Outage Probability Analysis of Single Energy Constraint Relay NOMA Network

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Abstract. In this paper, we investigate energy harvesting decode-and-forward relaying non-orthogonal multiple access (NOMA) networks. Specifically, one source node wishes to transmit two symbols to its two desired destinations directly and via the help of an intermediate energy constraint relay node, and the NOMA technique is applied in the transmission of both hops (from source to relay and from relay to destinations). For performance evaluation, we derive the closed-form expressions for the outage probability (OP) at D_1 and D_2. Our analysis is substantiated via Monte Carlo simulation. The effect of several parameters, such as power allocation factors in both transmissions in two hops, the power splitting ratio, the location of relay node, to the outage performances at two destinations is investigated.

Keywords: Non-orthogonal Multiple Access · Energy harvesting
Power splitting · Decode-and-forward

1 Introduction

Due to the significantly growing number of users and wireless devices, the future 5G networks are required to support the demand for low-latency, low-cost and diversified services, yet at higher quality and a thousand-time faster data rate. In the quest for new technologies, non-orthogonal multiple access (NOMA) technique has emerged as one of the most prominent candidates in meeting these requirements. The use of NOMA can ensure a significant spectral efficiency as it takes advantage of the power domain to serve multiple users at the same time/frequency/code. In addition, compared with conventional multiple access, NOMA offers better user fairness since even users with weak channel state information (CSI) can be served in a timely manner.

NOMA in cooperative and cognitive radio networks has been pursued by research groups from Princeton University, USA (Ding et al.) and Queen Mary University of London, UK (Elkashlan et al.) with a focus on cooperative communication protocols and performance analysis of cooperative networks [1] and large-scale underlay cognitive radio networks [2] taking into account users? geographical distribution. Specifically, in [1], the authors analyzed the outage probability and diversity order under the assumption that users with better

Y. Chen and T. Q. Duong (Eds.): INISCOM 2017, LNICST 221, pp. 28–39, 2018.
https://doi.org/10.1007/978-3-319-74176-5_3

channel conditions can decode the message for the others, and proposed a cooperative NOMA transmission protocol. The work in [2] presented the closed-form expression of outage probability to evaluate the system performance by using stochastic-geometry. Also in this research stream, Men and Ge (Xidian University, China) proposed a NOMA-based downlink cooperative cellular system, where the base station communicates with two paired mobile users through the help of a half-duplex amplify-and-forward (AF) relay [3]. To investigate the performance of the considered network, a closed-form expression of outage probability was derived and ergodic sum-rate was studied. By comparing NOMA with conventional multiple access, the authors showed that NOMA can offer better spectral efficiency and user fairness since more users are served at the same time/frequency/spreading code. Furthermore, J.-B. Kim and I.-H. Lee's research group has investigated NOMA in cooperative networks and derived exact and closed-form expressions of outage probability. The results showed that the system performance is improved significantly with NOMA. Their system model consists of one base station (BS) and two users, in which user 1 communicates directly to the BS while user 2 communicates with the BS through the help of user 1.

For NOMA with RF-EH, authors from Aristotle University have studied data rates optimization and fairness increase in NOMA systems with wireless energy harvesting based on time allocation [4]. The analytical and simulation results indicated that this proposed method is better than TDMA scheme. Moreover, the research group from Queen Mary University of London (UK) and Princeton University (USA) proposed NOMA scheme in simultaneous wireless information and power transfer (SWIPT) networks [5]. Specifically, near NOMA users that are close to the source act as energy harvesting relays to help far NOMA users. Furthermore, the authors investigated the performance of the considered systems by deriving the closed-form expressions for outage probability and system throughput under the random distribution of users' location. Analytical and simulation results showed that selecting users can reasonably reduce the outage probability. Moreover, by carefully choosing the parameters of the network such as transmission rate or power splitting coefficient, system performance can be guaranteed even if the users do not use their own batteries to power the relay transmission.

In this paper, we investigate energy harvesting DF NOMA relaying networks, in which one source nodes want to transmit its two symbols to two destinations directly and via the help of an energy constraint relay nodes. The relay harvests the energy and decode the radio frequency (RF) signal from the source and forward the encoded signal to two destinations. In addition, the NOMA technique is considered for transmission in both hops from the source to relay and from relay to destinations with two set of power allocation factor.

Notation: The notation $\mathcal{CN}(0, N_0)$ denotes a circularly symmetric complex Gaussian random variable (RV) with zero mean and variance N_0. $\mathcal{E}\{.\}$ denotes mathematical expectation. The functions $f_X(.)$ and $F_X(.)$ present the probability density function (PDF) and cumulative distribution function (CDF) of RV

X. The function $\Gamma(x, y)$ is an incomplete Gamma function (Eq. 8.310.1 of [6]). $C_b^a = \frac{b!}{a!(b-a)!}$. Notation Pr[.] returns the probability.

