A New Resampling Parameter Algorithm for Kullback-Leibler Distance with Adjusted Variance and Gradient Data Based on Particle Filter

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Abstract. In this paper, we propose a new resampling method of particle filter (PF) to monitor target position. The target location is to improve enhancing the effect of the received signal strength (RSS) variations. The key issue of our technique is to determine a new resampling parameter that finding the optimal bound error and lower bound variance values for Kullback-Leibler distance (KLD)-resampling adjusted variance and gradient data based on PF to ameliorate the effect of the RSS variations by generating a sample set near the high-likelihood region. To find these values, these optimal algorithms are proposed based on the maximum mean number of particles used of our proposal and other KLD-resampling methods. Our experiments show that the new technique does not only enhance the estimation accuracy but also improves the efficient number of particles compared to the traditional methods.

Keywords: $SIR \cdot Bound$ error \cdot KLD-resampling \cdot RSS

1 Introduction

The core challenge in wireless sensor network (WSN) is the estimation of target location in space, for instance, moving velocity of client and other physical parameter in [[1\]](#page-11-0). These parameters of target tracking are estimated by many different techniques. The recent technique is a recursive Bayesian filter that uses a set of particles with assigned primary weights serves as the basic idea of a particle filter (PF) in [\[2](#page-11-0)]. A set of weighted particles is theoretically qualified to represent any filtering distribution. Therefore, each particle conforms to the dynamic model and explains the observations, and then using this assessment to update set online for evaluating filter. As a result, Bayesian posterior estimate is formed.

Over the past decade, most research in WSN has emphasized the use of PFs in [\[3](#page-11-0)–[5](#page-11-0)] for solving many problems. Almost all of them are based on three operations. The first operation, called particle propagation, is to update the state of particles via the state transition process. The second one, namely weight computation, is the process that updates the weight of particles. The last one, called resampling, is to sample a new set of particles from the original particle population for better particle representation of the filtering distribution. Thanks the parallel processing to reduce the complexity of the former two processes and easily implement if parallel hardware is available. While the resampling step is a critical procedure for PF to avoid a degenerate set of particles (sample impoverishment) leading to the estimation inaccuracy. There are many methods are introduced such as initially employed to combat degeneracy in [\[6](#page-11-0)], generally replicating high-weighted particles to replace low-weighted particles in [\[4](#page-11-0)] for reducing the probability that the filter loses tracking.

A recent year, a number of authors have considered the effects of choosing metric and weight functional approach on PFs in $[3, 8, 10]$ $[3, 8, 10]$ $[3, 8, 10]$ $[3, 8, 10]$ $[3, 8, 10]$ $[3, 8, 10]$. The first approach, the PF based on Kullback-Leibler Distance (KLD)-sampling, determines the minimum number of particles needed to maintain the approximation quality in the sampling process. Meanwhile, adjusting standard deviation and then using gradient data for KLD-sampling in [\[10](#page-11-0)] is proposed to further improve the operation time and sample set size for target tracking thanks to the given upper bound error with fixed probability. In the KLD-sampling, the predictive belief state is used as the estimate of the underlying posterior in [[8\]](#page-11-0). In contrast to KLD-sampling algorithms, KLD-resampling algorithms in [\[3](#page-11-0)] also determines the number of particles to resample so that the KLD between the distribution of particles before resampling and after resampling does not exceed a prespecified error bound.

Our recent work in [[9,](#page-11-0) [14\]](#page-11-0) introduced an enhanced PF based on the finding lower bound variance for KLD-resampling adjusted variance and gradient data algorithm to improve the estimation error of target for WSN because of the variation of RSS measurement value is the diminished. Meanwhile, our research in [[12,](#page-11-0) [15\]](#page-11-0) also introduced another enhanced PF based on the finding upper bound error for KLD-resampling to further improve the estimation error of target for WSN in [[12\]](#page-11-0), especial in [[15\]](#page-11-0) show the enhancing estimation error in various power levels in localization and ubiquitous monitoring of patients system in [[11\]](#page-11-0).

