An Adaptive Step-Size Prediction Joint OMP Algorithm for Beam Tracking in Millimeter Wave Systems

Xu Yang[1], Xiaohui Li[1(✉)], Bin Zhou[2], Yanbin Zhao[3], and Ruiyang Yuan[1]

[1] State Key Laboratory of Integrated Service Networks,
Xidian University, Xi'an 710071, Shaanxi, China
xhli@mail.xidian.edu.cn

[2] AVIC Computing Technique Research Institute, Xi'an, Shaanxi, China

[3] Geographic Information Center, Yangquan, Shanxi, China

Abstract. For the millimeter wave (mmWave) time-varying system, beamforming poses a formidable challenge for huge power overhead. In this paper, an adaptive step-size prediction joint orthogonal matching pursuit (OMP) algorithm (ASP-OMP), is proposed for beam tracking. With the direction of arrival (DOA) information obtained by OMP algorithm, kalman filter is used to predict the beamforming vector during N time slots to avoid huge sampling loss. In addition, we propose to determine the prediction step-size with outlier identification and eliminate the beam tracking error through data fusion based on Bayesian estimation. The simulation results demonstrate that the proposed algorithm can converge quickly and have a good performance for mmWave beam tracking.

Keywords: Millimeter wave time-varying system · OMP algorithm
Beam tracking · Outlier identification · Bayesian estimation

1 Introduction

In the 5th Generation mobile communication (5G) technology standard, 60 GHz millimeter wave wireless communication technology has become a research hotspot. Millimeter wave multi-input multi-output (MIMO) systems can provide throughput for future communication systems to meet the expected requirements of mobile data [1,2]. Aiming at reducing the huge path loss in millimeter wave communication, beamforming technology is introduced in the IEEE 802.11ad standard [3], which can improve the range and quality of the communication by directionally enhancing signal and control the signal propagation direction.

In order to conduct adaptive beamforming, channel estimation is required generally for that optimal singular value decomposition (SVD) beamforming. Due to the beam training overhead, the compressive sensing (CS) theory is

B. Li et al. (Eds.): ChinaCom 2017, LNICST 237, pp. 227–238, 2018.
https://doi.org/10.1007/978-3-319-78139-6_24

adopted for the channel estimation [4]. [5] proposed a novel compressed SNR-and-channel estimation algorithm, which can significantly improve the estimation performance over the scheme in IEEE 802.11ad with reducing the pilot overhead of beam tracking. While these compression-based channel estimation algorithms further reduce the overhead, it is still a huge challenge for multiple estimations in the 60 GHz millimeter wave time-varying system.

The general solution is to use the uplink information to carry out DOA estimation, indicating the direction of the user. The current research aims to reduce the overhead and improve the estimation accuracy. The algorithm proposed in [6] suggests that DOA can be determined by beam scanning and the fast beam tracking can be realized through wide beam training and narrow beam alignment with the layered codebook and the antenna pattern [7–9]. However, this algorithm can only enable one data stream and face the outdated risks. Aiming at further improving spectral efficiency, [10] proposed to adopt the OMP algorithm to estimate the AoAs and AoDs of multi-paths, which can be used to construct the beamforming vector. Although this method can provide high estimation accuracy and support multi-stream transmission, it has to face with the unavoidable problem of high iteration complexity and sampling loss with the OMP algorithm.

In this paper, the ASP-OMP algorithm is proposed to achieve the fast beam tracking with the partial information of channel, which can meet the channel time-varying characteristic. Compared to the previous works, the complexity and validity of the proposed algorithm are both considered. Moreover, to carry out beam tracking in time-varying channel, kalman filter is used to predict the beamforming vector. For keeping the tracking accuracy of OMP algorithm and reducing the tracking complexity and power loss, this paper sets the error threshold through the outlier identification method, which can adaptively determine the prediction step-size to achieve fast beam alignment. In addition, it is Bayesian estimation that is used to eliminate the effect of outliers on the tracking accuracy. The theoretical analysis and simulation results demonstrate that the proposed algorithm can provide a close performance to the existing algorithms with low complexity.

