Optimal ZF Precoder Under per Antenna Power with Conjugate Beamforming for MU Massive MIMO Systems

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Abstract. In this paper, we deliberate on multiuser massive multiple-input multiple-output (MU-MIMO) system in designing optimal zero forcing (ZF) precoder under per antenna power constraint. MU massive MIMO with non-square matrix is restrained by the large channel matrix dimension, conjugate beamforming maximization approach is developed to align the channel matrix for the optimal ZF precoder. We further introduced complex lattice reduction (CLR) to transform the lattice bases of the channel matrix and shorten the basis vector, thus meliorates the orthogonality of the conjugate beamforming. Simulation results show LR-based optimal ZF precoder outperforms other precoding schemas. The LR-based optimal ZF precoder improved the beamforming for the base station (BS) to focus on the users, thus improving spatial multiplexing gain and diversity order. As BS antennas and users turn large, the sum rate over the subchannels depends on the dominance of users (that is BS antennas to user antennas ratio) for the channel gain. Thus performance of the LR-based precoder schema under per antenna power can help save power in practical massive MIMO implementation.

Keywords: MU massive MIMO · Zero forcing (ZF) precoder
Conjugate beamforming · Lattice reduction (LR) · Per antenna power

1 Introduction

Multiuser massive MIMO system is an emerging technology, the system have spatial multiplexing and diversity gains as distinct pair of channel vectors turn orthogonal as number of antennas increase [1,2]. However, the overall performance of MU massive MIMO requires efficient multi-user interference (MUI) elimination, hence transmit precoding is a strategy to study. Linear precoder such as zero forcing (ZF) can search domains of MU MIMO transmission over entire nullspace (nulling the space is a conventional method for interference elimination) of other users [3,4]. In this paper, ZF precoder is designed to search domain of MU MIMO transmission over entire null of other users with block

B. Li et al. (Eds.): ChinaCom 2017, LNICST 237, pp. 189–198, 2018.
https://doi.org/10.1007/978-3-319-78139-6_20

diagonalization (BD). In [3,5] studied (BD) transmissions, as each user is set to the entire null space of other adjacent users, thus parallels the user subchannels, this however does not involve optimization over the subchannels. Therefore sum rate of optimal ZF precoder with BD is maximized under two conditions: firstly by transmitting on the right eigenchannel (set of parallel non-interfering subchannels) and secondly by power allocations on each non-interfering subchannel through optimization [6–8]. In [5,9] studied square and non square channel matrices respectively under sum power constraint. In this paper, we consider a system with large non-square channel matrix where the BS antennas M are more than the combined user antennas K and users N (i.e. $M \geq NK$), we analyze the user selection with the precoder with conjugate beamform vector in the downlink. Furthermore we extend this work to investigate the non-square matrix under per antenna power constraint. Per antenna power constraint (diagonal operations) is a novel power allocations approach for achieving massive MIMO performance. In [10], the sum rate with BD under per-antenna power constraint is suggested to be less than sum power constraint, to resolve this sum rate limitations, we propose a solution that bounds (orthogonal) the lattice size of the transmit beamforming vectors [11] under per-antenna power constraint. Lattice reduction (LR) using the complex Lenstra, Lenstra and Lovasz (CLLL) algorithm is efficient [3] in transforming the bases of the channel matrix, thus meliorating the orthogonality of basis vectors. In practice, per antenna power allocation is very critical as power to power amplifiers (PA) can serve each antenna effectively as compared to the sum power allocation where power is arbitrarily distributed to the antennas. Thus sum rate of MU massive MIMO systems for under per-antenna power constraint is a great contribution to power saving.

The paper is outlined as: Sect. 2 Designs the System Model, Sect. 3 describes the optimal ZF Precoder Design. Section 4 provides the numerical Analysis and discussions. Section 5 draws the conclusion of the paper.