In this paper, we more extend and apply the finding bound error algorithm for our method in [[9](#page-11-0), [14](#page-11-0)]. Our technique is to propose the finding bound error method in [[12](#page-11-0), [15\]](#page-11-0), called adjusted bound error, and the finding lower bound variance algorithm in [[9,](#page-11-0) [14\]](#page-11-0), called adjusted lower bound variance, to further reduce the estimation error of target and maintain the proper KLD-resampling in [\[7](#page-11-0)]. By combining the adjusted bound error in [[12,](#page-11-0) [15](#page-11-0)] and adjusted lower bound variance in [[9,](#page-11-0) [14](#page-11-0)] for KLD-resampling adjusted variance and gradient data, our experiments show that the new technique not only enhances the estimation accuracy but also improves the efficient number of particles used when compared traditional methods for WSN based on RSS measurement.

The paper is organized as follows. Introduction to system is given in Sect. [2](#page-2-0). All related schemes, namely, SIR, KLD-resampling, KLD-resampling adjusted variance and gradient data are presented in Sect. [3](#page-3-0). Our proposal is introduced in Sect. [4](#page-5-0).

All experimental results based on MATLAB for tracking are shown in Sect. [5](#page-8-0). Finally, we conclude the paper in Sect. [6.](#page-10-0)

2 System Model

We consider the robot carrying the sensor node (e.g. mobile node) and the node at some position as static node or node. We assume that the mobile robot moves along the determined path and a velocity in the long and thin region. The mobile robot can send the data to anchors, and then it can receive the data from the anchors. Its position can be computed by PF algorithm. We also assume all sensor nodes that have own equal physical parameters; the movement velocity of mobile node remains the same at time; the random velocity follows normal uniform distribution and the anchors are deployed at a determined pattern and the position can not be changed. This system model is divided into three parts, including mobility, RSS statistical and system state models.

2.1 Mobility Model

When the robot moves along the direction at the constant value, the time is divided into equal time segment. The velocity of the robot has some random noise which conforms to normal distribution ($\mu = 0$, $\sigma = 1$). So the random velocity and the determined velocity can be expressed as Fig. 1. Let us denote v″ (the solid line) to be the determined velocity and v′ (the dot dash line) to be the random velocity. Then, the dash line is the last velocity of the robot. The random velocity v' is a random variable which conforms to normal distribution.

Fig. 1. Vector decomposition of determined and random velocity [\[16](#page-11-0)]

2.2 RSS Statistical Model

The RSS statistical model indicates the relation between the RSS and distance between nodes. The common mathematical model of WSN can be expressed as follows

$$
P(d) = P(d_0) - 10n \log \left(\frac{d}{d_0}\right) + v_\sigma, \qquad (1)
$$

where P(d) is the RSS at position d between transmitter and receiver, $P(d_0)$ is the RSS at reference position (typically $d_0 = 1$ m); n is the path loss parameter related to the specific application environment; and v_{σ} is a Gauss stochastic variable.

2.3 System State Model

The system state model for the mobile wireless sensor in $[13, 16]$ $[13, 16]$ $[13, 16]$ is defined as follows

$$
x_k = x_{k-1} + V_k \Delta t + w_k, \qquad (2)
$$

$$
z_k = Pref + Klog(x_k) + Rv_k, \tag{3}
$$

where x_k is the position of a mobile node from the anchor, Δt is the time segment, z_k is the RSS measurement; V_k is the current velocity which consists of determined velocity and random velocity in [\(1](#page-2-0)); the w_k and v_k denote the system state noise and measurement noises which obey Gauss distributions whose mean are 0 and variances are Q and R, respectively; *Pref* is reference value of RSS, and K is the factor in path loss.