The remainder of paper is organized as follows. We first provide the preliminary background and present the system model and problem formulation in Sect. 2 and summarize the proposed tracking algorithm based on adaptive stride prediction with kalman filter in Sect. 3. Section 4 presents simulation results. Conclusions are presented in Sect. 5.

2 System Model

We consider a single user massive MIMO mmWave time-varying system, where a base station (BS) is equipped with N_t antennas and a mobile station (MS) is equipped with N_r antennas. For the initial beam alignment, a beamforming scheme based on OMP algorithm is considered in [10]. The system model is as illustrated in Fig. 1. In order to simplify the model, it is assumed that the transmitter uses an antenna array for beamforming and the receiver uses an omnidirectional antenna. The downlink beamforming in 60 GHz millimeter wave system is considered in this paper with the BS tracking the MS. The transmit signal is expressed as s and the receive signal can be expressed as:

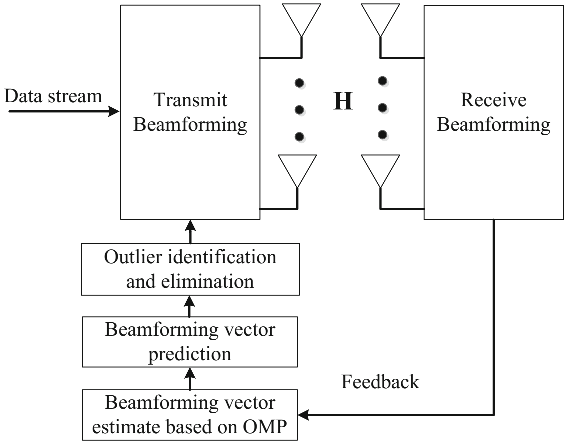

Fig. 1. System model

$$\mathbf{y} = \mathbf{hws} + \mathbf{n} \tag{1}$$

where $\mathbf{w}$ is the beamforming vector with the constraint $\|\mathbf{w}\| = 1$. The additive white Gaussian noise $\mathbf{n}$ is assumed to be i.i.d complex Gaussian with unit variance, i.e. $\mathbf{n} \sim CN(0, 1)$. Due to the high path loss characteristics of millimeter wave signals, millimeter wave channels are expected to exhibit limited spatial scattering. Millimeter wave channel $\mathbf{h}$ can be expressed as:

$$\mathbf{h} = \sqrt{\frac{1}{L}} \sum_{l=1}^{L} \alpha_l a_r(\theta_l) {a_t}^H(\phi_l) \tag{2}$$

where L denotes the number of transmission path, α_l represents the complex gain of the l_{th} path. θ_l and ϕ_l denote the AoA and AoD of the l_{th} path, respectively. $a_r(\theta_l)$ and $a_t(\phi_l)$ represent the receiver and transmitter array vector, respectively.

Millimeter wave beam tracking is to determine the millimeter wave beam direction between the BS and the MS to minimize energy loss and maximize received signal to noise ratio (SNR). The number of transmitted data stream is L_s. And the SNR of the i_{th} data stream can be calculated by:

$$SNR_i = \frac{1}{\sigma_n^2[(\|\mathbf{hw}\|^2 + \sigma_n^2 \mathbf{I}_{L_s})^{-1}]_i} - 1 \tag{3}$$