2 System Model

We consider a single cell downlink MU massive MIMO system with base station (BS) equipped with M-array antennas and N users, with each user equipped with K ($K \geq 1$) antennas. The nth user received signal is modeled as $\mathbf{y}_n = \mathbf{H}_n\mathbf{x}+\mathbf{z}_n$, where $\mathbf{H}_n \in \mathbb{C}^{K\times M}$ is channel matrix and is full row rank and $\mathbf{z}_n \in \mathbb{C}^{K\times 1}$ is the (i.i.d) complex Gaussian noise vector. The statistical information of the transmitted vector $\mathbf{x} \in \mathbb{C}^{M\times 1}$ is defined as

$$\mathbf{x} = \sum_{n=1}^{N} \mathbf{T}_n\mathbf{s}_n \tag{1}$$

$$\mathbb{E}\left[\left(\mathbf{x}\mathbf{x}^{\mathrm{H}}\right)\right]_{ii} = \left[\sum_{n=1}^{N} \mathrm{tr}(\mathbf{T}_n\mathbf{T}_n^{\mathrm{H}})\right]_{ii} \leq p_i \quad \forall_i = 1, ..., M \tag{2}$$

where $\mathbf{T}_n \in \mathbb{C}^{M\times K}$ and $\mathbf{s}_n \in \mathbb{C}^{K\times 1}$ denote the precoder matrix and transmit data vector respectively, $\mathbb{E}\left[\left(\mathbf{s}_n\mathbf{s}_n^{\mathrm{H}}\right)\right] = \mathbf{I}_K$ and p_i is power of ith transmit antenna. The nth user received signal $\mathbf{y}_n$ is expanded (1) as

$$\mathbf{y}_n = \mathbf{H}_n\mathbf{T}_n\mathbf{s}_n + \underbrace{\sum_{\substack{j=1\\ j\neq n}}^{N}\mathbf{H}_n\mathbf{T}_j\mathbf{s}_j + \mathbf{z}_n} \tag{3}$$

with the underlined term as the interference plus noise. As the transmitted signal, noise and interference signals are uncorrelated, we adopt a model to remove the interference in the next section.

3 Optimal ZF Precoder Design

Let assume the transmitter have perfect CSI, then estimation of nth user effective channel $\mathbf{H}_n\mathbf{T}_n$ is achieved by precoding the pilots of $\mathbf{T}_n$. The nth user (3) downlink MUI is mitigated by ZF condition enforced as

$$\mathbf{H}_n\mathbf{T}_j = 0 \quad \text{for} \quad j \neq n \tag{4}$$

where (4) perfectly zeros the interference component in (3). The columns of $\mathbf{H}_n\mathbf{T}_n$ corresponding to singular values equal to the zero interference. Therefore invoking condition (4) into (3) is given by

$$\mathbf{y}_n = \mathbf{H}_n\mathbf{T}_n\mathbf{s}_n + \mathbf{z}_n \tag{5}$$

As MUI is annihilated, a practical multiuser ZF is achieved. Condition (4) forces $\mathbf{T}_n$ to be located in the nullspace of $\bar{\mathbf{H}}_n = \left(\mathbf{H}_1^{\mathrm{H}}, \mathbf{H}_2^{\mathrm{H}}, \mathbf{H}_{n-1}^{\mathrm{H}}\mathbf{H}_{n+1}^{\mathrm{H}}, ..., \mathbf{H}_N^{\mathrm{H}}\right)^{\mathrm{H}}$ from reception by nth user due transmission from other users. Block diagonalization thus decomposes the MIMO channel into multiple parallel subchannels, the singular value decomposition (SVD) is performed [12] as

$$\bar{\mathbf{H}}_n = \mathbf{U}_n\ \mathbf{\Sigma}_n\ \mathbf{V}_n^{\mathrm{H}} \tag{6}$$

where $\mathbf{U}_n$ and $\mathbf{V}_n$ are $(N-1)K \times (N-1)K$ and $(M \times M)$ unitary matrices respectively, $\mathbf{\Sigma}_n$ is $(N-1)K \times M$ component of diagonal matrix consisting of the ordered singular values. Since rank $\left(\bar{\mathbf{H}}_n\right) = (N-1)K$, then columns of $\bar{\mathbf{H}}_n$ are constructed in $\mathbf{V}_n$ for the precoder $\mathbf{T}_n$, we set $\bar{\mathbf{V}}_n \in \mathbb{C}^{M\times m}$ for $m = M - (N-1)K$ [5] and is conditioned as $\bar{\mathbf{V}}_n^{\mathrm{H}}\bar{\mathbf{V}}_n = \mathbf{I}_m$ satisfying orthogonality. The precoder aggregation matrix is $\mathbf{T}_n = \bar{\mathbf{V}}_n\hat{\mathbf{V}}_n$, where $\hat{\mathbf{V}}_n \in \mathbb{C}^{m\times K}$ denotes arbitrary matrix of the power constraint, optimization over $\hat{\mathbf{V}}_n$ assumes computation of diagonal elements. Plugging (6) into (5), estimated signal nth user is expressed as

