{u}_i^T\right|^2 + \left(\frac{1}{P_t/N_t}\right) z_{n,k}}\end{aligned} \tag{3}$$

Assuming $E\left[\left|s_{i,j}\right|^2\right] = 1,\ \forall i,j$, then the rate of the k-th users is

$$R_{n,k} = B\log_2\left|1 + \mathrm{SINR}_{n,k}\right| = B\ log_2\left|1 + \frac{g_{n,k}a_{n,k}}{g_{n,k}\sum_{j=1}^{k-1} a_{n,j} + 1}\right| \tag{4}$$

Where B represents the bandwidth of each transmitted beam, and

$$g_{n,k} = \frac{\left|\chi \boldsymbol{h}_{n,k}\boldsymbol{u}_n\right|^2}{B\left(\sum_{i=1,i\neq n}^{N}\left|\chi \boldsymbol{h}_{n,k}\boldsymbol{u}_i\right|^2 + \left(\frac{1}{P_t/N_t}\right)\chi z_{n,k}\right)} \tag{5}$$

Here, $g_{n,k}$ represent the normalized channel, the channel is able to simplify the calculation of the normalized objective, then type (3) can be written as follows

$$\mathrm{SINR}_{n,k} = \frac{g_{n,k}a_{n,k}}{g_{n,k}\sum_{j=1}^{k-1} a_{n,j} + 1} \tag{6}$$

The capacity of the whole secondary user system is:

$$\hat{R}_{SBS-sus} = \sum_{n=1}^{N}\sum_{k=1}^{K} B\log_2\left|1 + \frac{g_{n,k}a_{n,k}}{g_{n,k}\sum_{j=1}^{k-1} a_{n,j} + 1}\right| \tag{7}$$

3 Problem Description and Solution

The PUs are equipped with two antennas, the SBS equipped with 2 antennas, and there are N SUs in total and each SU is equipped with one antenna, the entire SUs are divided into two large clusters, and each cluster has K SUs, that is, $N = 2K$.

3.1 Cognitive System

Because of the cognitive MIMO system based on NOMA underlay model, in this system, apart from the interference from SBS to PUs, we mainly consider the inter-cluster and intra-cluster interference of secondary users system. Other interference is not main interference, so for the sake of simplicity here, they are not being discussed.

In MIMO cognitive network, if researchers selected underlay spectrum sharing paradigm, then the interference between SBS and PUs need to be less than predefined threshold I_p, and if interference is larger than this threshold, then SUs can not to work properly [8], so in other words, the presence of secondary user is based on the premise that the normal communication of the primary user is not influenced at all. The number of PUs are modeled based on homogeneous poisson point processes (PPPs), denoted by ϕ_l with density of λ_l, $|m_l|^2$ is equivalent gain from SBS to PUs. In this case, SBS's actual maximum transmission power P_t can be described as follows

$$P_t = \min\left\{ \frac{I_p}{\max\limits_{l\in\phi_l}|m_l|^2}, P_s \right\} = \min\left\{ \frac{I_p}{\max\limits_{l\in\phi_l}(|\widehat{m}_l|^2 L(d_l))}, P_s \right\} \tag{8}$$

Where P_s is the maximum transmit power from SBS. $\widehat{m}_l$ is the channel gain of Rayleigh fading channel from SBS to PUs, and $L(d_l) = 1/(1 + d_l^{\delta})$ is large-scale path loss where d_l is the distance between k-th PUs and SBS, and δ is path-loss coefficient.

This is a MIMO cognitive radio, and the transmit power of SBS takes into account the performance of the primary system and the SBS itself as well. Figure 2 can describe this relationship between them. From this figure, it is obvious that the transmission power of the SBS is maintained at its maximum transmission power P_s when the interference threshold reached a certain value.