To evaluate the required number of particles, the system state model in (2) and in (3) can be rewriten as follows

$$
x_{k} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} (x_{k-1} + V_{k}\Delta t) + \begin{bmatrix} 0.5 & 0 \\ 1 & 0 \\ 0 & 0.5 \\ 0 & 1 \end{bmatrix} Q \begin{bmatrix} w_{1,k} \\ w_{2,k} \end{bmatrix}.
$$
 (4)

$$
z_{k} = Pref + Klog(arctan(x_{1,k}, x_{3,k})) + Rv_{k}.
$$

3 Particle Filters

In this section, we review all related PFs based on SIR algorithm in [[5\]](#page-11-0), KLD-resampling algorithm in [\[7](#page-11-0)], and KLD-resampling adjusted variance and gradient data algorithm in [\[9](#page-11-0)].

3.1 SIR PF

The concept of the SIR PF introduced in [\[5](#page-11-0)] is the simplest of all PFs for tracking which known as the bootstrap filter, condensation algorithm, interacting particle approximations and survival of the fittest. The core problem represents the required posterior density function by a set of random samples (particles) with associated weights, and computes the estimates by these samples and weights. When the number of samples increases very large, the Monte Carlo characterization becomes an equivalent representation of the posterior probability function.

The so-called sequential important sampling (SIS) algorithm for the PF in [[5\]](#page-11-0) which includes a resampling step at each instant, as described in detail in references. The SIS algorithm serves as an important density (or a proposed density to represent another one that cannot be exactly computed), that is, the sought posterior density in the present case. Then, samples are drawn from the important density instead of the actual density. A common problem with the SIS PF is the degeneracy phenomenon, where after a few states all but one particle will have negligible weight in [\[4](#page-11-0)]. This degeneracy implies that a large computational effort is devoted to updating particles whose contribution to the approximation of the posterior density function is almost zero. By increasing the number of particles (or more efficiently by approximately selecting the important density), this problem can be overcome. In addition, the use of resampling technique in [\[5](#page-11-0)] is recommended to avoid the degeneracy phenomenon (Table 1).

| 1: procedure SIR $\left({\left\{\mathbf{x}_{k-1}^{i}, \mathbf{w}_{k-1}^{i}\right\}}_{i=1}^{N}, \mathbf{z}_{k}\right)$ | | | |
|---|---|-------------------------------------|--|
| 2: | for $i = l \cdot N$ do | | |
| | 3: $x_k^i - p(x_k x_{k-1}^i)$ | \triangleright draw the signal | |
| | 4: $w_k^i = p(z_k x_k^i)$ | \triangleright the weights | |
| 5: | end for | | |
| | 6: $t = SUM \left[\{ w_k^i \}_{i=1}^N \right]$ | \triangleright total weights | |
| | 7: for $i = l : N$ do | | |
| | 8: $w_i^i = t^{-1} w_i^i$ | \triangleright normalised weights | |
| | $9:$ end for | | |
| | 10: end procedure | | |

Table 1. SIR PF

3.2 KLD-resampling

In the sampling process, as these individuals in the population are sorted by nondomination, the use of fast KLD-sampling technique in [[8\]](#page-11-0), called an adaptive PF at each iteration of the PF, determine the number of samples such that, with probability $1 - \delta$, the error between the true posterior and the sample-based approximation is less than ε . KLD is used to show how to determine the number of samples so that the distance between the sample-based maximum likelihood estimate and the true posterior does not exceed a pre-specified threshold.

