Resource Allocation in OFDM-Based Cognitive Radios Under Proportional Rate Constraint

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Abstract. The objective of this paper is to optimize the power allocation to maximize the total rate of secondary users (SUs) under the SU total transmit power constraint and primary user (PU) interference temperature constraint, a proportional rate constraint is also used to assure that each SU can achieve fairness. The power allocation problem is not a convex optimization problem, which can be converted to a convex optimization problem without introducing auxiliary variables. To reduce the burden of information exchange and computational complexity, PU interference temperature constraint can be decoupled to an average interference constraint. The Lagrangian duality method is used to solve the optimal transmission power. Numerical results show that the proposed algorithm not only improves the rate fairness of each SU, but also guarantees the quality of service (QoS) of PU.

Keywords: Cognitive radio
Orthogonal frequency division multiplexing (OFDM)
Resource allocation · Proportional fairness · Spectrum sharing

1 Introduction

As the broadband wireless communications field continues to grow at a rapid rate, the emergence of new technologies, i.e., WiMAX [1] Bluetooth and LTE [2] lead to a rapidly escalating demand for radio spectrum resources. In [3], as a result of the static spectrum usage policy, some global authorized frequency bands are fixed which results in a lower spectral efficiency. Spectrum resources have been assigned to an existing wireless system which has a lot of idle time and space. The issue of spectrum scarcity has been exacerbated due to inefficient use of the fixed spectrum resources.

The grim spectrum utilization situation urgently requires a way of communication by opportunistic access the underutilized radio spectrum. This form of communication in dynamic spectrum access to the characteristics is cognitive radio. The concept of cognitive radio was first proposed by Joseph Mitola III in a seminar at KTH Royal Institute of Technology in Stockholm in 1998 and published in an article by Mitola and Gerald Q. Maguire, Jr. in 1999. To enhance the spectral utilization efficiency of wireless communication technology, it can detect spectrum holes and allows the unlicensed secondary user to access idle cognitive spectrum without affecting the communication quality of the primary user, to effectively reduce the waste of spectrum.

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OFDM [5] divides transmission bandwidth into a series of non-overlapping orthogonal subcarriers, by assigning different sets of subcarriers to different users. OFDM not only enables efficient use of the limited spectrum resources, but also cuts back a variety of adverse environmental impact of the channel due to multipath fading. Thereby greatly improve system performance to meet the needs of different users. OFDM is regarded as an ideal alternative technology of realizing CR system which can highly improve the performance of cellular system by effectively utilizing the characteristic of multiuser diversity to distribute the sub-channel, bit and power.

According to the literatures on resource allocation in OFDM-based CR systems [6–11], it attracts more attentions by the development of CR technology in recent years. In [6], the resource allocation (RA) problem is formulated as a dynamic selection of spectrum patterns and power allocations that are better suited to the available spectrum range. However, the interference introduced by SUs is not mentioned. An optimal RA algorithm is proposed in [7], which ensures the rates of SUs are maintained in proportion to predefined target rates. However, the algorithm is designed for non-real-time applications. In [8], both optimal and suboptimal algorithms are developed to maximize the sum capacity of a CR network without consider the transmission power limitation. In [9], the authors want to maximize the sum capacity of the CR system while keeping proportional rate constraints satisfying to guarantee the fairness among SUs. In [10], a linear water-filling scheme is proposed. This algorithm maximizes the overall transmitted data rate of the CR system while keeping the interference introduced to the PU bands below a threshold. The fairness among users is ignored. The equal power allocation-proportional rate greedy (EPA-PRG) [11] algorithm is proposed to maximize the system throughput while keeping the fairness. The cooperative transmission technology isn't applied in this algorithm.

In this paper, we study the algorithm to optimize the total rate of SUs in OFDM-CR networks by considering the proportional fairness constraint under the underlay spectrum access method. Three sets of constraints are considered: interference power constraints to the PU, maximum transmission power constraints to the SU, the proportional rate constraint. The Lagrange decomposition algorithm and sub-gradient algorithm are used to solve the problem of power allocation. Simulation results show that the algorithm proposed in this paper improves the rate fairness of each SU effectively and it can keep the performance of the system.

2 System Model

We consider the underlay spectrum access method of an OFDM-based CR network. There are M CR user links which are distributed in an area that is away from the one primary user link. And the different CR users are allocated to the K subcarriers.