2 Network and Channel Models

As illustrated in Fig. 1, we consider a system model of a NOMA EH DF relaying network, where a source node S want to transmit its two symbols x_1 and x_2 to two destination nodes D_1 and D_2, respectively, directly and via the help of an intermediate EH relay nodes R. All nodes are equipped with single antenna operating in half-duplex mode. In Fig. 1, (h_1, d_1), (h_2, d_2), (h_3, d_3), (h_4, d_4), and (h_5, d_5) denote the Rayleigh channel coefficients over the distances for the links between S and R, R and D_1, R and D_2, S and D_1, and S and D_2, respectively. The corresponding channel gain $g_\Omega \triangleq |h_\Omega|^2$ is exponential random variable (RV) with parameter $\lambda_\Omega = (d_\Omega)^\beta$, with $\Omega \in \{1, 2, 3, 4, 5\}$ and β denote path-loss exponent. The channel state information (CSI) is assumed to be known at all nodes. The corresponding probability density function (PDF) and cumulative distribution function (CDF) of each RV is $f_{g_\Omega}(x) = \lambda_\Omega e^{-\lambda_\Omega x}$ and $F_{g_\Omega}(x) = 1 - e^{-\lambda_\Omega x}$, respectively. The power splitting architecture is apply at relay for harvesting the energy with power splitting ratio ρ and $(1-\rho)$ for decoding the source information.

The channels from S to R, from R to D_1, and from R to D_2 are denoted by h_1, h_2 and h_3, respectively. In the first phase, the source node S broadcast its signal containing two symbols x_1 and x_2 as a form $x = \sqrt{a_1 P} x_1 + \sqrt{a_2 P} x_2$, with $\mathcal{E}\left\{|x|^2 = 1\right\}$, P is a transmit power of source node, a_1 and a_2 respectively denote the power allocation coefficient for symbols x_1 and x_2, and $a_1 + a_2 = 1$, $a_1 \geq a_2$. The received signals at relay R and two destinations D_1 and D_2, respectively, given as

$$y_1 = h_1(\sqrt{a_1 P} x_1 + \sqrt{a_2 P} x_2) + n_1^a \tag{1}$$

$$y_4 = h_4(\sqrt{a_1 P} x_1 + \sqrt{a_2 P} x_2) + n_4^a \tag{2}$$

$$y_5 = h_5(\sqrt{a_1 P} x_1 + \sqrt{a_2 P} x_2) + n_5^a \tag{3}$$

where n_1^a, n_4^a, and $n_5^a \sim \mathcal{CN}(0, N_0)$ denote the additive white Gaussian noise (AWGN) at R, D_1, and D_2, respectively.

At relay R, the received signal y_1 in (1) is split into two parts for energy harvesting $(y_{1,eh})$ and information decoding $(y_{1,id})$:

$$y_{1,eh} = \sqrt{\rho} y_1 = h_1(\sqrt{\rho a_1 P} x_1 + \sqrt{\rho a_2 P} x_2) + \sqrt{\rho} n_1^a \tag{4}$$

$$y_{1,id} = \sqrt{(1-\rho)} y_1 = h_1(\sqrt{(1-\rho) a_1 P} x_1 + \sqrt{(1-\rho) a_2 P} x_2) + \sqrt{(1-\rho)} n_1^a \tag{5}$$

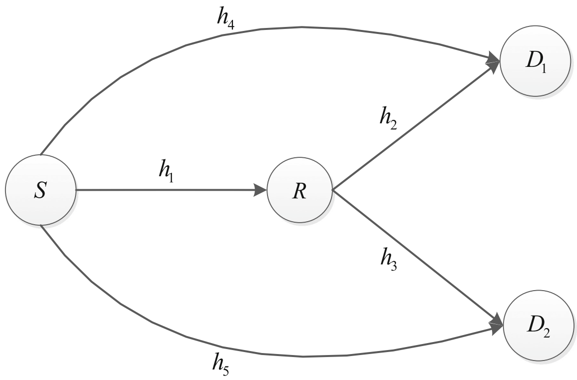

Fig. 1. Network model for NOMA energy constraint DF relaying.

The harvested power at R can be obtained from (4) as:

$$P_R = \eta\rho a_1 P|h_1|^2 + \eta\rho a_2 P|h_1|^2 = \eta\rho P|h_1|^2 = \eta\rho P g_1 \tag{6}$$

The received RF signals is sampled by RF-to-baseband conversion units. Thus, the signals in (2), (3) and (5) are added with the noise $n^c \sim \mathcal{CN}(0, \mu N_0)$, with $\mu > 0$, as

$$y_4^c = h_4(\sqrt{a_1 P}x_1 + \sqrt{a_2 P}x_2) + n_4^a + n_4^c \tag{7}$$

$$y_5^c = h_5(\sqrt{a_1 P}x_1 + \sqrt{a_2 P}x_2) + n_5^a + n_5^c \tag{8}$$

$$y_{1,id}^c = h_1(\sqrt{(1-\rho)a_1 P}x_1 + \sqrt{(1-\rho)a_2 P}x_2) + \sqrt{(1-\rho)}n_1^a + n_1^c \tag{9}$$

First, to decode symbol x_1, the relay R and two destinations D_1 and D_2 treat x_2 as noise. We obtain the signal to interference plus noise (SINR) for x_1 at R, D_1, and D_2, respectively, as

$$\gamma_1^{x_1} = \frac{a_1(1-\rho)P|h_1|^2}{a_2(1-\rho)P|h_1|^2 + (1-\rho+\mu)N_0} = \frac{a_1(1-\rho)\gamma_0 g_1}{a_2(1-\rho)\gamma_0 g_1 + (1-\rho+\mu)} \tag{10}$$

$$\gamma_4^{x_1} = \frac{a_1 P|h_4|^2}{a_2 P|h_4|^2 + (1+\mu)\,N_0} = \frac{a_1\gamma_0 g_4}{a_2\gamma_0 g_4 + 1 + \mu} \tag{11}$$

$$\gamma_5^{x_1} = \frac{a_1 P|h_5|^2}{a_2 P|h_5|^2 + (1+\mu)\,N_0} = \frac{a_1\gamma_0 g_5}{a_2\gamma_0 g_5 + 1 + \mu} \tag{12}$$

where $\gamma_0 \overset{\Delta}{=} \frac{P}{N_0}$ denote the transmit signal to noise (SNR).