It is well known that the optimal beamforming matrix $\mathbf{w}$ maximizing SNR_i is determined by the SVD of the channel matrix $\mathbf{h}$. It is Assumed that the SVD of $\mathbf{h}$ bear the form below:

$$\mathbf{h} = \mathbf{U}\Lambda\mathbf{V}^H \tag{4}$$

where $\mathbf{U} = [\alpha_r(\phi_1), \alpha_r(\phi_2), \cdots, \alpha_r(\phi_L)]$ denotes a unitary matrix with the size of $N_r \times L$, Λ is a $L \times L$ diagonal matrix. $\mathbf{V} = [\alpha_t(\theta_1), \alpha_t(\theta_2), \cdots, \alpha_t(\theta_L)]$ denotes a $N_t \times L$ unitary matrix. Each array propagation vector in $\mathbf{V}$ can be expressed as:

$$\alpha_t(\theta_k) = \frac{1}{\sqrt{N_t}}\left[1, e^{j\frac{2\pi}{\lambda}d_t \sin\theta_k}, \cdots, e^{j\frac{2\pi}{\lambda}(N_t-1)d_t \sin\theta_k}\right] \tag{5}$$

where d_t is the antenna element spacing for the TX antenna array, and λ is the wavelength at the carrier frequency.

In order to track the channel change and reduce the energy loss, the adaptive beamforming vector can be calculated by:

$$\begin{aligned} \mathbf{w}_{opt}(i) &= \arg\underset{\mathbf{w}}{max}\, SNR_i \\ &= \arg\min_{\mathbf{w}} \sigma_n^2[(\|\mathbf{hw}\|^2 + \sigma_n^2\mathbf{I}_{L_s})^{-1}]_i \\ &= \arg\underset{\mathbf{w}}{max}\, \|\mathbf{hw}_i\|^2 \end{aligned} \tag{6}$$

When $\mathbf{w}_{opt} = \mathbf{V}_i$, $\|\mathbf{hw}_i\|^2$ reaches the maximal value theoretically and it equals to Λ^2. Thus, $\mathbf{w}_{opt}$ can be derived as:

$$\begin{aligned} \mathbf{w}_{opt}(i) &= \arg\max_{\mathbf{w}\in\mathbf{V}_i} \|\mathbf{hw}\|^2 \\ &= \arg\max_{\alpha_t(\theta_k)} \|\mathbf{h}\alpha_t(\theta_k)\|^2 \end{aligned} \tag{7}$$

The millimeter-wave beam tracking problem is transformed into the problem of determining the millimeter-wave transmitter array vector.

3 Beam Tracking Algorithm

According to the analysis above, it can be seen that the beam tracking only need to predict beamforming vector maximizing SNR. We can consider that the change of the DOA in the polar coordinates is in a linear state and we can introduce the kalman filter method for beam tracking.

Beamforming vector can be calculated through Eq. (5) with the DOA information obtained by OMP algorithm. The detailed steps of the OMP algorithm is given in [10]. The beamforming complexity based on the OMP algorithm is $O\left({N_t}^3\right)$ for one iteration. To avoid a large number of matrix calculation involved in the beam tracking process with OMP algorithm, this paper proposes to predict the beamforming vector of the time $n+N$ by that of the time n. Thus, the beamforming complexity of the proposed method is $O\left({N_t}^3/N\right)$ and the sampling times are reduced by N times.

Adaptive step-size prediction joint OMP algorithm: In order to reduce the complexity of the beam tracking with small tracking error, the adaptive step-size prediction joint OMP tracking algorithm is summarized as follows. The beam direction is kept constant within the prediction step, which reduces the complexity of beam tracking. For detail, the ASP-OMP algorithm can be concluded in three steps.

Step 1: Initialize DOA estimation$\{\theta_l\}$ based OMP algorithm and construct the beamforming vector $\mathbf{w}$ according to the Eq. (5);

Step 2: Adaptive step-size prediction based on multi-step kalman filter. The heuristic algorithm is used to determine the predicted step-size, that is, by comparing the prediction error with the threshold δ, the prediction step-size is determined to achieve fast millimeter beam alignment.

Step 3: Outliers elimination based on Bayesian estimation. Through the fusion of the predicted beamforming vector and the measured beamforming vector, the effects of excessive prediction errors and measured errors are eliminated.

Prediction error threshold determination: The error threshold δ can be set up through outlier identification, the derivation is as follows.

