

# Optimizing Vehicle Routing with Path and Carbon Dioxide Emission for Municipal Solid Waste Collection in Ha Giang, Vietnam

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**Abstract.** Municipal solid waste (MSW) management issues emerged in many countries due to the steadily increasing population over the last decade, followed by the rising amount of solid waste generated. In most of the urban areas, current waste collection are already overloaded arising from the lack of facilities and insufficient resources. Mathematical optimization models are known to propose useful solutions that get multi-objectives and save cost for decision-makers. In this paper, Geographic Information System (GIS) analysis, integer linear programming (ILP) and mixed integer linear programming (MILP) for optimizing vehicle routing and carbon dioxide emission of municipal solid waste collection will be proposed. Firstly, GIS analysis for the real urban data is handled. Then vehicle routing optimization models considering path and carbon dioxide emission using ILP, MILP are developed. Finally, the results of proposal optimized models have been implemented in a case study in Ha Giang City, Vietnam. Concretely, the total cost the MSW collection using the ILP proposal model is reduced by from 7% to 13.7%, and MILP proposal model is reduced by from 15.1% to 21.5%.

**Keywords:** Municipal solid waste  
Mixed Integer linear programming · Optimization · Simulation  
GIS · Waste management

## 1 Introduction

Daily activities at homes, hospitals, schools, businesses, industries and so on are primarily sources, that generated municipal solid waste (MSW). Multiple components such as population, waste generation rates, technology, resident behavior,

the state of the economy relate to the municipal solid waste management [16]. In many municipalities of cities in developing countries, the transportation cost of the waste to different facilities such as transfer stations, temporary storage sites, landfills and also the fixed costs and operational of these facilities are the total cost of the solid waste management [10].

Thus, many problems about route optimization of waste collection were asserted in order to achieve effective waste management system. Reduction of operational expenses and optimization of vehicle fleet size need to be researched. The aim of optimizing waste collection vehicle routing is to minimize on fuel consumption rather than finding shortest distances. This can be explained that in many city areas, some of the shorter distances may be inconvenient for driving leading to more fuel consumption, pollution, and/or congestion [10].

Routing involves the use of extensive spatial data making it possible to use new technologies such as Geographic Information System (GIS). GIS is able to provide effective handling, display and manipulation of both geographic and spatial information. GIS also plays a potential role for solving various types of engineering and management problems in siting waste disposal sites. GIS supports development of a multi objective model, especially in collection vehicle routing and scheduling for solid waste management systems such as reduction of travel time, cost of site selection and provides a data bank for future monitoring [10].

Researchers highlighted that mathematical optimization models can propose useful solutions that obtain multi-objectives and save cost for decision-makers. Integer linear programming (ILP) and mixed-integer linear programming (MILP) empowered many researches in optimizing vehicle routing (see [4, 10, 12, 16]). Concretely, Archetti *et al.* [2] proposed an integer programming formulation to solve the orienteering vehicle routing problem. They maximized the total collected profit while satisfying a maximum time duration of the route where ILP solutions are obtained by branch-and-cut algorithm [3]. Moreover, according to Lee *et al.* [11] if a user only has some preliminary data which are restricted only to integers, ILP is appropriate to use. If other specific data are available, which includes both integers and non-integers, the user can use mixed integer programming approach that is expected to give a more accurate solution. Decision makers can choose different approaches based on their preferences and actual situations.

### 1.1 Case Study: Ha Giang Province, Vietnam

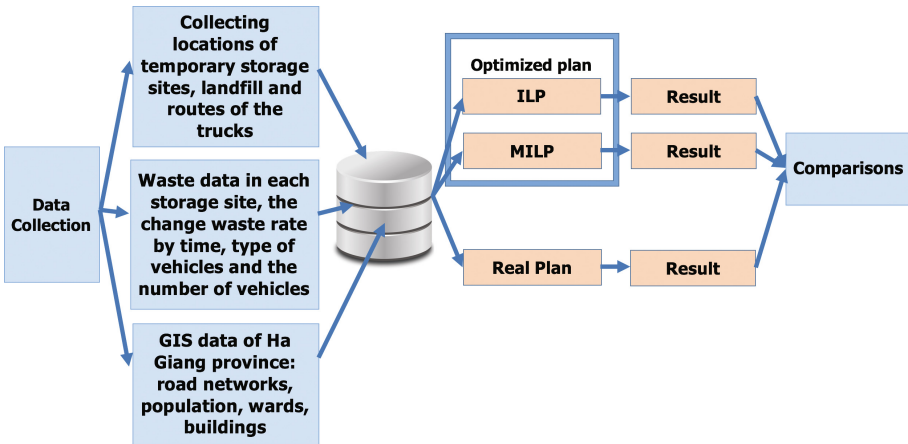
Ha Giang is located in the northernmost region of Vietnam, with mountainous and rocky topography. It shares a common international border of 270 Km with China. Ha Giang has an area of nearly 8000 km<sup>2</sup> and a population of 806,702 (2015). It includes one city, ten districts, five wards, 13 towns, and 177 communes.