Second, the relay R and destination D_2 decode symbol x_2 by cancelling x_1 with successive interference cancellation (SIC) from (9) and (8). The received SNRs for x_2 at R and D_2 are respectively given as

$$\gamma_1^{x_2} = \frac{a_2(1-\rho)P|h_1|^2}{(1-\rho)N_0 + \mu N_0} = \frac{a_2(1-\rho)\gamma_0 g_1}{1-\rho+\mu} \tag{13}$$

$$\gamma_5^{x_2} = \frac{a_2 P|h_5|^2}{(1+\mu)N_0} = \frac{a_2\gamma_0 g_5}{1+\mu} \tag{14}$$

In the second phase, after successfully decoded the symbols x_1 and x_2, relay R forwards them to D_1 and D_2 as a form $(\sqrt{b_1 P_R}x_1 + \sqrt{b_2 P_R}x_2)$ with the transmit power P_R in (6), with b_1 and b_2 denote the power allocation coefficient ($b_1+b_2 = 1$, $b_1 \geq b_2$). The base-band received signals at D_1 and D_2 are expressed as

$$y_2 = h_2(\sqrt{b_1 P_R}x_1 + \sqrt{b_2 P_R}x_2) + n_2^a + n_2^c \tag{15}$$

$$y_3 = h_3(\sqrt{b_1 P_R}x_1 + \sqrt{b_2 P_R}x_2) + n_3^a + n_3^c \tag{16}$$

where $n_2^a, n_3^a \sim \mathcal{CN}(0, N_0)$, $n_2^c, n_3^c \sim \mathcal{CN}(0, \mu N_0)$.

D_1 decode its desired symbol (x_1) by treating x_2 as noise. From (15), the SINR for decoding x_1 at D_1 is given as

$$\gamma_2^{x_1} = \frac{b_1 P_R|h_2|^2}{b_2 P_R|h_2|^2 + (1+\mu)N_0} = \frac{b_1\eta\rho\gamma_0 g_1 g_2}{b_2\eta\rho\gamma_0 g_1 g_2 + 1 + \mu} \tag{17}$$

D_2 decode its desired symbol (x_2) after decoding x_1 (with SINR $\gamma_3^{x_1} = \frac{b_1 P_R g_3}{b_2 P_R g_3 + (1+\mu)N_0} = \frac{b_1\eta\rho\gamma_0 g_1 g_3}{b_2\eta\rho\gamma_0 g_1 g_3 + 1 + \mu}$) and cancelling it. The SNR for decoding x_2 at D_2 is given as

$$\gamma_3^{x_2} = \frac{b_2 P_R|h_3|^2}{(1+\mu)N_0} = \frac{b_2\eta\rho\gamma_0 g_1 g_3}{1+\mu} \tag{18}$$

3 Outage Probability Analysis

In this paper, the receiver decodes successfully the information if its SINR or SNR satisfies the pre-defined threshold γ_t. In this section, we will derive the outage probabilities at D_1, D_2 both cases of one relay and multiple relays under relay selection scheme.

3.1 Outage Probability at D_1

An outage event happens when D_1 unsuccessfully decodes the symbol x_1 both from S in the first phase and from R in the second phase. The outage probability at D_1 can be formulated as

$$\begin{aligned} OP_{D_1}^{1relay} &= \underbrace{\Pr\left[\min\left(\gamma_1^{x_1},\gamma_1^{x_2}\right)<\gamma_t,\gamma_4^{x_1}<\gamma_t\right]}_{OP_1} \\ &+\underbrace{\Pr\left[\min\left(\gamma_1^{x_1},\gamma_1^{x_2}\right)\geqslant\gamma_t,\max\left(\gamma_4^{x_1},\gamma_2^{x_1}\right)<\gamma_t\right]}_{OP_2} \end{aligned} \quad (19)$$

Particularly, OP_1 is the outage event for the case that R can not decode successfully both x_1 and x_2 ($\min\left(\gamma_1^{x_1},\gamma_1^{x_2}\right)<\gamma_t$), leading to R does not forward the signal ($\sqrt{b_1 P_R}x_1+\sqrt{b_2 P_R}x_2$) to destinations, and the destination D_1 can not decode successfully symbol x_1 directly from S in the first phase ($\gamma_4^{x_1}<\gamma_t$). OP_2 is the outage event for the case that R decodes correctly both symbol x_1 and x_2 ($\min\left(\gamma_1^{x_1},\gamma_1^{x_2}\right)\geq\gamma_t$), but D_1 can not decodes successfully x_1 both from S and R in the first and second phase, respectively ($\max\left(\gamma_4^{x_1},\gamma_2^{x_1}\right)<\gamma_t$).