$$\hat{\mathbf{s}}_n = \mathbf{U}_n^{\mathrm{H}}\mathbf{y}_n = \mathbf{U}_n^{\mathrm{H}}\mathbf{U}_n\ \mathbf{\Sigma}_n\ \mathbf{V}_n^{\mathrm{H}}\bar{\mathbf{V}}_n\hat{\mathbf{V}}_n\mathbf{s}_n + \tilde{\mathbf{z}}_n \tag{7}$$

with $\tilde{\mathbf{z}}_n = \mathbf{U}_n^{\mathrm{H}}\mathbf{z}_n$ as the additive Gaussian noise and $\mathbf{U}_n\mathbf{\Sigma}_n\mathbf{V}_n^{\mathrm{H}}\bar{\mathbf{V}}_n\hat{\mathbf{V}}_n\mathbf{U}_n^{\mathrm{H}}$ is the parallelized non-interfering SU-MIMO channels. The precoder rotation

$\mathbf{T}_n = \bar{\mathbf{V}}_n \hat{\mathbf{V}}_n$ for the transmitted power[1] must be properly align with the sub-channels. Optimization over $\hat{\mathbf{V}}_n$ with SVD-ZF (7) often assumes water-filling to align the power to the parallelized eigenchannels.

3.1 Optimal ZF Precoder Optimization

To construct the precoder rotations $\mathbf{T}_n = \bar{\mathbf{V}}_n \hat{\mathbf{V}}_n$, we set $\hat{\mathbf{V}}_n$ as $\hat{\mathbf{V}}_n \hat{\mathbf{V}}_n^{\mathrm{H}} = \Theta_n$ $(m \times m)$ positive semi-definite matrix of the precoder power. The sum rate maximization problem under per antenna power constraint is formulated as

$$\begin{aligned} &\max_{\mathbf{\Theta_n}} \text{imize } C_n(\mathbf{P}_n) = \sum_{n=1}^{N} \log\det(\mathbf{I} + \mathbf{B}\mathbf{P}_n) \\ &\text{subject to } \left[\sum_{n=1}^{N} \mathrm{tr}\left|\bar{\mathbf{V}}_{\mathbf{n}} \mathbf{\Theta_n} \bar{\mathbf{V}}_n^{\mathrm{H}}\right|\right]_{ii} \le p_i \quad \forall_i = 1, ..., M \\ &\qquad \mathbf{\Theta_n} \succeq 0 \quad n = 1, ..., N \\ &\qquad \text{rank } (\mathbf{\Theta_n}) \le K \end{aligned} \tag{8}$$

where $\mathbf{P}_n = \left|\mathbf{U}_n\ \mathbf{\Sigma}_n\ \mathbf{V}_{\mathbf{n}} \bar{\mathbf{V}}_{\mathbf{n}} \mathbf{\Theta_n} \bar{\mathbf{V}}_{\mathbf{n}}^{\mathrm{H}} \mathbf{V}_n^{\mathrm{H}} \mathbf{\Sigma}_n^{\mathrm{H}} \mathbf{U}_n^{\mathrm{H}}\right|$ and $\mathbf{B}$ is any arbitrary matrix. The per antenna power constraint (8) gives the sum rate maximization over the diagonal entries of $\mathbf{\Theta_n}$. Considering the objective of this study in $M \ge NK$ (non-square) regime, the domain search for optimization (8) limits the span of diagonal $[.]_{ii}$ in choosing $\mathbf{\Theta_n}$ entries. Thus optimal precoder $\left(\bar{\mathbf{V}}_n \hat{\mathbf{V}}_n\right)$ can not achieved best optimal solution, as dimensions of $\bar{\mathbf{V}}_n \in \mathbb{C}^{M \times m}$ is large or equal to the precoder $\mathbf{T}_n$ [5] resulting in deficiency. This dimension restrained is easily optimized with square matrix $(M = NK)$ under sum power constraint [9]. To solve this problem under per antenna power constraint, we propose conjugate beamforming approach to resize the matrix dimension. We define channel matrix as $\mathbf{X}_n = \mathbf{\Sigma}_n\ \mathbf{V}_{\mathbf{n}} \bar{\mathbf{V}}_n \in \mathbb{C}^{(N-1)K \times m}$ and conjugate transmit beamform matrix $\mathbf{W}_n \in \mathbb{C}^{M \times (N-1)K}$ that enforces the per antenna power constraint as