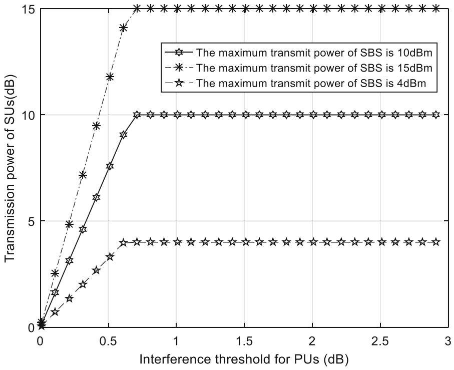

Fig. 2. The relationship between interference threshold (IT) transmit power of SBS

3.2 Precoding Selection

Nowadays, many literatures use zero-forcing (ZF) precoding method to precode the signal. The literature [8] adopt the zero-forcing precoding method, which is simple, and when there is only one user in each cluster, this method can eliminates inter-cluster interference ideally when neglect system noise. However, in actual communication systems, the number of users in each group is larger than or equal to two, this approach is no longer simple anymore. And if we precode through minimum mean square error (MMSE), the bit error rate (BER) is lower, and MMSE is more realistic than ZF method. The precoding matrix of MMSE method is closely related to the channel, and the expression is: $\mathbf{U} = \left(\mathbf{H}^H\mathbf{H} + \sigma^2\mathbf{I}\right)^{-1}\mathbf{H}^H$. In the NOMA cognitive MIMO system, since the NOMA receiver uses the SIC to receive desired signals, the BER can be further reduced. Figure 3 is the comparison of BER.

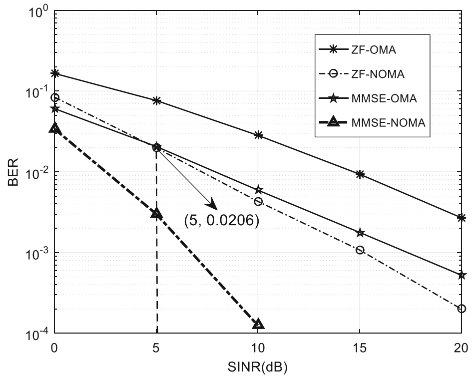

Fig. 3. The comparison of BER for different precoding methods

3.3 Clustering

Compared with conventional orthogonal multiple access (MA), non-orthogonal multiple access (NOMA) can enlarge the performance gap between them through applying user pairing [9]. Compared with conventional orthogonal multiple access (MA), non-orthogonal multiple access (NOMA) can enlarge the performance gap between them through applying user pairing [10]. In order to suppress interference and receive useful signals as much as possible, NOMA system typically cluster users. Clustering needs to consider the users in each cluster which have a neutralization between channel diversity and channel correlation [11]. Some researchers are simply distributed two clusters which the channel gain is strong and divided into one cluster while the channel gain is weak into another group, so that the group can merely guarantee the correlation between the channels without considering the difference between them fully [12]. Some researchers study signal antenna in NOMA system and cluster by head and tail [12]. We cluster SUs according to [11, 12] which consider a neutralization between channel diversity and channel correlation. The SUs channel gains are being sorted descendingly firstly, which is $\boldsymbol{h}_1 > \boldsymbol{h}_2 > \boldsymbol{h}_3 > \cdots > \boldsymbol{h}_{N-1} > \boldsymbol{h}_N$, and then perform clustering. There, we divide the SUs into two clusters, and discuss two situations which is the number of users in each cluster are odd or even, that is $n = \{1, 2\}$ The clustering is shown as follows.