The term OP_1 and OP_2 can be obtain by substituting the SINRs and SNRs in (10), (13), (11) and (17) into (19) as follows

$$\begin{aligned} OP_1 &= \Pr\left[\min\left(\frac{a_1(1-\rho)\gamma_0 g_1}{a_2(1-\rho)\gamma_0 g_1+(1-\rho+\mu)},\frac{a_2(1-\rho)\gamma_0 g_1}{1-\rho+\mu}\right)<\gamma_t,\frac{a_1\gamma_0 g_4}{a_2\gamma_0 g_4+1+\mu}<\gamma_t\right] \\ &= \Pr\left[\frac{a_1\gamma_0 g_4}{a_2\gamma_0 g_4+1+\mu}<\gamma_t\right]\Pr\left[\min\left(\frac{a_1(1-\rho)\gamma_0 g_1}{a_2(1-\rho)\gamma_0 g_1+(1-\rho+\mu)},\frac{a_2(1-\rho)\gamma_0 g_1}{1-\rho+\mu}\right)<\gamma_t\right] \\ &= \Pr\left[g_4<\frac{(1+\mu)\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}\right]\left\{1-\Pr\left[g_1\geqslant\frac{(1-\rho+\mu)\gamma_t}{(a_1-a_2\gamma_t)(1-\rho)\gamma_0},g_1\geqslant\frac{(1-\rho+\mu)\gamma_t}{a_2(1-\rho)\gamma_0}\right]\right\} \\ &= F_{g_4}\left(\frac{(1+\mu)\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}\right)\times\begin{cases} F_{g_1}\left(\frac{(1-\rho+\mu)\gamma_t}{(a_1-a_2\gamma_t)(1-\rho)\gamma_0}\right) & if\ a_1-a_2\gamma_t<a_2 \\ F_{g_1}\left(\frac{(1-\rho+\mu)\gamma_t}{a_2(1-\rho)\gamma_0}\right) & if\ a_1-a_2\gamma_t\geqslant a_2 \end{cases} \\ &= \left(1-e^{-\frac{\lambda_4\omega_1\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}\right)\times\begin{cases} \left(1-e^{-\frac{\lambda_1\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}\right) & if\ a_1<a_2(1+\gamma_t) \\ \left(1-e^{-\frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}}\right) & if\ a_1\geqslant a_2(1+\gamma_t) \end{cases} \end{aligned} \quad (20)$$

where $\omega_1\triangleq 1+\mu$, $\omega_2\triangleq\frac{1-\rho+\mu}{1-\rho}$.

$$\begin{aligned} OP_2 &= \Pr\left[\begin{array}{l}\min\left(\frac{a_1(1-\rho)\gamma_0 g_1}{a_2(1-\rho)\gamma_0 g_1+(1-\rho+\mu)},\frac{a_2(1-\rho)\gamma_0 g_1}{1-\rho+\mu}\right)\geqslant\gamma_t, \\ \max\left(\frac{a_1\gamma_0 g_4}{a_2\gamma_0 g_4+1+\mu},\frac{b_1\eta\rho\gamma_0 g_1 g_2}{b_2\eta\rho\gamma_0 g_1 g_2+1+\mu}\right)<\gamma_t\end{array}\right] \\ &= \Pr\left[g_4<\frac{(1+\mu)\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}\right]\Pr\left[\begin{array}{l} g_1\geqslant\frac{(1-\rho+\mu)\gamma_t}{(a_1-a_2\gamma_t)(1-\rho)\gamma_0},g_1\geqslant\frac{(1-\rho+\mu)\gamma_t}{a_2(1-\rho)\gamma_0} \\ \frac{b_1\eta\rho\gamma_0 g_1 g_2}{b_2\eta\rho\gamma_0 g_1 g_2+1+\mu}<\gamma_t\end{array}\right] \\ &\overset{(21.1)}{=}\left(1-e^{-\frac{\lambda_4(1+\mu)\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}\right) \\ &\times\begin{cases}\Pr\left[g_1\geqslant\frac{(1-\rho+\mu)\gamma_t}{(a_1-a_2\gamma_t)(1-\rho)\gamma_0},g_2<\frac{(1+\mu)\gamma_t}{(b_1-b_2\gamma_t)\eta\rho\gamma_0 g_1}\right] & if\ a_1-a_2\gamma_t<a_2 \\ \Pr\left[g_1\geqslant\frac{(1-\rho+\mu)\gamma_t}{a_2(1-\rho)\gamma_0},g_2<\frac{(1+\mu)\gamma_t}{(b_1-b_2\gamma_t)\eta\rho\gamma_0 g_1}\right] & if\ a_1-a_2\gamma_t\geqslant a_2\end{cases} \end{aligned}$$

$$\overset{(21.2)}{=} \left(1 - e^{-\frac{\lambda_4 \omega_1 \gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}}\right) \times \begin{cases} e^{-\frac{\lambda_1 \omega_2 \gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}} - \sum_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_2\omega_3\gamma_t}{(b_1 - b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}\right) & if\ a_1 < (1+\gamma_t)\,a_2 \\ e^{-\frac{\lambda_1 \omega_2 \gamma_t}{a_2\gamma_0}} - \sum_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_2\omega_3\gamma_t}{(b_1 - b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}\right) & if\ a_1 \geqslant (1+\gamma_t)\,a_2 \end{cases} \tag{21}$$

where $\omega_3 \triangleq \frac{1+\mu}{\eta\rho}$.

where (21.2) is obtained from (21.1) by using the result in Appendix A. Note that we allocate the power coefficients a_1, a_2, b_1, and b_2 that $(a_1 - a_2\gamma_t) > 0$ and $(b_1 - b_2\gamma_t) > 0$ for OP_1 and OP_2 not equal to 0.