The transmit power of the cognitive user transmitter is deterministic, the sum of the transmit powers of the transmitter of user i on all subcarriers must be less than or equal to the total power of the transmitter of the cognitive user i

$$\sum_{k=1}^{K} p_k^i \leq p_{\max}^i \tag{1}$$

where $p_{\max}^i$ denotes the maximum total transmit power (MTTP) of user i. p_k^i is the transmit power of user i on the subcarrier k.

Without affecting the communication quality of PU, the most important constraint is as follows that the interference power generated by all secondary user transmitter (SU-TX) on all subcarriers should not exceed the threshold which the PU can tolerate, i.e.,

$$\sum_{i=1}^{M} \sum_{k=1}^{K} p_k^i h_k^i \leq I^{MTI} \tag{2}$$

where I^{MTI} denotes maximum tolerated interference (MTI) at primary user receiver (PU-RX). h_k^i denotes the channel gain between the ith SU-TX to the PU-RX at the kth subcarrier.

Fairness is an important issue to be considered in the resource allocation of OFDM cognitive radio. The benefit of imposing a proportional fairness constraint is to assure that we can control the rate ratios among users, and generally ensure that each user is able to achieve a required data rate. The proportional rate constraints are expressed as

$$R_1 : R_2 : \cdots : R_M = r_1 : r_2 : \cdots : r_M \tag{3}$$

where r_M is a positive real number, $r_1, r_2, \cdots, r_M$ represents a predetermined proportionality constraint factor. R_M expresses the data rate of cognitive user M, when considering an ideal optimization scheme and using the Shannon rate formula [12], it can be expressed as

$$R_M = \sum_{k=1}^{K} \log\left(1 + \frac{p_k^i g_k^{ii}}{\sum\limits_{j \neq i} p_k^j g_k^{ji} + I_k^{ps} + \sigma_k^i}\right) \tag{4}$$

where I_k^{ps} indicates the interference of PU to cognitive user M. σ_k^i denotes the background noise of the cognitive user on subcarrier K.

3 The Proposed Power Allocation Algorithm

Our objective is to maximize the total rate of SUs under the SU total transmit power constraint and PU interference temperature constraint. The proportional fairness is introduced into the frame that we can control the rate ratio among users.

The optimization problem for OFDM-CR power allocation can be expressed as

$$\begin{aligned} & \max \sum_{i=1}^{M} \sum_{k=1}^{K} \log\left(1 + \frac{p_k^i g_k^{ii}}{\sum\limits_{j \neq i} p_k^j g_k^{ji} + I_k^{ps} + \sigma_k^i}\right) \\ s.t. \quad & C1 : \sum_{k=1}^{K} p_k^i \leq p_{\max}^i \\ & C2 : \sum_{i=1}^{M} \sum_{k=1}^{K} p_k^i h_k^i \leq I_k^{MTI} \\ & C3 : R_1{:}R_2{:} \cdots R_M = r_1{:}r_2{:} \cdots r_M \end{aligned} \tag{5}$$

The problem (5) is not convex optimization problem. We may rewrite the optimization problem as

$$\begin{aligned} & \min \; - \sum_{i=1}^{M} \sum_{k=1}^{K} \log\left(1 + \frac{p_k^i g_k^{ii}}{\sum\limits_{j \neq i} p_k^j g_k^{ji} + I_k^{ps} + \sigma_k^i}\right) \\ s.t. \quad & C1 : \sum_{k=1}^{K} p_k^i - p_{\max}^i \leq 0 \\ & C2 : \sum_{i=1}^{M} \sum_{k=1}^{K} p_k^i h_k^i - I_k^{MTI} \leq 0 \\ & C3 : R_1{:}R_2{:} \cdots R_M = r_1{:}r_2{:} \cdots r_M \end{aligned} \tag{6}$$

From $C2$, the transmission power of each cognitive user in the network is coupled in the interference to the primary user during the resource sharing. To ensure the normal communication of the primary users, the channel gain h_k^i between the cognitive user and the primary user is indispensable, but it requires a large amount of information exchange. To reduce the system burden which is caused by the large amount of information exchange, constraint $C2$ can be divided into $K * M$ parts, the total interference is transformed into the interference for each cognitive user. Then the cognitive user does not need to exchange the information, the constraint $C2$ will change into

$$p_k^i h_k^i \leq \frac{I^{MTI}}{K * M} \tag{7}$$

Equation (7) shows that the channel gain h_k^i has been decoupled. If the transmission power of each cognitive user meets the requirements of Eq. (7), the primary user communication quality will be guaranteed.