Kalman filter measurement residuals is calculated as:

$$\mathbf{e}_n = Z_n - \mathbf{H}\mathbf{w}_{n|n-N} \tag{8}$$

where $\mathbf{w}_{n|n-N}$ is a $N_t \times 1$ dimensional vector, representing the predicted beamforming vector. Z_n denotes a vector with the size of $N_t \times 1$, which represents the measured value, $\mathbf{H}$ is the observation matrix with the size of $N_t \times N_t$. $\mathbf{e}_n$ obeys the Gauss random distribution with mean value of 0, its covariance is expressed as:

$$\begin{aligned} & E\left(\mathbf{e}_n \cdot \mathbf{e}_n{}^T\right) \\ & = E\left\{\left(Z_n - \mathbf{H}\mathbf{w}_{n|n-N}\right)\left(Z_n - \mathbf{H}\mathbf{w}_{n|n-N}\right)^T\right\} \\ & = E\left\{\left|\mathbf{H}\mathbf{w}_n + \mathbf{g}_n - \mathbf{H}\mathbf{w}_{n|n-N}\right|^2\right\} \\ & = \mathbf{H}\mathbf{P}_{n|n-N}\mathbf{H}^T + R \end{aligned} \tag{9}$$

where $\mathbf{g}_n$ is a $N_t \times 1$ dimensional vector, which represents the observation noise vector, and R donates the variance of the observation noise. The outlier in adaptive step-size prediction process is discriminated according to the Eq. (9):

$$\|\mathbf{e}_{n+N}\| \le r \cdot \left\| \sqrt{diag_i\left(\mathbf{H}\mathbf{P}_{n+N|n}\mathbf{H}^T + R\right)} \right\| \tag{10}$$

where r is the outlier identification coefficient. $diag_i(\cdot)$ denotes the vector composed of the diagonal elements about beamforming vector in the matrix.

If Eq. (10) is satisfied, then Z_n is the normal observation value. Otherwise, it is assumed that Z_n is the outlier. The value of r is determined by the actual channel. The error threshold can be set according to the method of the outlier identification.

Algorithm 1. ASP-OMP algorithm for beam tracking

1. Initialize the prediction step-size $N = 1$;
2. The possible azimuth set is established by discrete sampling: $\{\theta_K\}$;
3. Initialize DOA estimation based OMP algorithm : $\{\theta_l\}$;
4. Given $\mathbf{F}, \mathbf{H}, \mathbf{Q}, \mathbf{R}, \mathbf{P_0}$;
5. Construct the beamforming vector according to the Eq. (5):$\mathbf{w}$;
6. **If** $e_N \leq \delta$ then
7. Update prediction step-size: $N = N + 1$;
8. **Else**
9. Update prediction step-size: $N = 1$;
10. **End**
11. **For** $i \leq N$
12. $\mathbf{w}_{n+i|n} = \mathbf{F}^{[N/2]}\mathbf{w}_n$;
13. **End**
14. Outlier elimination and update the prediction value $\mathbf{w}_{n+N|n}$;
15. Update the measured residuals: $e_{n+N} = \left\| Z_{n+N} - \mathbf{H}\mathbf{w}_{n+N|n} \right\|$;
16. Calculate kalman gain: $K_{n+N} = \frac{\mathbf{P}_{n+N|n} \cdot \mathbf{H}^T}{\mathbf{H} \cdot \mathbf{P}_{n+N|n} \cdot \mathbf{H}^T + \mathbf{R}_n}$;
17. State correction: $\mathbf{w}_{n+N} = \mathbf{w}_{n+N|n} + K_{n+N} \cdot e_{n+N}$;
18. Correct error covariance: $\mathbf{P}_{n+N} = (\mathbf{I} - K_{n+N} \cdot \mathbf{H}) \cdot \mathbf{P}_{n+N|n}$;
19. The azimuth set is established by discrete sampling: $\{\theta_{3l}\}$;
20. Construct the beamforming vector matric Ψ_θ;
21. DOA estimation based OMP algorithm:$\{\theta_l\} = \arg\min \|y - \Psi_\theta \hat{x}\|, \hat{x} = \left({\Psi_\theta}^T \Psi_\theta\right)^{-1} {\Psi_\theta}^T y$;
22. Go to 5;

$$\delta = r \cdot \left\| \sqrt{diag_i \left(\mathbf{H}\mathbf{P}_{n+N|n}\mathbf{H}^T + R\right)} \right\| \tag{11}$$