The KLD between the proposal (q) and (p) distributions can be defined in discrete form as follows

$$
d_{KL}(p||q) \triangleq \sum_{x} p(x)log\left(\frac{p(x)}{q(x)}\right)
$$

=
$$
\sum_{x} W(x)q(x)log W(x),
$$
 (6)

with $W(x) = p(x) / q(x)$. The required number N_r of samples can be determined as follows

$$
N_r = \frac{1}{2\varepsilon} \chi_{k-1, 1-\delta}^2,
$$
\n(7)

where k is the number of bins with support, the quantizes of Chi-square distribution can be computed as follows

$$
P\left(\chi_{k-1}^2 \le \chi_{k-1,1-\delta}^2\right) = 1 - \delta. \tag{8}
$$

Based on Wilson-Hilferty transformation to compute the approximation of $\chi^2_{k-1,1-\delta}$ in (7) can be expressed as follows

$$
N_r = \frac{k-1}{2\varepsilon} \left(1 - \frac{2}{9(k-1)} + \sqrt{\frac{2}{9(k-1)} z_{1-\delta}} \right)^3,
$$
\n(9)

where $z_{1-\delta}$ is the upper quartile of the standard normal distribution.

According to the number of particles needed in (9) , an upper bound ε plays important role in statistical bounds of the approximation quality of samples that are actually drawn from the proposal rather than the true posterior distributions. To avoid the mismatch between the true and the proposal distributions, the result in (9) can be applied in the resampling process to determine the total number of particles to resample. The authors in [[7\]](#page-11-0) proposed the method which divides the particles of the posterior distribution into bins and count the number k of bins which at least one particle is resampled to determine the total number of particles to resample as follows

$$
N_{r,re} = min(N_{max}, cell(N_r)),
$$
\n(10)

where N_r is defined in (9).

4 Proposal Techniques

In this section, we propose the adjusted resampling algorithm, including the finding bound error and lower bound variance values, for KLD-resampling adjusted variance and gradient data can be presented in Table [3](#page-7-0).

4.1 KLD-resampling with Adjusted Resampling Parameter

With successful KLD-resampling via the help of adjusted variance and gradient data [[14,](#page-11-0) [17\]](#page-11-0) is to enhance the operation time and Root Mean Square Error (RMSE) criteria. This thanks to adjusting the variance size by increasing variance inversely proportional to the likelihood and creating new samples near the true distribution or the high likelihood region. By the similarity to this method, our propose is to incorporate the KLD-resampling with adjusted variance and gradient data as the following.

First, the adjusted variance can be computed by using the relationship between the maximum number of samples and the number of required samples as follows

$$
\sigma_{\rm ad} = \sigma_{\rm ld} + \varepsilon \frac{N_{r,re}}{N_{max}},\tag{11}
$$

where σ_{ad} and σ_{lb} are the adjusted and lower bound variances, respectively; Next, the new samples should be drawn as follows

$$
x_{k}^{i+N_{\text{r},\text{re}}} = \begin{cases} x_{k}^{i} + \sigma_{\text{ad}} \cdot \text{randn}, & \text{if}(\exists i) \frac{\partial p(h(x))}{\partial x} \Big|_{x=x_{k}^{i}} > 0 \\ x_{k}^{i} - \sigma_{\text{ad}} \cdot \text{randn}, & \text{otherwise} \end{cases}
$$
(12)

where $\frac{\partial p(h(x))}{\partial x}\Big|_{x=x_k^i} = \frac{\partial}{\partial x} \left[\frac{1}{\sigma \sqrt{2\pi}} exp \left\{ -\frac{(Z_k - h(x))^2}{2\sigma^2} \right\} \right]\Big|_{x=x_k^i}$ is the variance of a Gaussian probability distribution function.

Final, the new samples $x_k^{i+N_{r,\text{re}}}$ in (12) is used to update the weight. This leads to generate new samples with high likelihood region. As a result, operation time reduces and the accurate of tracking objects is high due to small sample set size. To sum up, our algorithm can be shown in Table 2.