Combining Eqs. (6) and (7), the power allocation problem becomes

$$\begin{aligned} & \min \; -\sum_{i=1}^{M}\sum_{k=1}^{K}\log\left(1+\frac{p_k^i g_k^{ii}}{\sum\limits_{j\neq i} p_k^j g_k^{ji}+I_k^{ps}+\sigma_k^i}\right) \\ s.t. \quad & C1:\sum_{k=1}^{K} p_k^i - p_{\max}^i \le 0 \\ & C2':p_k^i h_k^i - \frac{I_k^{MTI}}{K*M}\le 0 \\ & C3:R_1{:}R_2{:}\cdots R_M = r_1{:}r_2{:}\cdots r_M \end{aligned} \tag{8}$$

where the constraints $C1$, $C2'$ are linear constraints. The convexity of the utility function is verified in the previous part. The optimization Eq. (**8**) is a standard convex problem that can be solved in a straightforward manner by using the Lagrange multiplier method [13]. Lagrangian basic idea converts inequality optimization problem to a weighted total objective function through a certain number of Lagrange multipliers, then use the extreme method to solve.

Combining with the Karush-Kuhn-Tucker (KKT) conditions [14], the paper constructs the Lagrangian function of the formula (**8**) by using the convex optimization theory and categorizes the same item on the right side of the equation, the function is simplified as

$$\begin{aligned} & L\left(\{\lambda_i\} \quad \{\mu_n\} \quad \{v_i\} \quad \{p_k^i\}\right) \\ & = \sum_{i=1}^{M}\left[\sum_{k=1}^{K}\left(-1+v_i\frac{r_1}{r_i}\right)\log\left(1+\frac{p_k^i g_k^{ii}}{\sum\limits_{j\neq i} p_k^j g_k^{ji}+I_k^{ps}+\sigma_k^i}\right)\right. \\ & \left.\quad+\sum_{k=1}^{K}\lambda_i p_k^i+\sum_{k=1}^{K}\sum_{n=1}^{N}\mu_n p_k^i h_k^i\right]-\sum_{i=1}^{M}\lambda_i p_{\max}^i-\sum_{n=1}^{N}\mu_n\frac{I^{MTI}}{K*M} \end{aligned} \tag{9}$$

where $\{\lambda_i\}$, $\{\mu_n\}$, $\{v_i\}$ are the Lagrangian coefficients of constraints $C1$, $C2'$, $C3$.

Using the Lagrangian dual decomposition method, the Eq. (9) is transformed into the minimum value problem of principal variables p_k^i and the dual optimization problem with the Lagrangian multiplier as the optimal variable, which as follows

$$\begin{aligned} & D(\{\lambda_i\}\{\mu_n\}\{v_i\}) \\ & =\sum_{i=1}^{M}\min L_i\left(p_k^i,\lambda_i,\mu_n,v_i\right) = -\sum_{i=1}^{M}\lambda_i p_{\max}^i-\sum_{n=1}^{N}\mu_n\frac{I^{MTI}}{K*M} \end{aligned} \tag{10}$$

where

$$L_i\left(p_k^i,\ \lambda_i,\ \mu_n,\ v_i\right) = \sum_{k=1}^{K}\left(-1 + v_i\frac{r_1}{r_i}\right)\log\left(1 + \frac{p_k^i g_k^{ii}}{\sum\limits_{j\neq i} p_k^j g_k^{ji} + I_k^{ps} + \sigma_k^i}\right) + \sum_{k=1}^{K}\lambda_i p_k^i + \sum_{k=1}^{K}\sum_{n=1}^{N}\mu_n p_k^i h_k^i \tag{11}$$

Using to the KKT condition, the optimal power control solution of the cognitive user can be solved by the following equation

$$\frac{\partial L_i(p_i,\ \lambda_i,\ \mu_n,\ v_i)}{\partial p_k^i} = 0 \tag{12}$$

The optimal power solution can be obtained as

$$p_k^{i^*} = \left[\frac{1 - v_i\frac{r_1}{r_i}}{\ln 2\left(\lambda_i + \sum\limits_{n=1}^{N}\mu_n h_k^i\right)} - \frac{\sum\limits_{j\neq i} p_k^j g_k^{ji} + I_k^{ps} + \sigma_k^i}{g_k^{ii}}\right]^+ \tag{13}$$

where, $[X]^+ = \max\{0,\ X\}$ denotes a projection in a non-negative quadrant.