When K is even, the channel gain distribution in the first cluster is

$$\boldsymbol{h}_{1,k} = \{\boldsymbol{h}_1, \boldsymbol{h}_3, \boldsymbol{h}_5, \cdots, \boldsymbol{h}_{K-1}, \boldsymbol{h}_{K+2}, \boldsymbol{h}_{K+4}, \cdots, \boldsymbol{h}_{N-2}, \boldsymbol{h}_N\};$$

And the channel gain distribution in the second cluster is

$$\boldsymbol{h}_{2,k} = \{\boldsymbol{h}_2, \boldsymbol{h}_4, \boldsymbol{h}_6, \cdots, \boldsymbol{h}_K, \boldsymbol{h}_{K+1}, \boldsymbol{h}_{K+3}, \cdots, \boldsymbol{h}_{N-3}, \boldsymbol{h}_{N-1}\}.$$

When K is odd, the channel gain distribution in the first cluster is

$$\boldsymbol{h}_{1,k} = \{\boldsymbol{h}_1, \boldsymbol{h}_3, \boldsymbol{h}_5, \cdots \boldsymbol{h}_K, \boldsymbol{h}_{K+3}, \boldsymbol{h}_{K+5}, \cdots, \boldsymbol{h}_{N-2}, \boldsymbol{h}_N\};$$

And the channel gain distribution in the second cluster is

$$\boldsymbol{h}_{2,k} = \{\boldsymbol{h}_2, \boldsymbol{h}_4, \boldsymbol{h}_6, \cdots, \boldsymbol{h}_{K+1}, \boldsymbol{h}_{K+2}, \boldsymbol{h}_{K+4} \cdots, \boldsymbol{h}_{N-3}, \boldsymbol{h}_{N-1}\}.$$

After clustering and precoding, the sum rate of the cluster is obtained:

$$\bar{\hat{R}}_n = \sum_{k=1}^{K} B \log_2 \left| 1 + \frac{\bar{g}_{n,k} a_{n,k}}{\bar{g}_{n,k} \sum_{j=1}^{k-1} a_{n,j} + 1} \right| \tag{9}$$

The goal of this paper is to find the maximum of formula (9).

3.4 Power Allocation and Problem Solving

According to document [12], the power between each user needs to be satisfied when the serial interference cancellation of each subsystem is repeated:

$$\frac{P_t}{2} a_{n,k} \bar{g}_{n,k-1} - \frac{P_t}{2} \bar{g}_{n,k-1} \sum_{j=1}^{k-1} a_{n,j} \geq p_{tol}$$

Let $\frac{p_{tol}}{p_t/2} = \beta$, $\frac{R_0}{B} = \gamma$, and β, γ are constant. R_0 is the minimum rate of SUs need to meet the rate. When the secondary users' rate are greater than or equal to R_0, then they can work properly, otherwise, they can not work normally. After the above analysis, according to the system model, we can get the optimization equation of the coefficient distribution:

$$
\begin{aligned}
&\mathbf{Q}: \quad \underset{\{a_{n,k}, \forall k=1,2,\cdots,K\}}{\text{maximize}} \quad \sum_{k=1}^{K} B \log_2 \left| 1 + \frac{\bar{g}_{n,k} a_{n,k}}{\bar{g}_{n,k} \sum_{j=1}^{k-1} a_{n,j} + 1} \right| \\
&S.t. \\
&\qquad \mathbf{C1}: \sum_{k=1}^{K} a_{n,k} \le 1, \quad \mathrm{n} = \{1,2\} \\
&\qquad \mathbf{C2}: \log_2 \left(1 + \frac{\bar{g}_{n,k} a_{n,k}}{\bar{g}_{n,k} \sum_{j=1}^{k-1} a_{n,j} + 1} \right) \ge \gamma, \quad \mathrm{n} = \{1,2\}, \forall \mathrm{k} = 1,2,\cdots,\mathrm{K} \\
&\qquad \mathbf{C3}: \left(a_{n,k} - \sum_{j=1}^{k-1} a_{n,j} \right) \bar{g}_{n,k-1} \ge \beta, \quad \mathrm{n} = \{1,2\}, \forall k = 2,\cdots,\mathrm{K} \\
&\qquad \mathbf{C4}: a_{n,k} \in [0,1], \forall n, \mathrm{k}
\end{aligned}
$$

Here, constraint **C1** indicates that the power allocation coefficient is less than or equal to one; Constraint **C**2 limits the rate of the secondary user, indicating that the rate of each user in the secondary user system satisfies greater than a certain value, thus ensuring the user service quality of the secondary user system; Constraint **C**3 indicates that the minimum power difference between the SUs that has been decoded and the un-decoded is greater than p_{tol}; Constraint **C**4 represents the condition that the power distribution coefficient is to be satisfied.