Table 2. Pseudo-code of proposal

| 1: procedure Pro $(\varepsilon, \delta, N_{max}, \sigma)$ | | | | |
|--|---|---|--|--|
| 2: | Assign R, Q | | | |
| 3: | ε | \triangleright based on line 19 of Table 3 | | |
| 4: | σ_{lb} | \triangleright based on <i>Algorithm 6</i> in [9] | | |
| 5: | Assign δ , bin size, N _{max} | | | |
| 6: | Assign $k=0$, $i=0$; $N_{r,re}=0$, all bins are zero-resampled: b | | | |
| 7: | while ($i \leq N_{r, re}$ and $i \leq N_{max}$) do | | | |
| 8: | Randomly select one particle from the underlying particle set according to the weight, $i=i+1$ | | | |
| 9: | if (new resampled particle comes from bin b) then | | | |
| 10: | Calculate σ_{ad} in (11) | | | |
| 11: | Draw $x_k^{i+N_{\rm r,re}}$ | \triangleright draw the samples in (12) | | |
| 12: | Update $k=k+1$ | \triangleright the number of resampled bin | | |
| 13: | Resampled | | | |
| 14: | Update $N_{r,re}$ | \triangleright the required number in (10) | | |
| 15: | end if | | | |
| 16: | end while | | | |
| 17: end procedure | | | | |

4.2 Resampling Parameter Bound Error (Proposal)

The key issue of KLD-resampling with adjusted variance and gradient data is to determine the bound error in (7) (7) and the lower bound variance in (11) (11) to evaluate the number of particles used-based resampling process. Our works provided the finding lower bound variance algorithm with fixed bound error (0.65) in [\[9](#page-11-0)] through the available value in [\[7](#page-11-0)]. Meanwhile, our current works in [\[12](#page-11-0)] also provided the bound error algorithm for KLD-resampling adjusted variance and gradient data, we extend and apply this algorithm to determine the bound error for KLD-resampling with adjusted variance and gradient data in [[9\]](#page-11-0) as shown in Table 3.

1:procedure $\varepsilon_{opt}(min,max,\Delta\varepsilon)$ 2: Assign $R=Q$ \triangleright fixed probability $3: \delta$ 4: $i=1, \varepsilon_1 = min, \varepsilon_{i^*} = \Delta N = \{\emptyset\}$ \triangleright initialized 5: $\Delta RMSE_{Pro}^{KLD-adj} = \Delta RMSE_{Pro}^{SIR} = 0$ 6: while (ε \leq max) do 7: $\varepsilon_i = \varepsilon_i + \Delta \varepsilon$ \triangleright update bound error $RMSE_{\epsilon}^{Pro}$ \triangleright RMSE of proposal $8:$ $\textit{RMSE}^{\text{KLD-adj}} \quad \triangleright \text{RMSE of KLD-resampling adjusted variance and gradient data at}$ $9:$ $\varepsilon = 0.65$ $RMSE^{SIR}$ \triangleright RMSE of SIR
 $N_{r,\text{re}}^{KLD-\text{adj}}$ \triangleright the number of used particles KLD-resampling adjusted variance $RMSE^\text{SIR}$ $10:$ $11:$ and gradient data at $\varepsilon = 0.65$ in [9] $(N_{\tau,\text{re}})_{\varepsilon}^{Pro}$ \triangleright Proposal used particles at ε $12:$ 13: if $(\triangle \text{RMSE}_{Pro}^{\text{SIR}} > 0 \&\& \triangle \text{RMSE}_{Pro}^{\text{KLD-adj}} > 0)$ then $\varepsilon_{i^*} = \varepsilon_{i^*} \cup \{\varepsilon_i\}$ \triangleright update bound error $14:$ $\Delta N = \Delta N \cup \{ \Delta N_{\epsilon_i, k} \}$ \triangleright update the gap particles used between proposal and $15:$ method in [9] $16:$ end if 17: $i=i+1$ 18: end while $\varepsilon_{opt} = max(\Delta N)$ \triangleright find ε_{out} 20:end procedure

Let us denote ε_i the bound error of *i*th, $RMSE_{\varepsilon_i}^{Pro}$ which is the RMSE value of proposal at ε_i ; RMSE^{KLD-adj} and RMSE^{SIR} are respectively the RMSE values of KLD-resampling adjusted variance and gradient data (fixed bound error 0.65) in [\[9](#page-11-0)] and SIR in [\[5](#page-11-0)]. Let us denote \triangle RMS E_{Pro}^{SIR} and \triangle RMS $E_{Pro}^{KLD-adj}$ are the gap of RMSE values between SIR in [\[5](#page-11-0)] and proposal; between KLD-resampling adjusted variance and gradient data in [\[9](#page-11-0)] and proposal, respectively.