The Lagrange multipliers can be updated by sub-gradient search method [15] in a parallel way as follows

$$\lambda_i^{t+1} = \left[\lambda_i^t + a_1\left(\sum_{k=1}^{K} p_k^i - p_{\max}^i\right)\right]^+ \tag{14}$$

$$\mu_n^{t+1} = \left[\mu_n^t + a_2\left(p_k^i h_k^i - \frac{I^{MTI}}{K * M}\right)\right]^+ \tag{15}$$

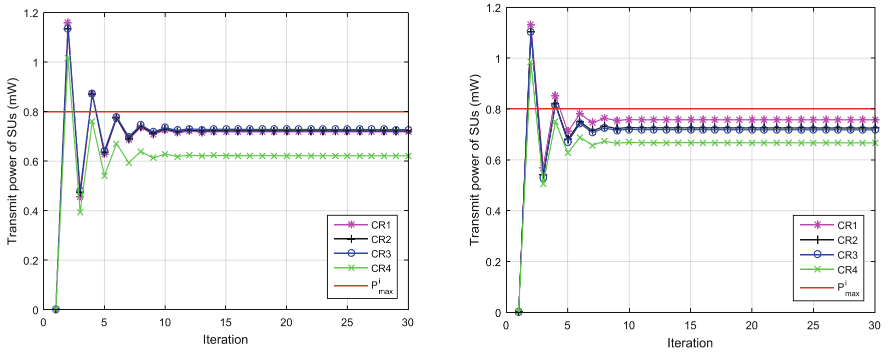
$$v_i^{t+1} = \left[v_i^t + a_3\frac{r_1}{r_i}\log\left(1 + \frac{p_k^i g_k^{ii}}{\sum\limits_{j\neq i} p_k^j g_k^{ji} + I_k^{ps} + \sigma_k^i}\right)\right]^+ \tag{16}$$

where t is the iteration number, $a_1,\ a_2,\ a_3$ denote the positive step sizes. By choosing the appropriate step size, the stability and convergence of the dual algorithm can be guaranteed. As the gradient method can converge to the dual optimal solution $(\lambda^*,\ \mu^*,\ v^*)$, thus ensures the convergence of the optimal transmission power $p_k^{i^*}$. Note that $a_1,\ a_2,\ a_3$ are sufficiently small, $(\lambda^*,\ \mu^*,\ v^*)$ can converge to the optimal point $\left(\lambda_i^*,\ \mu_n^*,\ v_i^*\right)$, when $t \to \infty$.

4 Simulations and Discussions

In this section, we present some numerical results to illustrate the performance of our proposed algorithms. We consider the OFDM system with 4 SU pairs ($M = 4$) and 1 PU pair, the number of available subcarriers for the CR network is 2 ($k = 2$). As Ref. [15], the nominal values of g_{ii}, g_{ij}, and h_i are randomly chosen from the intervals [0.7, 1], [0, 0.3], and [0, 0.2]. The maximum transmission power of each SU is $p^i_{\max} = 0.8\,\text{mW}$. The interference of PU to cognitive user M is $I_k^{ps} = 0.93\,\text{mW}$. The background noise power of every subcarrier is 0.003 mW.

Figure 1 depicts the transmit power for each user at two subcarriers. We determine the transmit power of each SU according to the maximum transmitting power of the SU i. It can be seen that the curves tend to be stable point near the 13^{th} iteration and the total power of each cognitive user does not exceed the maximum total power specified.



(a) Transmit power for SUs at subcarrier 1 (b) Transmit power for SUs at subcarrier 2

Fig. 1. Transmit power of each cognitive user at two subcarriers

Figure 2 describes the relationship between the data rate and the number of iterations. It shows the curves of the total rate of each cognitive user. The purpose of this paper is to allocate resources to users based on proportional fairness and to bring all of these users to the similar data transmission rate. From Fig. 2, the data rate continuously improves in the initial stage, followed into a horizontal state by the fifth iteration. When $r_1{:}r_2{:}r_3{:}r_4 = 1{:}1{:}1{:}1$, it indicates that the cognitive user meets the minimum proportional rate requirement and the reachable rate between cognitive users is the same, all users at the same rate to achieve the most fairness.

Figure 3 shows the capacity distribution among cognitive users. It can be seen that the capacity of each cognitive user is uneven under non-proportional constraint, however, each cognitive user get the same capacity level to transmit when the rate ratio is 1:1:1:1, this reflects the fairness that each cognitive user gets the same opportunity to transmit under the premise of considering the proportional rate constraint of the cognitive user.