A simplified analysis of constrained **C**2 is performed as

$$
\mathbf{C}2 \Leftrightarrow \bar{g}_{n,k} a_{n,k} \ge (2^{\gamma} - 1) \left(\bar{g}_{n,k} \sum_{j=1}^{k-1} a_{n,j} + 1 \right)
$$

Then the Lagrange function is available as

$$
\begin{aligned}
\mathbf{L}(a_{n,k}, \lambda, \eta, \zeta) = {} & \sum_{k=1}^{K} B \log_2 \left| 1 + \frac{\bar{g}_{n,k} a_{nk}}{\bar{g}_{n,k} \sum_{j=1}^{k-1} a_{n,j} + 1} \right| + \lambda \left(1 - \sum_{k=1}^{K} a_{n,k} \right) \\
& + \sum_{k=1}^{K} \eta_k \left(\bar{g}_{n,k} a_{n,k} - (2^{\gamma} - 1) \left(\bar{g}_{n,k} \sum_{j=1}^{k-1} a_{n,j} + 1 \right) \right) \\
& + \sum_{k=2}^{K} \zeta_k \left(\left(a_{n,k} - \sum_{t=1}^{k-1} a_{n,t} \right) \bar{g}_{n,k-1} - \beta \right)
\end{aligned}
$$

According to KKT conditions available

$$
\begin{aligned}
\frac{\partial \mathbf{L}}{\partial a_{n,k}{}^{*}} = {} & \frac{B \bar{g}_{n,k}}{\bar{g}_{n,k} \sum_{j=1}^{k} a_{n,j} + 1} - \sum_{j=k+1}^{K} \frac{B a_{n,j} \bar{g}_{n,j}{}^{2}}{\left(\sum_{t=1}^{j} a_{n,t} \bar{g}_{n,j} + 1 \right) \left(\sum_{t'=1}^{j-1} a_{n,t'} \bar{g}_{n,j} + 1 \right)} - \lambda \\
& + \eta_k \bar{g}_{n,k} - \sum_{j=k+1}^{K} (2^{\gamma} - 1) \eta_j \bar{g}_{n,j} + \zeta_k \bar{g}_{n,k-1} - \sum_{j=k+1}^{K} \zeta_j \bar{g}_{n,j} \le 0, \\
& 0 \le a_{n,k}{}^{*} \le 1, \forall \mathrm{k} = 2,3,\cdots,\mathrm{K},
\end{aligned} \tag{10}
$$

$$\frac{\partial \mathbf{L}}{\partial \lambda^*} = 1 - \sum_{k=1}^{K} a_{n,k} \geq 0,\ \lambda^* \geq 0,\ {\eta_k}^* \geq 0, \forall \mathrm{k} = 2, 3, \cdots, \mathrm{K} \tag{11}$$

$$\frac{\partial \mathbf{L}}{\partial {\eta_k}^*} = \overline{g}_{n,k} a_{n,k} - (2^{\gamma} - 1)\left(\overline{g}_{n,k} \sum\nolimits_{j=1}^{k-1} a_{n,j} + 1\right) \geq 0 \tag{12}$$

$$\frac{\partial \mathbf{L}}{\partial {\zeta_k}^*} = \left(a_{n,k} - \sum_{j=1}^{k-1} a_{n,j}\right) \overline{g}_{n,k-1} - \beta \geq 0, {\zeta_k}^* \geq 0, \forall \mathrm{k} = 2, 3, \cdots, \mathrm{K} \tag{13}$$

The original problem is a non-convex NP-hard problem [13], so it is difficult to solve the optimal value. In this paper, the Lagrange function and the KKT condition are used to solve the above problems by combining mathematical induction method, and verify that the value of the solution meets the requirement.