Finally, a closed-form expression for $OP_{D_1}^{1relay}$ is derived by substituting (20) and (21) into (19) as

$$OP_{D_1}^{1relay} = \left(1 - e^{-\frac{\lambda_4 \omega_1 \gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}}\right) \times \begin{cases} 1 - \sum_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_2\omega_3\gamma_t}{(b_1 - b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_3\gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}\right) & if\ a_1 < (1+\gamma_t)\,a_2 \\ 1 - \sum_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_2\omega_3\gamma_t}{(b_1 - b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_3\gamma_t}{a_2\gamma_0}\right) & if\ a_1 \geqslant (1+\gamma_t)\,a_2 \end{cases} \tag{22}$$

3.2 Outage Probability at D_2

In this paper, the desired symbol for destination is x_2, thus D_2 has to successfully decode x_1 first then using SIC to obtain x_2. There are two cases for outage happening at D_2 that (i) both x_1 and x_2 can not be decoded successfully from S and D_2 in the first time slot $((\min(\gamma_1^{x_1}, \gamma_1^{x_2}) < \gamma_t, \min(\gamma_5^{x_1}, \gamma_5^{x_2}) < \gamma_t))$, the probability for this event is denoted by OP_5; (ii) R detects correctly x_1 and x_2 transmitted from S in the first phase but D_2 does not from both S and R in the first and second phases, repsectively.

$(\Pr[\min(\gamma_1^{x_1}, \gamma_1^{x_2}) \geqslant \gamma_t, \min(\gamma_5^{x_1}, \gamma_5^{x_2}) < \gamma_t, \min(\gamma_3^{x_1}, \gamma_3^{x_2}) < \gamma_t])$,this probability denoted by OP_6. The outage probability at D can be formulated by:

$$\begin{aligned} OP_{D_2}^{1\,relay} &= \underbrace{\Pr[(\min(\gamma_1^{x_1}, \gamma_1^{x_2}) < \gamma_t, \min(\gamma_5^{x_1}, \gamma_5^{x_2}) < \gamma_t)]}_{OP_5} \\ &+ \underbrace{\Pr[\min(\gamma_1^{x_1}, \gamma_1^{x_2}) \geqslant \gamma_t, \min(\gamma_5^{x_1}, \gamma_5^{x_2}) < \gamma_t, \min(\gamma_3^{x_1}, \gamma_3^{x_2}) < \gamma_t]}_{OP_6} \end{aligned} \tag{23}$$

The probabilities OP_5 and OP_6 can be obtained by substituting the SINRs and SINRs $\gamma_1^{x_1}$, $\gamma_1^{x_2}$, $\gamma_5^{x_1}$, $\gamma_5^{x_2}$, $\gamma_3^{x_1}$, and $\gamma_3^{x_2}$ into (23) as follows

$$\begin{aligned} OP_5 &= \underbrace{\Pr\left[\min\left(\frac{a_1(1-\rho)\gamma_0 g_1}{a_2(1-\rho)\gamma_0 g_1+(1-\rho+\mu)}, \frac{a_2(1-\rho)\gamma_0 g_1}{1-\rho+\mu}\right)<\gamma_t\right]}_{OP_{5.1}} \\ &\quad\times\underbrace{\Pr\left[\min\left(\frac{a_1\gamma_0 g_5}{a_2\gamma_0 g_5+1+\mu}, \frac{a_2\gamma_0 g_5}{1+\mu}\right)<\gamma_t\right]}_{OP_{5.2}} \end{aligned} \tag{24}$$

$$\begin{aligned} OP_6 &= \underbrace{\Pr\left[\min\left(\frac{a_1\gamma_0 g_5}{a_2\gamma_0 g_5+1+\mu}, \frac{a_1\gamma_0 g_5}{1+\mu}\right)<\gamma_t\right]}_{OP_{6.1}} \\ &\quad\times\underbrace{\Pr\left[\begin{array}{l}\min\left(\frac{a_1(1-\rho)\gamma_0 g_1}{a_2(1-\rho)\gamma_0 g_1+(1-\rho+\mu)}, \frac{a_2(1-\rho)\gamma_0 g_1}{1-\rho+\mu}\right)\geqslant\gamma_t, \\ \min\left(\frac{b_1\eta\rho\gamma_0 g_1 g_3}{b_2\eta\rho\gamma_0 g_1 g_3+1+\mu}, \frac{b_2\eta\rho\gamma_0 g_1 g_3}{1+\mu}\right)<\gamma_t\end{array}\right]}_{OP_{6.2}} \end{aligned} \tag{25}$$

where $OP_{5.1}$ can be obtained from OP_1 as

$$OP_{5.1} = \begin{cases} \left(1-e^{-\frac{\lambda_1\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}\right) & if \quad a_1 < a_2\left(1+\gamma_t\right) \\ \left(1-e^{-\frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}}\right) & if \quad a_1 \geqslant a_2\left(1+\gamma_t\right) \end{cases} \tag{26}$$

$OP_{5.2} = OP_{6.1}$ is expressed as

$$\begin{aligned} OP_{5.2} = OP_{6.1} &= 1-\Pr\left[g_5 \geqslant \frac{(1+\mu)\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}, g_5 \geqslant \frac{(1+\mu)\gamma_t}{a_2\gamma_0}\right] \\ &= \begin{cases} F_{g_5}\left(\frac{(1+\mu)\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}\right) & if\ a_1-a_2\gamma_t < a_2 \\ F_{g_5}\left(\frac{(1+\mu)\gamma_t}{a_2\gamma_0}\right) & if\ a_1-a_2\gamma_t \geqslant a_2 \end{cases} \\ &= \begin{cases} \left(1-e^{\frac{-\lambda_5\omega_1\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}\right) & if \quad a_1 < a_2\left(1+\gamma_t\right) \\ \left(1-e^{\frac{-\lambda_5\omega_1\gamma_t}{a_2\gamma_0}}\right) & if \quad a_1 \geqslant a_2\left(1+\gamma_t\right) \end{cases} \end{aligned} \tag{27}$$