Error judgment is based on beamforming vector obtained through OMP algorithm in BS. By comparing the prediction error and the threshold, the prediction step-size is determined adaptively.

Outlier elimination based on Bayesian estimation: Since that the parameters of the beam tracking algorithm are Gauss normal distribution, the predicted beamforming vector $\mathbf{w}_{n|n-N}$ and the measured vector Z_n, which are recorded as x_1, x_2, can be fused based on Bayesian estimation to predict the new beamforming vector to eliminate the influence of the excessive prediction errors and measured errors.

$$P\left(\mu|x_1, x_2\right) = \frac{P\left(\mu; x_1, x_2\right)}{P\left(x_1, x_2\right)} \tag{12}$$

where $\mu \sim N\left(\mu_0, {\sigma_0}^2\right)$ and $x_k \sim N\left(\mu, {\sigma_k}^2\right), k = 1, 2$. For the adaptive step-size prediction beam tracking algorithm proposed in this paper, the parameters satisfy that $\mu_0 = \frac{1}{2}\sum_{k=1}^{2} x_k$, ${\sigma_0}^2 = \frac{1}{2}\sum_{k=1}^{2} {\sigma_k}^2$ and ${\sigma_k}^2 = \begin{cases} \mathbf{P}_{n|n-N}, & k=1 \\ \mathbf{P}_n, & k=2 \end{cases}$. We assume that $\alpha = \frac{1}{P(x_1,x_2)}$, which is independent of the μ. Equation (11) can be rewritten as:

$$\begin{aligned} &P\left(\mu|x_1, x_2\right) \\ &= \alpha \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left\{-\frac{1}{2}\left(\frac{\mu-\mu_0}{\sigma_0}\right)^2\right\} \prod_{k=1}^{2} \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left\{-\frac{1}{2}\left(\frac{x_k-\mu}{\sigma_k}\right)^2\right\} \\ &= \alpha \frac{1}{\sqrt{2\pi}\sigma_0} \prod_{k=1}^{2} \frac{1}{\sqrt{2\pi}\sigma_k} \exp\left\{-\frac{1}{2}\left(\frac{\mu-\mu_0}{\sigma_0}\right)^2 - \frac{1}{2}\sum_{k=1}^{2}\left(\frac{x_k-\mu}{\sigma_k}\right)^2\right\} \end{aligned} \tag{13}$$

The exponential part in (12) is a quadratic function of μ, so Eq. (12) can be rewritten as follows in the assumption that $\left(\mu|x_1, x_2\right) \sim N\left(\mu_n, {\sigma_n}^2\right)$.

$$P\left(\mu|x_1, x_2\right) = \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left\{-\frac{1}{2}\left(\frac{\mu-\mu_n}{\sigma_n}\right)^2\right\} \tag{14}$$

Equation (15) can be obtained in comparison of Eqs. (12) and (13):

$$\begin{cases} \sum_{k=1}^{2} \frac{\mu^2}{{\sigma_k}^2} + \frac{\mu^2}{{\sigma_0}^2} = \frac{\mu^2}{{\sigma_N}^2} \\ \sum_{k=1}^{2} \frac{\mu x_k}{\sigma_k} + \frac{\mu\mu_0}{\sigma_0} = \frac{\mu\mu_N}{\sigma_N} \end{cases} \tag{15}$$