Letting $O = \{\lambda\}$; $\mathbf{\Omega} = \{\eta_1, \eta_2, \eta_3, \cdots, \eta_K\}$; $\mathbf{\Theta} = \{\zeta_1, \zeta_2, \zeta_3, \cdots, \zeta_K\}$, since $K \geq 2$, so the set of optimal solution of Lagrange function $\mathbf{L}\left(a_{n,k}, \lambda, \eta, \zeta\right)$ may be assumed as $\mathbf{\Psi} = \{\lambda, \eta_2/\zeta_2, \eta_3/\zeta_3, \eta_4/\zeta_4, \cdots, \eta_K/\zeta_K\}$. Suppose that the minimum rate of the cognitive system is satisfied $\mathbf{\Psi} = \{\lambda, \zeta_2, \zeta_3, \zeta_4, \cdots, \zeta_K\}$ and $\zeta_1 = \eta_1 = \eta_2 = \eta_3 = \cdots = \eta_K = 0$.

Then

$$\sum_{k=1}^{K} a_{n,k} = 1 \tag{14}$$

$$\left(a_{n,k} - \sum_{t=1}^{k-1} a_{n,t}\right) \overline{g}_{n,k-1} - \beta = 0, \forall k = 2, 3, 4 \tag{15}$$

$$\overline{g}_{n,k} a_{n,k} - (2^{\gamma} - 1)\left(\overline{g}_{n,k} \sum\nolimits_{j=1}^{k-1} a_{n,j} + 1\right) > 0, \forall k = 1, 2, 3, 4 \tag{16}$$

From the above formula (14)–(15), the power allocation coefficient can be solved when the number of users in each group is 2, 3, 4, as shown in Table 1.

By mathematical induction (MI), at this time, can get each cluster head power allocation coefficient:

$$a_{n,1} = \frac{1}{2^{K-1}} - \sum_{j=2}^{K} \frac{\beta}{\overline{g}_{n,j-1} 2^{j-1}}$$

The power allocation factor of the other $K - 1$ users in this cluster is:

$$a_{n,k} = \frac{1}{2^{(K-k+1)}} - \sum_{j=k}^{K} \frac{\beta}{\overline{g}_{n,j-1} 2^{(j-k+1)}} + \frac{\beta}{\overline{g}_{n,k-1}}, 2 \leq k \leq K.$$

Where $n = \{1, 2\}$.

Table 1. Power allocation coefficient of SUs

The number of users in each cluster	Power allocation coefficient
2	$a_{n,1} = \frac{1}{2} - \frac{\beta}{2\overline{g}_{n,1}}$ $a_{n,2} = \frac{1}{2} + \frac{\beta}{2\overline{g}_{n,1}}$
3	$a_{n,1} = \frac{1}{4} - (\frac{\beta}{2\overline{g}_{n,1}} + \frac{\beta}{4\overline{g}_{n,2}})$ $a_{n,2} = \frac{1}{4} + \frac{\beta}{2\overline{g}_{n,1}} - \frac{\beta}{4\overline{g}_{n,2}}$ $a_{n,3} = \frac{1}{2} + \frac{\beta}{2\overline{g}_{n,2}}$
4	$a_{n,1} = \frac{1}{8} - \frac{\beta}{2\overline{g}_{n,1}} - \frac{\beta}{4\overline{g}_{n,2}} - \frac{\beta}{8\overline{g}_{n,3}}$ $a_{n,2} = \frac{1}{8} + \frac{\beta}{2\overline{g}_{n,1}} - \frac{\beta}{4\overline{g}_{n,2}} - \frac{\beta}{8\overline{g}_{n,3}}$ $a_{n,3} = \frac{1}{4} + \frac{\beta}{2\overline{g}_{n,2}} - \frac{\beta}{4\overline{g}_{n,3}}$ $a_{n,4} = \frac{1}{2} + \frac{\beta}{2\overline{g}_{n,3}}$