$$\mathbf{W}_n = \bar{\mathbf{V}}_n \mathbf{X}_n^{\dagger} \tag{9}$$

where $\mathbf{X}_n^{\dagger} = \mathbf{X}_n^{\mathrm{H}} \left(\mathbf{X}_n \mathbf{X}_n^{\mathrm{H}}\right)^{-1}$ is the Moore-Penrose inverse of $\mathbf{X}_n$ and $\bar{\mathbf{V}}_n$ in $\mathbf{V}_{\mathbf{n}}$ (6) is for designing the precoder power. Capitalizing on $\mathbf{P}_n = \mathbf{\Sigma}_n$ $\mathbf{V}_{\mathbf{n}} \bar{\mathbf{V}}_{\mathbf{n}} \mathbf{\Theta_n} \bar{\mathbf{V}}_{\mathbf{n}}^{\mathrm{H}} \mathbf{V}_n^{\mathrm{H}} \mathbf{\Sigma}_n^{\mathrm{H}}$, the $\mathbf{U}_n$ matrix is dropped in the sequel, we recompute $\mathbf{\Theta_n} = \left(\mathbf{\Sigma}_n \mathbf{V}_{\mathbf{n}} \bar{\mathbf{V}}_{\mathbf{n}}\right)^{\dagger} \mathbf{P}_n \left(\bar{\mathbf{V}}_{\mathbf{n}}^{\mathrm{H}} \mathbf{V}_n^{\mathrm{H}} \mathbf{\Sigma}_n^{\mathrm{H}}\right)^{\dagger} = \left(\mathbf{X}_n^{\dagger}\right) \mathbf{P}_n \left(\mathbf{X}_n^{\dagger}\right)^{\mathrm{H}}$. Now plugging $\mathbf{\Theta_n}$ into (9), we rewrite (8) for optimal SVD-ZF with conjugate beamforming (BF) as

[1] For $\mathbf{T}_n = \left[\bar{\mathbf{V}}_{\mathbf{1}} \hat{\mathbf{V}}_1, \bar{\mathbf{V}}_{\mathbf{2}} \hat{\mathbf{V}}_2, \ldots \bar{\mathbf{V}}_{\mathbf{N}} \hat{\mathbf{V}}_N\right]$ as the transmit power constraint (2) is formulated in $\mathbf{tr}\left(\bar{\mathbf{V}}_{\mathbf{n}} \hat{\mathbf{V}}_n \hat{\mathbf{V}}_n^{\mathrm{H}} \bar{\mathbf{V}}_n^{\mathrm{H}}\right)$ for the power constraint.

$$\begin{aligned} &\max_{\mathbf{P}_n} \text{imize } C_n\left(\mathbf{P}_n\right) = \sum_{n=1}^{N} \log\det\left(\mathbf{I} + \mathbf{B}\mathbf{P}_n\right) \\ &\text{subject to } \sum_{n=1}^{N} \operatorname{tr}\left|\mathbf{W_n}\mathbf{P_n}\mathbf{W}_n^{\mathrm{H}}\right|_{ii} \leq p_i \quad \forall_i = 1,..M \\ &\mathbf{P}_n \succeq 0, \mathbf{W}_n \succeq 0 \\ &\text{rank }\left(\mathbf{W_n}\right) = \text{rank }\left(\mathbf{P}_n\right) \leq K \end{aligned} \tag{10}$$

thence optimal solution always has rank $(\mathbf{P}_n) \geq 1$ for $K \geq 1$ with the user antenna that allows the transmitted power to target user antenna. As $\mathbf{W}_n \succeq 0$ satisfies $\sum_{i=1}^{M} |\mathbf{W_n}|_{ii} \succeq 0$ for $p_i \geq 0$, the beamform is thus aligned with the channel matrix. The conjugate beamforming matrix $(\mathbf{W}_n)$ is suboptimal when the channel matrix $(\mathbf{X}_n)$ is orthogonal for the sum rate maximization.