Let us define $N_{\text{r},\text{re}}^{\text{KLD-adj}}$ and $(N_{\text{r},\text{re}})_{\varepsilon_{i*}}^{Pro}$ as the mean numbers of particles used of KLD-resampling with adjusted variance and gradient data (fixed bound error) [[9\]](#page-11-0) and proposal, respectively; and ε_{i*} (line 14) is the sets of the bound error that fulfills the condition of *Remark* 1 (line 13).

Remark 1: if \triangle RMSE^{SIR} and \triangle RMSE^{KLD-adj} are greater than zero (line 13), then a bound error value ε_{opt} (line 19) exits to maximize the function ΔN .

5 Simulation Results

We conduct a series of simulations to determine the bound error in ([9\)](#page-5-0) based on Table [3.](#page-7-0) Next, the performance of RMSE, mean number of particles used, and the comparison of the number of particles used between our proposal and for KLD-resampling adjusted variance and gradient data in [[9\]](#page-11-0) are considered.

5.1 Finding Resampling Parameter Bound Error

Setting up the range of bound error value is [0.8, 0.95] with $\Delta \varepsilon = 0.05$ as shown in Table 4 to satisfy the RMSE criteria between SIR, KLD-resampling adjusted variance and gradient data (fixed bound error $\varepsilon = 0.65$) in [[9\]](#page-11-0), and our algorithm under the same conditions. The parameters of system are assumed and simulated as follows [\[9](#page-11-0)] δ = 0.01; bin size as the smaller of the standard deviations of the dynamic and the measurement (R = Q = 0.5), N = 300, N_{max} = N; $V_{\text{max}} = 5$; $V_{\text{min}} = 1$; $V_{\text{init}} = 5$; $Pref =$ -23 ; K = -45 ; and length time is 40 for sample size variation in 20 trials with various bound error values ε from 0.8 to 0.95 in Table 4. Based on Remark 1, the value $\varepsilon_{\text{opt}} =$ 0.95 is the optimal bound error because the function ΔN is to maximize at value 9.7 (*the italic row*, Table 4). Thus, we set up this value for our proposed method in the next section, and simulations are conducted to compare the RMSE criterion and the number of particles used for all approaches as shown in Figs. [2](#page-9-0) and [3,](#page-9-0) respectively.

| ε | $\sigma_{\rm lb}$ | Mean RMSE | | | | $\Delta N_{Pro}^{KLD-adj}$ | $\Delta RMSE_{Pro}^{SIR}$ | $\Delta RMSE_{Pro}^{KLD-adj}$ | |
|------|-------------------|------------|-------------------|----------------|-------------------|----------------------------|---------------------------|-------------------------------|--------|
| | (Algorithm | SIR | KLD- | KLD-resampling | KLD-resampling | Proposal | | | |
| | 6 in $[9]$ | | resampling | adjusted bound | adjusted variance | | | | |
| | | | $\lceil 7 \rceil$ | error $[12]$ | and gradient data | | | | |
| | | | | | $\lceil 9 \rceil$ | | | | |
| 0.8 | 0.8 | 0.0530 | 0.0812 | 0.0287 | 0.0607 | 0.0181 | 8.7 | 0.0349 | 0.0426 |
| 0.85 | 1.05 | 0.0992 | 0.0906 | 0.0413 | 0.0638 | 0.0207 | 9.2 | 0.0785 | 0.0431 |
| 0.9 | 1.4 | 0.1226 | 0.0986 | 0.0651 | 0.0918 | 0.0705 | 9.4 | 0.0521 | 0.0213 |
| 0.95 | 1.3 | 0.0267 | 0.0245 | 0.0194 | 0.0191 | 0.0109 | 9.7 | 0.0158 | 0.0082 |