By the nature of the KKT conditions when $N = 8$ verify the situation, at this time, there are four secondary users in each cluster: $K = 4$. At this point, assuming $\Psi = \{\lambda, \zeta_2, \zeta_3, \zeta_4\}$, $\Theta = \{\zeta_2, \zeta_3, \zeta_4\}$ and setting formulation (10) is equal to zero, then the following relation can be obtained according to the geometric operations:

$$\frac{B\overline{g}_{n,k}}{\overline{g}_{n,k}\sum_{j=1}^{k} a_{n,j} + 1} - \sum_{j=k+1}^{K} \frac{Ba_{n,j}\,\overline{g}_{n,j}^{\,2}}{\left(\sum_{t=1}^{j} a_{n,t}\overline{g}_{n,j} + 1\right)\left(\sum_{t'=1}^{j-1} a_{n,t'}\overline{g}_{n,j} + 1\right)} - \lambda + \zeta_k\overline{g}_{n,k-1} - \sum_{j=k+1}^{K} \zeta_j\overline{g}_{n,j} = 0$$

$$\Leftrightarrow \frac{B\overline{g}_{n,4}}{\overline{g}_{n,4}\sum_{j=1}^{k} a_{n,j} + 1} - \sum_{j=1}^{3} \frac{Ba_{n,j}(\overline{g}_{n,j-1} - \overline{g}_{n,j})}{\left(\sum_{t=1}^{j} a_{n,t}\overline{g}_{n,j} + 1\right)\left(\sum_{t'=1}^{j-1} a_{n,t'}\overline{g}_{n,j} + 1\right)} - \lambda + \zeta_4\overline{g}_{n,3} - \sum_{j=k}^{3} \zeta_j\overline{g}_{n,j} = 0$$

Based on the nature of the equation, it can be solved:

$$\lambda = \frac{B\overline{g}_{n,4}}{\overline{g}_{n,4}\sum_{j=1}^{4} a_{n,j} + 1} + \zeta_4\overline{g}_{n,3}$$

$$\zeta_2 = \frac{Ba_{n,2}(\overline{g}_{n,1} - \overline{g}_{n,2})}{\overline{g}_{n,1}\left(a_{n,1}\overline{g}_{n,1} + 1\right)\left(a_{n,1}\overline{g}_{n,2} + 1\right)}$$

$$\zeta_k = \frac{Ba_{n,k}(\overline{g}_{n,k-1} - \overline{g}_{k,j})}{\overline{g}_{n,k}\left(\sum_{t=1}^{j-1} a_{n,t}\overline{g}_{n,j-1} + 1\right)\left(\sum_{t'=1}^{j-1} a_{n,t'}\overline{g}_{n,j} + 1\right)} + \zeta_{k-1}\overline{g}_{n,k-2},\ k = 3, 4$$

$$\zeta_1 = \eta_k = 0, \quad k = 1, 2, 3, 4$$

In the above formula, $\overline{g}_{n,k-1} > \overline{g}_{n,k}$, η_k, λ, ζ_k are larger than zero, so the solution satisfy KKT conditions. What's more, when the user system uses the average power distribution mode, $a_{n,k} = \frac{1}{K}$, then the system capacity is

$$
\begin{aligned}
C_{apa} &= \tilde{R}_1 + \tilde{R}_2 \\
&= \sum_{k=1}^{K} B\log_2\left(1 + \frac{\frac{1}{K}\bar{g}_{1,k}}{\bar{g}_{1,k}\sum_{j=1}^{k-1}\frac{1}{K} + 1}\right) + \sum_{k=1}^{K} B\log_2\left(1 + \frac{\frac{1}{K}\bar{g}_{2,k}}{\bar{g}_{2,k}\sum_{j=1}^{k-1}\frac{1}{K} + 1}\right) \\
&= \sum_{k=1}^{K} B\log_2\left(1 + \frac{K\bar{g}_{1,k}}{(k-1)\bar{g}_{1,k} + K}\right) + \sum_{k=1}^{K} B\log_2\left(1 + \frac{K\bar{g}_{2,k}}{(k-1)\bar{g}_{2,k} + K}\right)
\end{aligned}
$$