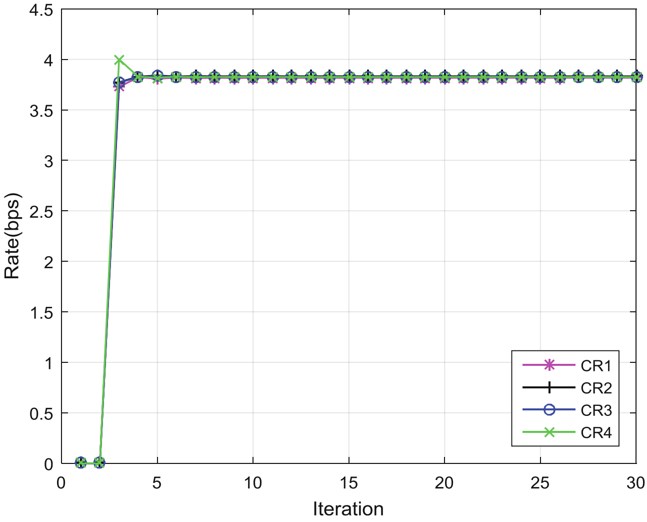


Fig. 2. The rate of each cognitive user.

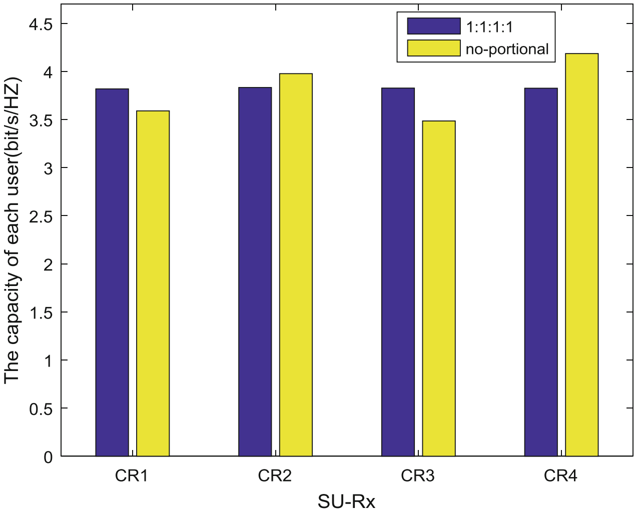


Fig. 3. The capacity distribution among cognitive users

Figure 4 describes the interference power at the PU receiver. With the increase of iterations, the interference power from SUs has increased dramatically at the beginning. When the transmission power reaches at Nash equilibrium point state, the interference power at PU receiver is stable.

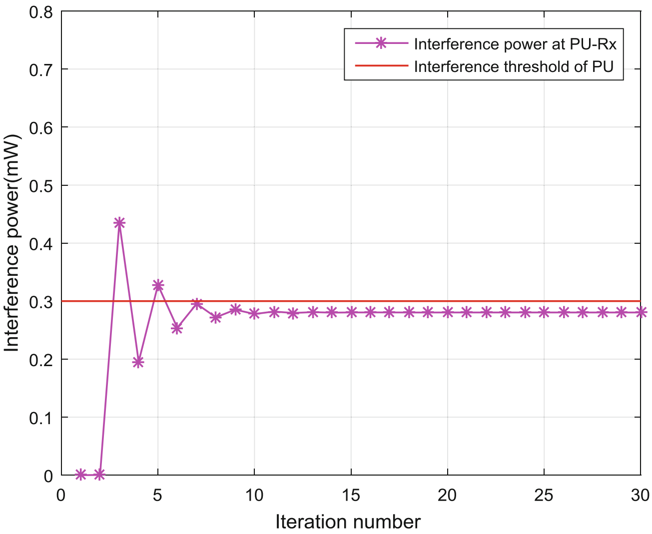


Fig. 4. The relationship of the interference power and the number of iterations.

5 Conclusion

In this paper, we have studied the problem of maximize the total channel rate of SUs and power allocation under the SU total transmit power constraint and PU interference temperature constraint in OFDM-based CR networks. Using the proportional fairness as the constraint, we can improve proportional fairness of resource allocation and achieve substantial transmitted data rate gain. We transform non-convex optimization objective function into convex form, and use the Lagrange decomposition method and sub-gradient method to derive the subcarrier allocation. It is shown by simulation results that the transmission power of SUs can quickly reach the optimal point and the data rate of each SU is almost equal, the interference introduced to the PU bands below interference power threshold, the computational complexity of this paper is relatively lower.

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