The solution of the Eq. (15) is shown as:

$$\begin{cases} \sigma_N = \frac{1}{\sqrt{\sum_{k=1}^{2} \frac{1}{{\sigma_k}^2} + \frac{1}{{\sigma_0}^2}}} \\ \mu_N = \frac{\sum_{k=1}^{2} \frac{x_k}{\sigma_k} + \frac{\mu_0}{\sigma_0}}{\sqrt{\sum_{k=1}^{2} \frac{1}{{\sigma_k}^2} + \frac{1}{{\sigma_0}^2}}} \end{cases} \tag{16}$$

The Bayesian estimation of μ can be calculated as:

$$\begin{aligned}\hat{\mu} &= \int\limits_{\Omega} \mu \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left\{-\frac{1}{2}\left(\frac{\mu-\mu_n}{\sigma_n}\right)^2\right\} d\mu \\ &= \mu_N = \frac{\sum\limits_{k=1}^{2} \frac{x_k}{\sigma_k} + \frac{\mu_0}{\sigma_0}}{\sqrt{\sum\limits_{k=1}^{2} \frac{1}{\sigma_k{}^2} + \frac{1}{\sigma_0{}^2}}}\end{aligned} \tag{17}$$

The predicted beamforming vector after eliminating outliers can be calculated as:

$$\mathbf{w}_{n|n-N} = \hat{\mu} \tag{18}$$

With the outlier recognition and elimination, we can set up reasonable prediction step-size and eliminate the influence of the outlier in the beam tracking process at the same time, solving the problem of the large prediction error caused by the various types of sudden errors and avoiding the tracking divergence.

4 Simulation Results

Here we give some performance comparison with the simulation results of several beam tracking algorithms, the simulation compared the proposed beam tracking algorithm with that in [7,10] in the millimeter wave channel. The tracking performance of several algorithms is compared for 30 consecutive beam tracking cycles in the case of $N_t = 32$ and $N_r = 32$, user's speed is 5 m/s and the initial distance between BS and MS is 50 m.

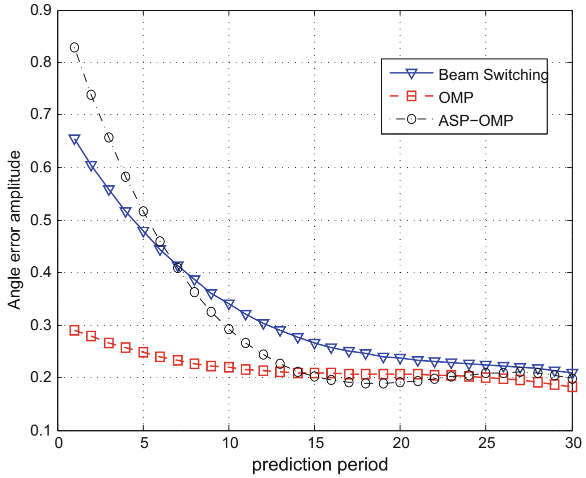

Fig. 2. Beam tracking angle error amplitude

To verify the effectiveness of the beam tracking algorithm proposed in this paper, Fig. 2 shows the average error amplitude curve of the DOA based on seval beam tracking algorithms. When the tracking time is short, it can obviously be seen from the curve that the tracking error of the algorithm proposed is slightly higher than that based on OMP algorithm proposed in [10] and is almost the same as that proposed in [7], which is based on beam switching algorithm with the wide beam training and the narrow beam alignment. As time goes on, the tracking performance of the ASP-OMP algorithm gradually approaches the OMP algorithm and is better than the beam switching algorithm. The tracking error of all three beam tracking algorithm is within the acceptable range.