4 Simulation Results and Analysis

This section mainly verifies the capacity problem of the MIMO cognitive secondary users system based on NOMA. System parameters: the cognitive base station has two transmit antennas, the number of SUs in the cognitive system is greater than or equal to 4, which each user is configured with a single transceiver antenna. The authorization system has two main users and are configured with two transceiver antennas. The maximum power that can be sustained by each antenna of the cognitive base station is 50 dBm, and the power difference between the sub-users satisfying the NOMA condition is 10 dBm, and the system bandwidth is 1 MHz (Table 2).

Table 2. Simulation parameters

Parameters	Values
Maximum transmit power of base station P_s	47 dBm
Difference in power between SUs p_{tol}	10 dBm
System bandwidth	8 MHz
The distance between PU1 and SBS	50 m
The distance between PU2 and SBS	100 m
The antennas of SBS N_t	2
The effective antennas of SUs	1
The number of SUs	4, 6, 8
Interference threshold I_p	5 W
Gaussian white noise power σ^2	0.1 W
Path loss exponent δ	3

In the first experiment, the Monte Carlo simulation was used to compare the change in the capacity of the secondary user system under two different ways: the dynamic power allocation mode and the average power mode. It can be seen that the use of dynamic power distribution, when the number of secondary users increased, the system capacity will increase also, and it is worth noting that the number of SUs from four increased to six, the system capacity to improve more than the number of users from 6 increased to 8. However, in the average power allocation method, the more the secondary users, the lower the capacity. It can be seen that the dynamic power allocation method has more system capacity than the average power allocation method. As shown in Fig. 4.

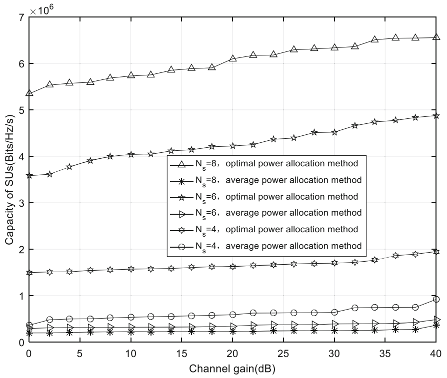

Fig. 4. The variation of secondary users' number with channel gain

The second experiment compare the capacity changes of the cognitive system under different interference thresholds. When the IT is small, the user's interference to PUs is small, relatively, and the system capacity of the SUs is larger. As shown in Fig. 5.

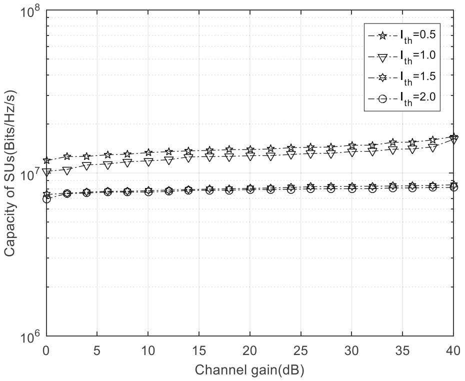

Fig. 5. Capacity comparison of cognitive system with PUs' IT

5 Conclusion

With the further development of integrated circuits, NOMA technology is an important technology to increase the capacity of the system for the future mobile communication, which is based on the increase of the receiver's complexity. In this paper, the problem of improving the capacity of secondary users system is studied by combining the cognitive MIMO and NOMA technology. The simulation results show that the proposed algorithm can improve the system capacity. The next step is to consider the new pre-coding method combined with other clustering and power allocation methods to further optimize the state of the NOMA cognitive MIMO secondary users system.

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