$OP_{6.2}$ is derived from Appendix B as

$$OP_{6.2} = \begin{cases} e^{-\frac{\lambda_1 \omega_2 \gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}} - \sum\limits_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{(b_1 - b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}\right), & if \begin{cases} b_1 < b_2(1+\gamma_t) \\ a_1 < a_2(1+\gamma_t) \end{cases} \\ e^{-\frac{\lambda_1 \omega_2 \gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}} - \sum\limits_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{b_2\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}\right), & if \begin{cases} b_1 \geqslant b_2(1+\gamma_t) \\ a_1 < a_2(1+\gamma_t) \end{cases} \\ e^{-\frac{\lambda_1 \omega_2 \gamma_t}{a_2\gamma_0}} - \sum\limits_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{(b_1 - b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}\right), & if \begin{cases} b_1 < b_2(1+\gamma_t) \\ a_1 \geqslant a_2(1+\gamma_t) \end{cases} \\ e^{-\frac{\lambda_1 \omega_2 \gamma_t}{a_2\gamma_0}} - \sum\limits_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{b_2\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}\right), & if \begin{cases} b_1 \geqslant b_2(1+\gamma_t) \\ a_1 \geqslant a_2(1+\gamma_t) \end{cases} \end{cases} \tag{28}$$

The outage probability at D_2 in the case of one relay can be obtained by substituting the equations from (24) to (28) into (23) as

$$OP_{D_2}^{1\ relay} = \begin{cases} \left(1 - e^{-\frac{\lambda_5\omega_1\gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}}\right)\left[1 - \sum\limits_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{(b_1 - b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}\right)\right], & if \begin{cases} b_1 < b_2(1+\gamma_t) \\ a_1 < a_2(1+\gamma_t) \end{cases} \\ \left(1 - e^{-\frac{\lambda_5\omega_1\gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}}\right)\left[1 - \sum\limits_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{b_2\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{(a_1 - a_2\gamma_t)\gamma_0}\right)\right], & if \begin{cases} b_1 \geqslant b_2(1+\gamma_t) \\ a_1 < a_2(1+\gamma_t) \end{cases} \\ \left(1 - e^{-\frac{\lambda_5\omega_1\gamma_t}{a_2\gamma_0}}\right)\left[1 - \sum\limits_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{(b_1 - b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}\right)\right], & if \begin{cases} b_1 < b_2(1+\gamma_t) \\ a_1 \geqslant a_2(1+\gamma_t) \end{cases} \\ \left(1 - e^{-\frac{\lambda_5\omega_1\gamma_t}{a_2\gamma_0}}\right)\left[1 - \sum\limits_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{b_2\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}\right)\right], & if \begin{cases} b_1 \geqslant b_2(1+\gamma_t) \\ a_1 \geqslant a_2(1+\gamma_t) \end{cases} \end{cases} \tag{29}$$

4 Result and Disscussion

This section provide result and discussion of the outage performance at both D_1 and D_2 in both cases of one and N relays via Monte Carlo simulation and theoretical results. In a two-dimensional plane, the corrdinates of the source S, the destinations D_1, D_2, and the cluster of relays are $(0, 0)$, $(1, 0.3)$, $(0.8, -0.3)$, and $(x_R, 0)$, respectively. Hence, we obtain the normalize distances $d_1 = |x_R|$, $d_2 = \sqrt{(1 - x_R)^2 + 0.3^2}$, $d_3 = \sqrt{(0.8 - x_R)^2 + 0.3^2}$, $d_4 = \sqrt{1 + 0.3^2}$, $d_5 = \sqrt{0.8^2 + 0.3^2}$. We assume that the path-loss exponent $\beta = 3$, the target $\gamma_t = 1$, and $\mu = 1$.

In Fig. 2, the outage probabilities at D_1 and D_2 versus power splitting ratio $\rho \in (0.1, 0.9)$ (for relay located between source and destinations) are investigated. It can be seen that at ρ around 0.7, the outage performances of almost cases in this scenario are obtained the best performance because it is the optimal position for relay to decode information and harvest energy from source in the first phase and forward information to destination in the second phase. We note that D_2 locates nearer the source and relay than D_1 does, therefore the outage performance at D_2 is better than D_1 in the case of the power allocation for symbol x_2 and x_1 is nearly similar like $(a_1, a_2) = (b_1, b_2) = (0.6, 0.4)$, but in the case that the power allocation for x_1 is much higher than that for x_2, the outage performance at D_1 is higher than that at D_2.