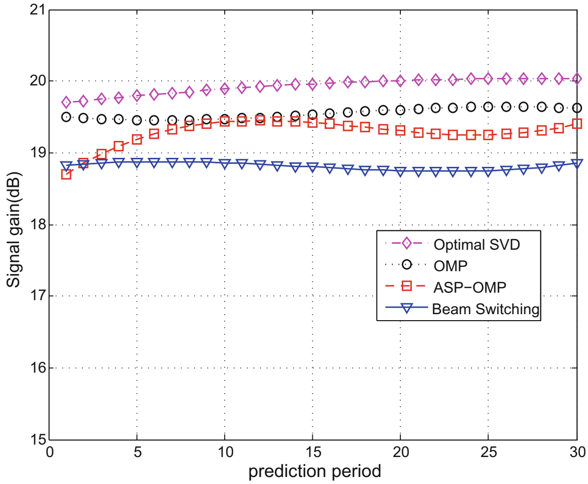

Fig. 3. Received signal gain of different beam tracking algorithms

Figure 3 shows the receiver gain curve for the beam tracking algorithm based on adaptive step-size prediction with kalman filter. It obviously can be seen that the performance of the proposed ASP-OMP algorithm is close to that of the OMP algorithm in the literature [10], which is better than that of the [7] based on beam switching algorithm. Compared with the beamforming method based on the optimal SVD, the receiver gain of the proposed method is reduced by about 0.6 dB. The complexity of the proposed ASP-OMP algorithm is greatly reduced with little effect on the communication.

Figure 4 shows the spectral frequency of different beam tracking methods under different transmission streams. It obviously can be seen that the spectral frequency of the proposed ASP-OMP algorithm is close to that of the OMP algorithm in both single and multiple streams. Due to the absence of the channel estimation, the proposed ASP-OMP algorithm and the OMP algorithm in [10] cannot provide a high spectral frequency as same as the optimal SVD beamforming method. This performance gap between the optimal SVD beamforming

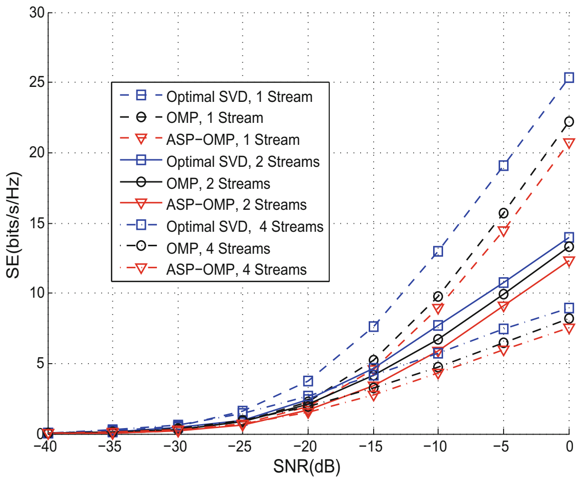

Fig. 4. Spectral frequency of different beam tracking algorithms

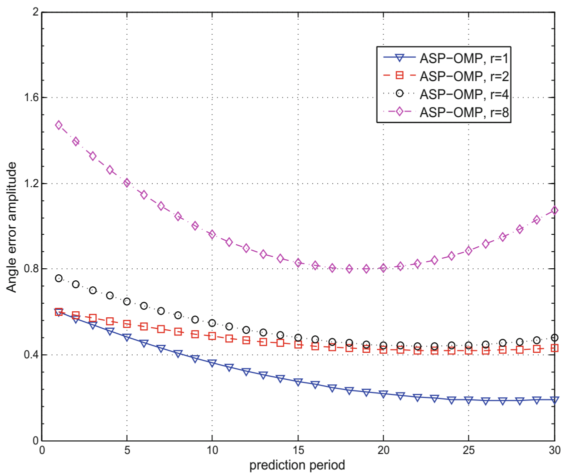

Fig. 5. Beam tracking angle error amplitude under different outlier identification coefficients

method and the proposed ASP-OMP algorithm will become increasingly large as the number of the transmission stream increases, which is a inevitable result of the decline in complexity.