Optimal SVD-ZF with conjugate beamforming (BF) Relaxation. To resolve the inequality constraint (10) for the fixed point p_i contained in the undetermined $|\mathbf{P_n}|_{ii}$, we let eigenvector of $\mathbf{P_n}$ be $\mathbf{p}_n = (k, 1)$ for $1 \leq k \leq K$ and beamform vector $\mathbf{w}_n = (w_{1,k}, .., w_{M,k})$ with entry basis (i, k) form the Hermitian matrix $\mathbf{W_n} = (w_{i,k})$ as the k-dimensional volume of the parallelepiped form the basis vectors over the M antennas. By Shur's inequality [12], beamform coefficient is given as $\mathbf{W_n} = |\mathbf{w}_n|^2 \leq \left(\mathbf{w}_n^{\mathrm{H}}\mathbf{w}_n\right)$, thus bounds of sum rate $C_n(\mathbf{P}_n)$ is reflection of linear inequality constraint (10) as

$$\sum_{n=1}^{N} \left[\mathbf{p}_n |\mathbf{w}_n|^2\right]_{ii} \leq p_i \quad \forall_i = 1,..M \tag{11}$$

where $\left[\mathbf{p}_n |\mathbf{w}_n|^2\right]_{ii}$ is obtained from $\operatorname{tr}\left|\mathbf{W_n}\mathbf{P_n}\mathbf{W}_n^{\mathrm{H}}\right|_{ii}$ for the ith transmit antenna. Generally, the relaxation of constraint $\left(\mathbf{w}_n^{\mathrm{H}}\mathbf{w}_n\right)$ [7] is rank-one or standard basis $|\mathbf{w}_n|^2 = \mathbf{W_n} = 1$, hence beamforming has unit norm vector. Since (10) is convex constraint, the optimal solution has rank$(\mathbf{p}_n) \geq 1$ as $k \geq 1$, is achieved with water-filling. However, the relaxation constraint is not tight if $1 < k \leq rank\left(\mathbf{W_n}\right)$, as the users (user antennas) grow large, the basis of conjugate beamform $\mathbf{W_n}$ consisting of long $\mathbf{w}_n$ vectors allow combination off diagonal elements to appear in the diagonal $\mathbf{P_n}$. This lacks the objective of orthogonality to the user channels. Massive MIMO matrix dimension constrained is studied in [11] for the complexity of user dimension (NK) with the channel matrix $\mathbf{X}_n$ and $\bar{\mathbf{V}}_n$ precoding power constraints. In the next subsection evaluates the tightness of $\mathbf{W_n}$ by reducing the basis $\mathbf{w}_n$ consisting of short vectors, i.e. dimension span in vector space of the channel bases is to eliminate vectors that are linear combinations of others vectors.

Optimal SVD-ZF with Lattice Reduction based conjugate BF. The BS transmits to the users using lattice reduction based conjugate beamforming. The conjugate beamform matrix $\mathbf{W_n}$ (9) is written in complex lattice form as

$$\begin{aligned} \mathbf{W}_n^* &= \bar{\mathbf{V}}_n^u \breve{\mathbf{X}}_n^\dagger \\ &= \{\ \kappa_1^u \breve{\mathbf{x}}_1 + ... +\ \kappa_n^u \breve{\mathbf{x}}_n :\ \kappa_n^u \in \mathbb{Z} + j\mathbb{Z} \ \text{ for } \ n \in N\} \end{aligned} \tag{12}$$

with $\bar{\mathbf{V}}_n^u \in \mathbb{C}^{M \times m}$ as a unimodular transformation matrix[2] satisfying det $\left|\bar{\mathbf{V}}_n^u\right| = 1$ and maintains the channel signal power during LR process. The (CLR) algorithm uses the Gram-Schmidt Orthogonalization (GSO) to transform $\left(\breve{\mathbf{X}}_n^*\right)$ in order to bound the orthogonality defect[3]. The GSO is initiated by setting $\left(\breve{\mathbf{X}}_n^*\right)^\dagger = \left(x_{i,k}^*\right)$ for $1 < k \leq K$ and $1 \leq i \leq M$, thus orthonormal basis for the ith BS antenna and kth user antenna is given [13] as

$$x_{i,k}^* = x_{i,k} - \sum_{i=1}^{k-1} \kappa_{i,k}^u x_{i,k}^* \ \text{ for } \ 1 \leq i < k \leq M \tag{13}$$