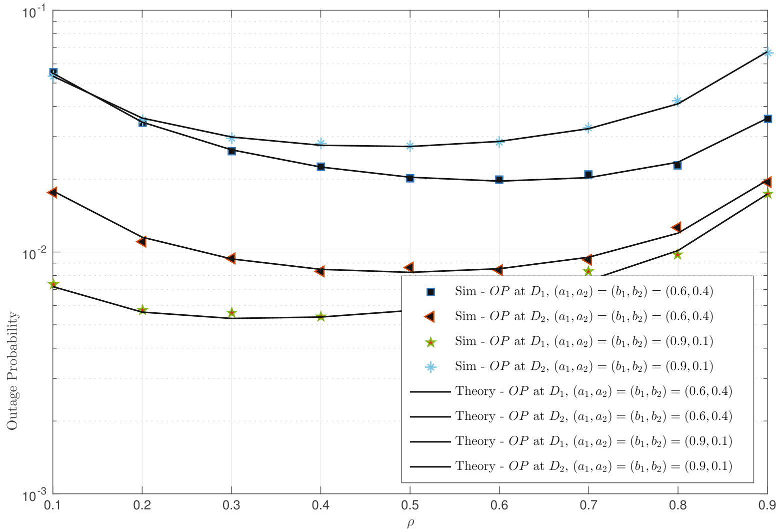

Fig. 2. Effect of power allocation a_1, a_2, b_1, and b_2 on the outage probability at D_1 and D_2 versus ρ, when $x_R = 0.4$, $\gamma_0 = 15$ dB, $\rho = 0.5$, and $\eta = 0.9$.

5 Conclusions

In this paper, we consider energy harvesting technique in the NOMA relaying networks. Partial relay selection scheme is applied to improve the system performance. The closed-form expressions of the outage probability are presented to evaluation and comparison of the performance at two destinations in both cases of single and multiple relays. These theoretical expressions are derived using the Monte Carlo simulation method. The theoretical results match the simulation results well.

A Appendix A: Finding the Closed-Form of Probability $\Pr\left[g_1 \geqslant u_1, g_2 < \frac{u_2}{g_1}\right]$

By using the PDF of RV g_1 and CDF of RV g_2, the probability $\Pr\left[g_1 \geqslant u_1, g_2 < \frac{u_2}{g_1}\right]$ can be obtained as

$$\begin{aligned}
\Pr\left[g_1 \geqslant u_1, g_2 < \frac{u_2}{g_1}\right] &= \int_{u_1}^{\infty} f_{g_1}(x) F_{g_2}\left(\frac{u_2}{x}\right) dx \\
&= \int_{u_1}^{\infty} \lambda_1 e^{-\lambda_1 x}\left(1 - e^{-\frac{\lambda_2 u_2}{x}}\right) dx \\
&= e^{-\lambda_1 u_1} - \underbrace{\int_{u_1}^{\infty} \lambda_1 e^{-\lambda_1 x} e^{-\frac{\lambda_2 u_2}{x}} dx}_{I_1}
\end{aligned} \tag{A.1}$$

To calculate the integral I_1, we first apply the Eq. 1.211 of [6]: $e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}$ to the term $e^{-\frac{\lambda_2 u_2}{x}}$ to obtain (A.2.1), then using Eq. 3.381.3 of [6]: $\int_u^{\infty} x^{v-1} e^{-\mu x} dx = \frac{1}{\mu^v}\Gamma(v, \mu u)$ to obtain (A.2.2) as follows

$$\begin{aligned}
I_1 &\overset{(A.2.1)}{=} \lambda_1 \sum_{k=0}^{\infty} \frac{1}{k!}(-\lambda_2 u_2)^k \int_{u_1}^{\infty} \frac{e^{-\lambda_1 x}}{(x)^k} dx \\
&\overset{(A.2.2)}{=} \sum_{k=0}^{\infty} \frac{1}{k!}(-\lambda_1 \lambda_2 u_2)^k \Gamma(1-k, \lambda_1 u_1)
\end{aligned} \tag{A.2}$$

By substituting (A.3) into (A.1), we obtain:

$$\Pr\left[g_1 \geqslant u_1, g_2 < \frac{u_2}{g_1}\right] = e^{-\lambda_1 u_1} - \sum_{k=0}^{\infty} \frac{1}{k!}(-\lambda_1 \lambda_2 u_2)^k \Gamma(1-k, \lambda_1 u_1) \tag{A.3}$$

B Appendix B: Proof of Eq. (28)

First, for the case of $a_1 < a_2(1+\gamma_t)$, the probability $OP_{6.2}$ in (25) can be rewritten as

$$\begin{aligned}
&OP_{6.2}|_{a_1 < a_2(1+\gamma_t)} = \Pr\left[\begin{array}{l} g_1 \geq \frac{(1-\rho+\mu)\gamma_t}{(a_1 - a_2\gamma_t)(1-\rho)\gamma_0} \\ \min\left(\frac{b_1 \eta\rho\gamma_0 g_1 g_3}{b_2 \eta\rho\gamma_0 g_1 g_3 + 1 + \mu}, \frac{b_2 \eta\rho\gamma_0 g_1 g_3}{1+\mu}\right) < \gamma_t \end{array}\right] \\
&= \Pr\left[\begin{array}{l} g_1 \geq \frac{(1-\rho+\mu)\gamma_t}{(a_1 - a_2\gamma_t)(1-\rho)\gamma_0} \\ \frac{b_1 \eta\rho\gamma_0 g_1 g_3}{b_2 \eta\rho\gamma_0 g_1 g_3 + 1 + \mu} < \frac{b_2 \eta\rho\gamma_0 g_1 g_3}{1+\mu}, \frac{b_1 \eta\rho\gamma_0 g_1 g_3}{b_2 \eta\rho\gamma_0 g_1 g_3 + 1 + \mu} < \gamma_t \end{array}\right] \\
&+ \Pr\left[\begin{array}{l} g_1 \geq \frac{(1-\rho+\mu)\gamma_t}{(a_1 - a_2\gamma_t)(1-\rho)\gamma_0} \\ \frac{b_1 \eta\rho\gamma_0 g_1 g_3}{b_2 \eta\rho\gamma_0 g_1 g_3 + 1 + \mu} \geq \frac{b_2 \eta\rho\gamma_0 g_1 g_3}{1+\mu}, \frac{b_2 \eta\rho\gamma_0 g_1 g_3}{1+\mu} < \gamma_t \end{array}\right] \\
&= \underbrace{\Pr\left[\begin{array}{l} g_1 \geq \frac{(1-\rho+\mu)\gamma_t}{(a_1 - a_2\gamma_t)(1-\rho)\gamma_0} \\ g_3 > \frac{(b_1 - b_2)(1+\mu)}{(b_2)^2 \eta\rho\gamma_0 g_1}, g_3 < \frac{(1+\mu)\gamma_t}{(b_1 - b_2\gamma_t)\eta\rho\gamma_0 g_1} \end{array}\right]}_{OP_{6.2.1}} + \underbrace{\Pr\left[\begin{array}{l} g_1 \geq \frac{(1-\rho+\mu)\gamma_t}{(a_1 - a_2\gamma_t)(1-\rho)\gamma_0} \\ g_3 \leq \frac{(b_1 - b_2)(1+\mu)}{(b_2)^2 \eta\rho\gamma_0 g_1}, g_3 < \frac{(1+\mu)\gamma_t}{b_2 \eta\rho\gamma_0 g_1} \end{array}\right]}_{OP_{6.2.2}}
\end{aligned} \tag{B.1}$$