Table 4. Mean RMSE vs. Mean Number of particles used

Fig. 2. RMSE of system for all approaches

Fig. 3. Mean number of particles used for sample size variation in 20 trials

| | N_{max} (ϵ based on Table 3; σ_{lb} based on Algorithm 6 in [9]) | | | | | |
|-----|---|---|--------------|--|--|--|
| | | $R = 0.5$, Q = 0.1 R = 0.5, Q = 0.3 R = 0.5, Q = 0.7 | | | | |
| 100 | (0.95; 0.3) | (0.95; 0.15) | (0.95; 0.05) | | | |
| 400 | (0.95; 0.5) | (0.95; 0.8) | (0.95; 1.2) | | | |
| 800 | (0.95; 1.4) | (0.95; 0.2) | (0.85; 0.3) | | | |

Table 5. Bound error and lower bound variance values vs. various Q

5.2 Performance for SIR, KLD-resampling, Proposal

Figure 2 shows the comparison of RMSE for SIR in [\[5](#page-11-0)], KLD-resampling (ε = 0.65) in [[7\]](#page-11-0), KLD-resampling adjusted variance and gradient data in [[9\]](#page-11-0) ($\sigma_{\text{lb}} = 1.3$, fixed bound error $\varepsilon = 0.65$), KLD-resampling adjusted bound error ($\varepsilon = 0.95$) in [[12](#page-11-0)] and our proposal with length time 40, sample size in 20 trials. Clearly, the RMSE value of our proposal is lower than that of the others. For example, from 8 to 36 s, the gap between the proposed approach and KLD-resampling adjusted variance and gradient data in [\[9](#page-11-0)]

regularly increases from about 0.05 to around 0.62, respectively, due to the adjusted lower variance algorithm (see Algorithm 6 in [\[9](#page-11-0)]). Furthermore, our method also improves the accuracy of target when compared KLD-resampling adjusted bound error.

The comparison of the mean number of particles used of KLD-resampling, KLD-resampling-based adjusted variance and gradient data in [\[9](#page-11-0)], KLD-resampling adjusted bound error in [[12\]](#page-11-0), and our proposal is shown in Fig. [3.](#page-9-0) It verifies that the curves of mean number of particles used with adjusted bound error are lower than that of without adjusted bound error. For instance, the curves of these methods maintain from 7 to 40 s. Otherwise, the number of particles used for our method is around 19 and sharply decreases after the next seconds it reaches about 4 particles used. Finally, our method slightly improves the number of particles used when compared KLD-resampling adjusted bound error in [[12\]](#page-11-0).

Similar to Sect. [5.1,](#page-8-0) the bound error and lower bound variance values can be found in Table [5](#page-9-0) in case of changing various $O(0.1; 0.3; 0.7)$ and different number of samples. The gap of mean number particles used for KLD-resampling adjusted variance between with and without adjusted bound error under various Q can be evaluated and shown in Fig. 4. When increasing the number samples from 100 to 800, the mean number of particles used of our method is less than that of KLD-resampling adjusted variance and gradient data about 9 particles with different Q.

Fig. 4. Gap of mean number particles used vs. various Q and sample size variation in 20 trials

6 Conclusion

This paper, we propose a new resampling parameter for KLD-resampling adjusted variance and gradient data based on PF to estimate the location of target in WSN for reducing the fluctuations of RSS samples. In addition, in using the finding bound error and lower bound variance algorithms, this approach reduces the number of particles used when compared traditional methods.

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