where $\kappa_{i,k}^u = \frac{\left\langle x_{i,k}, x_{i,k}^* \right\rangle}{\left\| x_{i,k}^* \right\|^2}$ is the GSO coefficient for the linear combination for any $k \in (1, n)$. As the reduction $|x_{1,k}|$,...,$|x_{k-1,k}|$ approach zero, the vector $\breve{\mathbf{x}}_n$ is more orthogonal in the subspace span $\breve{\mathbf{x}}_1, ..., \breve{\mathbf{x}}_{n-1}$ linearly independent vectors, hence $\kappa_{i,k}^u = 0$. The lattice basis is size reduced if $\left|\kappa_{i,k}^u\right| \leq 1/2$ [11], then

$$\left|x_{i,k}^*\right| = \frac{1}{2}\left|x_{i,i}^*\right| \ \text{ for } \ 1 \leq i < k \leq M \tag{14}$$

where the reduced basis ensures off-diagonal elements of the channel vectors are almost half the diagonal elements. The general size-reduced basis using Lovasz condition [13] is achieved by subtracting a suitable linear combination $\left(\rho - \left|\kappa_{k-1,k}^u\right|^2\right)$ for the consecutive basis $x_{k,k}^*$ and $x_{k-1,k-1}^*$, is given as

$$\left\|x_{k,k}^*\right\|^2 + \left\|\kappa_{k-1,k}^u x_{k-1,k-1}^*\right\|^2 \geq \rho \left\|x_{k-1,k-1}^*\right\|^2, 2 \leq k \leq M \tag{15}$$

where the reduction basis $\rho = \frac{3}{4}$ is standard value ($\frac{1}{4} < \rho < \mathbf{1}$) in achieving a better performance in (14). Thence the new shorter basis $x_{k,k}^* + \kappa_{k-1,k}^u x_{k-1,k-1}^*$ is transformation of $x_{k,k}$ onto the orthogonal vector space, similarly $x_{k-1,k-1}^*$ is component of $x_{k-1,k-1}$ beam vector basis. Thus $\breve{\mathbf{x}}_n^*$ is near orthogonal and shorter projection of $\breve{\mathbf{x}}_n$, then reduced vector $\mathbf{w}_n^* = \kappa_n^u \breve{\mathbf{x}}_n^*$ of conjugate beamform $\mathbf{W}_n^* = \bar{\mathbf{V}}_n^u \left(\breve{\mathbf{X}}_n^*\right)^\dagger$, the new basis $\left(\breve{\mathbf{X}}_n^*\right)^\dagger$ for a given $\mathbf{W}_n^*$ is near orthogonal and

[2] The basis vectors are multiplied by square vector and determinant of ± 1, the elements are complex integer entries κ_n^u.

[3] The orthogonality defect is used to measure the orthogonality basis vectors, formed by all the inner products as $\frac{\prod_{i=1}^{n} \|\breve{\mathbf{x}}_i\|}{\left\|\breve{\mathbf{X}}_n^\dagger\right\|^2}$.

shorter as compared with previous beamforming $\mathbf{W_n}$ (9). The implementation of the CLLL algorithm requires QR decomposition on $\mathbf{W}_n^* = \mathbf{Q}_n^* \mathbf{R}_n^*$, where $\mathbf{Q}_n^* = \bar{\mathbf{V}}_n^u$ is $M \times m$ matrix and $\mathbf{R}_n^* = \left(\breve{\mathbf{X}}_n^*\right)^\dagger$ is the $m \times (N-1)K$ upper triangular matrix, then followed by the iteration over polynomial time as

Algorithm 1

1. Initialize the GSO for $x_{1,k}, .., x_{i,k}$, calculate $x_{1,k}^*, .., x_{i,k}^*$ and coefficients κ_n^u
2. Form size reduction for the pairs $x_{k,k}$ and $x_{k-1,k-1}$ and update $\kappa_{k-1,k}^u$
3. Use Lovasz condition for the pair $x_{k,k}^*$ and $x_{k-1,k-1}^*$ and update $\kappa_{k-1,k}^u$
4. Else go to step 2.

The CLLL algorithm swaps pair $x_{k,k}$ and $x_{k-1,k-1}$ for $x_{k,k}^*$ and $x_{k-1,k-1}^*$as the size-reduction steps proceed. Applying the transformation for the conjugate beamform (12) and (10), the optimal precoder achieves the maximum sum rate as $C_n^* = \max_{\breve{\mathbf{X}}_n^\dagger \in \mathbf{W_n^*}} C_n(\mathbf{P}_n)$ with reduced basis of the transformed beamforming.