where $OP_{6.2.1}$ and $OP_{6.2.2}$ are given as

$$OP_{6.2.1} = \begin{cases} \int\limits_{\frac{\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}^{\infty} f_{g_1}(x)\left[F_{g_3}\left(\frac{\omega_3\gamma_t}{(b_1-b_2\gamma_t)\gamma_0 x}\right) - F_{g_3}\left(\frac{(b_1-b_2)\omega_3}{(b_2)^2\gamma_0 x}\right)\right]dx, & if\ b_1 < b_2(1+\gamma_t) \\ 0 & if\ b_1 \geq b_2(1+\gamma_t) \end{cases} \tag{B.2}$$

$$OP_{6.2.2} = \begin{cases} \Pr\left[\begin{array}{l} g_1 \geq \frac{\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0} \\ g_3 \leq \frac{(b_1-b_2)\omega_3}{(b_2)^2\gamma_0 g_1} \end{array}\right] & if\ b_1 < b_2(1+\gamma_t) \\ \Pr\left[\begin{array}{l} g_1 \geq \frac{\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0} \\ g_3 < \frac{\omega_3\gamma_t}{b_2\gamma_0 g_1} \end{array}\right] & if\ b_1 \geq b_2(1+\gamma_t) \end{cases}$$
$$= \begin{cases} \int\limits_{\frac{\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}^{\infty} f_{g_1}(x)\left[F_{g_3}\left(\frac{(b_1-b_2)\omega_3}{(b_2)^2\gamma_0 g_1}\right)\right]dx, & if\ b_1 < b_2(1+\gamma_t) \\ \int\limits_{\frac{\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}^{\infty} f_{g_1}(x)\left[F_{g_3}\left(\frac{\omega_3\gamma_t}{b_2\gamma_0 g_1}\right)\right]dx, & if\ b_1 \geq b_2(1+\gamma_t) \end{cases} \tag{B.3}$$

By substituting (B.2) and (B.3) into (B.1), and using the result in Appendix A, we obtain

$$OP_{6.2}|_{a_1<a_2(1+\gamma_t)} = OP_{6.2.1} + OP_{6.2.2}$$
$$= \begin{cases} \int\limits_{\frac{\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}^{\infty} f_{g_1}(x)\left[F_{g_3}\left(\frac{\omega_3\gamma_t}{(b_1-b_2\gamma_t)\gamma_0 g_1}\right)\right]dx, & if\ b_1 < b_2(1+\gamma_t) \\ \int\limits_{\frac{\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}}^{\infty} f_{g_1}(x)\left[F_{g_3}\left(\frac{\omega_3\gamma_t}{b_2\gamma_0 g_1}\right)\right]dx, & if\ b_1 \geq b_2(1+\gamma_t) \end{cases} \tag{B.4}$$
$$= e^{-\frac{\lambda_1\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}} - \begin{cases} \sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{(b_1-b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}\right), & if\ b_1 < b_2(1+\gamma_t) \\ \sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{b_2\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{(a_1-a_2\gamma_t)\gamma_0}\right), & if\ b_1 \geq b_2(1+\gamma_t) \end{cases}$$

Next, we can obtain the result for $OP_{6.2}$ in the case of $a_1 \geq a_2(1+\gamma_t)$ from (B.4) with replacing '$(a_1 - a_2\gamma_t)$' by 'a_2' as

$$OP_{6.2}|_{a_1\geq a_2(1+\gamma_t)} = e^{-\frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}} - \begin{cases} \sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{(b_1-b_2\gamma_t)\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}\right), & if\ b_1 < b_2(1+\gamma_t) \\ \sum_{k=0}^{\infty}\frac{1}{k!}\left(-\frac{\lambda_1\lambda_3\omega_3\gamma_t}{b_2\gamma_0}\right)^k \Gamma\left(1-k, \frac{\lambda_1\omega_2\gamma_t}{a_2\gamma_0}\right), & if\ b_1 \geq b_2(1+\gamma_t) \end{cases} \tag{B.5}$$

By combining (B.4) and (B.5), we finish the proof for Eq. (28).

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