Figure 5 shows the receiver gain curves under different outlier identification coefficients. When $r = 1$, which means that the threshold is low, the tracking error is small with the small prediction step-size. The tracking error gradually converges to the same value in case of that $r = 2$ and $r = 4$, which means that r should be in the range of 2 to 4 to predict the relatively accurate beamforming vector. The curve gradually diverges due to the existence of outliers when $r = 8$. In this case, The outlier identification method is out of action due to that the error threshold is too large. Obviously, the selection of the outlier identification coefficient has a severe effect on the beam tracking accuracy.

All in all, the results point out that the proposed ASP-OMP algorithm can be adapted to the change of the channel and has a close tracking performance to the existing beam tracking algorithm with the algorithm complexity decreasing.

5 Conclusions

In this paper, an adaptive beam tracking algorithm is proposed for the mobile millimeter wave communication system. For the complexity of downlink beam tracking and the requirement of the coherent communication, the proposed ASP-OMP algorithm uses the kalman filter to carry out the adaptive step-size prediction and beam tracking with the DOA information obtained by OMP algorithm. By setting the error threshold with the outlier identification, the prediction step-size is determined adaptively.

The optimal beam is kept constant within the prediction step, which reduces the complexity of beam tracking. In practice, it only needs to predict the optimal beamforming vector in advance before the communication link suffers from significant fading. The beam tracking method proposed in this paper can be adapted to the change of the channel and the mobility of the users, which reduces the complexity of the traditional beam tracking algorithm and helps to reduce the path loss of millimeter wave communication. Finally, the theoretical analysis and simulation results demonstrate that the proposed algorithm has no apparent performance loss with the complexity being significantly reduced.

Acknowledgment. This work was supported by the State Major Science and Technique Project (MJ-2014-S-37), National Natural Science Foundation of China (61201134) and 111 Project (B08038).

References

1. Zhu, D., Choi, J., Heath, R.W.: Auxiliary beam pair enabled aod and aoa estimation in closed-loop large-scale mmwave mimo system. IEEE Trans. Wirel. Commun. **16**, 4770–4785 (2017)
2. Pu, W., Li, X., Lin, Y., et al.: Low complexity hybrid beamforming based on orthogonal constraint and phase extraction. In: Wireless Communications and Networking Conference (WCNC), pp. 1–5 (2017)
3. Yaman, Y., Spasojevic, P.: Reducing the LOS ray beamforming setup time for IEEE 802.11 ad and IEEE 802.15. 3c. In: Military Communications Conference (MILCOM) (2016)
4. Han, Y., Lee, J.: Asymmetric channel estimation for multi-user millimeter wave communications. In: International Conference on Information and Communication Technology Convergence (ICTC), pp. 4–6 (2016)

5. Bo, G., Changming, Z., Depeng, J., et al.: Compressed SNR-and-channel estimation for beam tracking in 60-GHz WLAN. Chin. Commun. **12**(6), 46–58 (2015)
6. Wang, Y., Shi, Z., Zeng, K., et al.: Two layers of beam alignment for millimeter-wave communications. In: 2016 Third International Conference on Digital Information Processing, Data Mining, and Wireless Communications (DIPDMWC), pp. 262–267 (2016)
7. Xiao, Z., Xia, P., Xia, X.G.: Codebook design for millimeter-wave channel estimation with hybrid precoding structure. IEEE Trans. Wirel. Commun. **16**(1), 141–153 (2017)
8. Gao, K., Cai, M., Nie, D., et al.: Beampattern-based tracking for millimeter wave communication systems. In: Global Communications Conference (GLOBECOM), pp. 1–6 (2016)
9. Thejaswi, P.S.C., Bynam, K., Nair, J.P., et al.: Low-complexity direction estimation techniques for fast beamforming training in millimeter wave multiantenna systems. In: IEEE International Conference on Communications (ICC), pp. 1–6 (2016)
10. Xiaohui, L., Meimei, M., Yingchao, L., et al.: Low complexity asymptotically unitary algorithm for hybrid beamforming in mmWave communication systems. J. Chin. Univ. Posts Telecommun. **24**(1), 18–25 (2017)