Proposition 1. *Considering* $(M \geq NK)$ *with constant user antennas* $1 < k \leq K$ *for all* N *users, then (15) depends on user selection* $(N-1)K$, *assuming* $M \to \infty$, $N \to \infty$, *then* $0 < k \leq \frac{M}{N} < \infty$ *is constant with* k *values. Thus the singular values of* $\mathbf{X}_n \mathbf{X}_n^{\mathrm{H}} \in \mathbb{C}^{(N-1)K \times (N-1)K}$ *converge to constant value* $k \to \infty$, *hence given as*

$$M \geq (N-1)K \tag{16}$$

for $(M - N + 1)$ varies as $M \geq N$, thus objective function under per antenna power constraint is optimal (waterfilling) in achieving maximum sum rate for large $M \to \infty$, $N \to \infty$ in $M \geq NK$ regime.

4 Numerical Analysis and Discussions

In this section, numerical analysis and discussions are provided to validate performance of per-antenna power constraint for MU massive MIMO. The theoretical tightness of the study is validated with Monte Carlo simulations of 10000 realization. The precoder is constructed from the $\bar{\mathbf{V}}_n$ $(M \times m)$ for $m = M - (N-1)K$ and the LR standard basis is $\rho = \frac{3}{4}$. The figures compare schemas such as direct SVD-ZF-BF (10), SVD-ZF-BF with conjugate BF matrix with inner product $|\mathbf{w}_n|^2 = [\mathbf{W_n}] = 1$ (11) and the LR-based SVD-ZF-BF, all the schemas are analyzed under per antenna power constraint.

Figure 1, shows the sum rate with SNR for all the schemas. LR-based SVD-ZF-BF achieves higher sum rate as users (N) selection increases, this validate tightness through orthogonal channel for the distinct pairs $x_{k,k}$ and $x_{k-1,k-1}$ and also the direct SVD-ZF-BF improve with user selections whilst SVD-ZF-BF with BF $= [\mathbf{W_n}] = 1$ shows worse performance, this is due to the rank one assumption of $\mathbf{W}_n$ (unit norm vector) which constrained the beamforming diagonalization

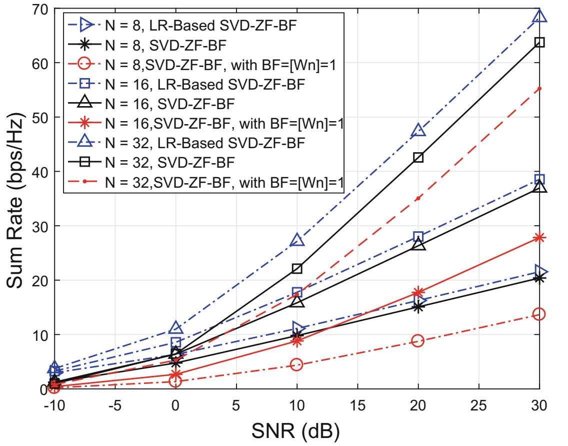

Fig. 1. Sum rate with SNR (dB) values, for M antennas = 128 and K antennas = 2.

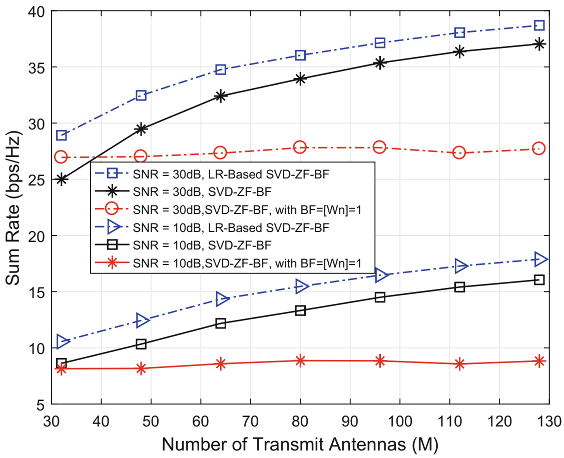

Fig. 2. Sum rate versus BS (M) antennas, for N users = 16 and K antennas = 2.

as user selections increases. The overall sum rate of our LR-based SVD-ZF-BF schema improved the performance than in [5, 10].

The sum rate versus transmit antennas M is presented in Fig. 2. Clearly sum rate increase with M for LR-based SVD-ZF-BF and direct LR-based SVD-ZF-BF, that argues an increase in channel gain for the subchannels as $M \geq NK$, the rate gain in LR-based SVD-ZF-BF is due to elimination of vectors which are linear combinations of others vectors. However as M turns large, the sum rate becomes stable suggesting limited gain due to the spread over the large $\mathbf{H}_n\mathbf{T}_n$ [5]. Subsequently sum rate of SVD-ZF-BF with BF = $[\mathbf{W_n}] = 1$ schema is constant regardless of channel randomness, thus the BF = $[\mathbf{W_n}] = 1$ restricts the diagonalize singular vectors of beamforming.

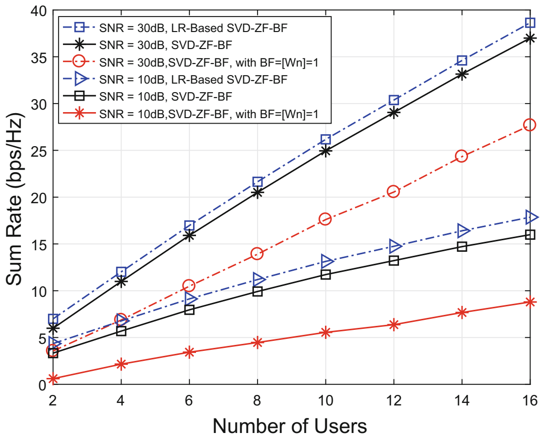

Fig. 3. Sum rate versus Users N, for M antennas = 128 and K antennas = 2.

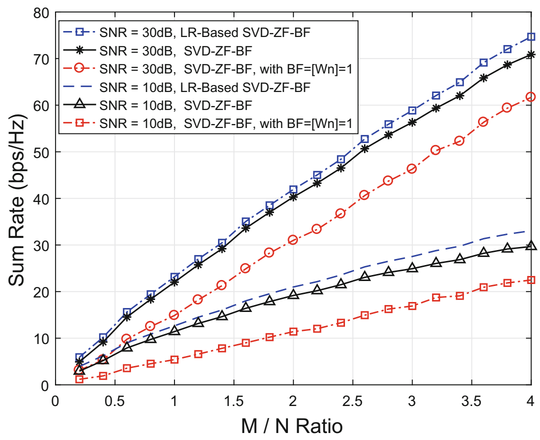

Fig. 4. Sum rate with the $0 < k \leq \frac{M}{N} < \infty$, the ratio k is equivalent user antennas.

Figure 3 plots the sum rate against the number of users N, i.e. selection of the SU-MIMO channels. The number of users increase with SNR gain hence increase sum rate in all schemas. Our LR-based SVD-ZF-BF shows high gain in the equivalent selection of SU-MIMO channels with the orthogonal bases justifying our argument that $1 < NK \leq rank\,(\mathbf{W_n})$ is not tight for relaxation (less orthogonal), as $\text{BF} = [\mathbf{W_n}] = 1$ suffers from the assumption.

Figure 4 presents the sum rate compared with the ratio $k \leq \frac{M}{N} < \infty$ (as $1 \leq k \leq K$) for multiplexing gain and diversity order, hence sum rate increases with user antennas k for all schemas. As $M = (N-1)K$ grows larger, the sum rate due to (16) turns to dominance of $M - N + 1$ channels. Thus increase in optimal power by the schemas for eigenchannel ($M \geq NK$). Then Fig. 3 is consistence with Fig. 4 justifying Proposition 1. Moreover increase in transmit antennas M results in increase multiplexing gain $\mathbf{\Sigma_n}$ as in $(N-1)K$ and compensate increase in the optimal power allocation in our LR-based SVD-ZF-BF.

5 Conclusion

We present optimal ZF precoder with conjugate beamforming under per antenna power constraints with MU massive MIMO system. An optimal SVD-ZF precoder is designed for the per antenna power. The conjugate beamforming maximization efficiently aligned the channel matrix for constrained MU massive MIMO matrix dimension. Furthermore, conjugate beamforming with lattice reduction transform the lattice basis of the channel matrix. Optimal ZF precoder with LR-based SVD guaranteed higher sum rate (multiplexing gain and diversity order) in meliorating the orthogonality of the distinct vector basis as compared with other precoding schemas. This theoretical analysis fulfills practical issues for optimal ZF precoder with per antenna power in MU massive MIMO systems.

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