Ha Giang city is located in the centre of Ha Giang province with the total area is 133,5 km<sup>2</sup> are habitat of 54,240 people. It is also the economic, cultural

and politic center of the province. Tourism is especially developed in recent years. The sharp development in the number of tourists, followed by economic growth and uncontrolled urbanization, has greatly magnified Ha Giang with many problems regarding the exponential amounts of solid waste generation.

The current waste collection and transportation is already overtaxed due to the lack of physical facilities and insufficient human and technical resources as can be evidenced by low collection rates and inefficiency waste transportation. How to deal with its solid waste in Ha Giang city will only become more and more critical.

Hagiang city is currently facing challenges of solid waste management. On an average, about from 1500 to 2500 metric tons of waste is generated in the City per day. The city has a single recognized landfill A. The majority of the remaining waste is indiscriminately disposed of in drainage channels or open land spaces, where it is later burned. Only a small proportion of the waste comprising of plastics and metals is re-used or recycled (Fig. 1).



**Fig. 1.** Methodology structure to find the routes for waste collection in municipal waste management.

This paper is divided into four sections: the introduction represents the important role of MSW and research objects in municipal waste management with related works being investigated; Sect. 2 describes the problem from mathematical view. Two models are built for finding the optimal solution for waste collection problem: MILP and ILP models; we show some computational experiments applying proposed approach for particular cases in Ha Giang, Vietnam; the last subsection is conclusion and discussion.

## 2 Optimization Formulation for Municipal Solid Waste Collection Problem

### 2.1 Research Problem of Vehicle Collection Routing

Two main processes of municipal solid waste (MSW) collection are described in Fig. 2. Firstly, the waste are collected from different sources (such as offices, schools, hospitals, ...) and gathered to the nearest collection centre. Each collection centre temporarily stores a huge amount of waste from its nearby sources. Twice per day, there are trucks starting from a depot, traveling through these collection centres on an assigned schedule to collect all waste in its route and finishing the route by returning to the depot.

We are interested in the latter part of the process when municipal solid waste is collected by the trucks to the depot. Because, as proposed by [6], in contemporary MSW management systems, the total management cost is mainly used for waste collection and transportation, namely 80–90% in low-income countries, and 50–80% in middle-income countries.

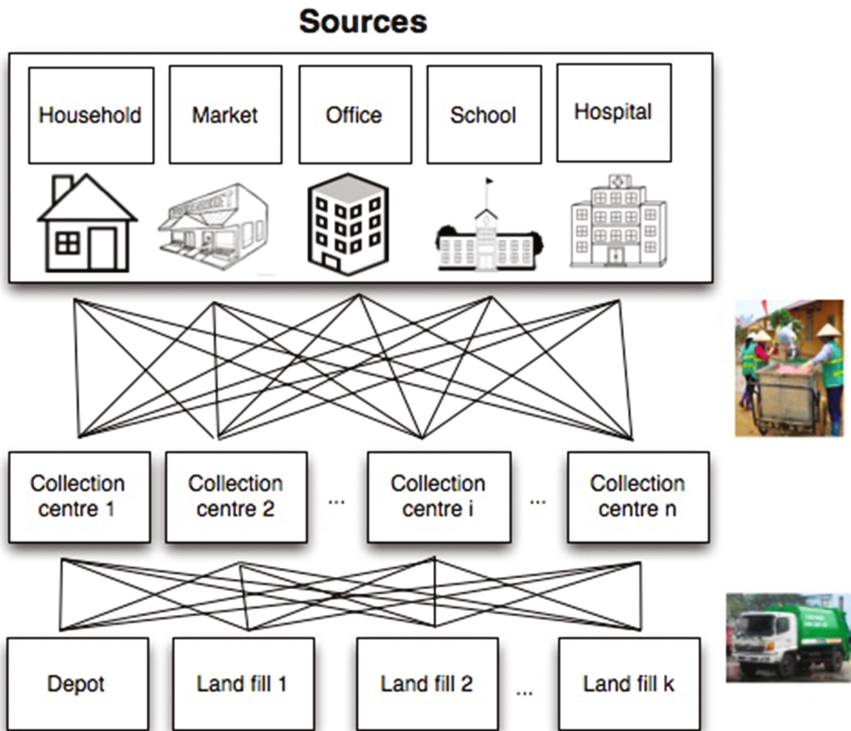


Fig. 2. Architecture of municipal solid waste collection and transportation.

In this paper, we assume these following statements:

- There exists feasible collecting tours of which total volume of waste is less than or equal to the capacity of a truck. By this assumption, our problem is also considered as a Vehicle Routing Problem (VRP). For the VRPs which violate this rule, we can separate the problem into many classic VRPs;
- The landfill is also the depot;
- The distance between any two nodes is well-defined;
- The demand of each waste collection centre is known.

In this section, we consider two equation-based models to identify the optimized plan. The first model is an integer linear programming problem, and the second is a mixed integer programming problem. Let  $G = (V, A)$  be the directed graph that indicates the route of a vehicle, where  $V$  is a set of collection centres, i.e.  $V = \{v_0, \dots, v_n\}$ , and  $A$  is a set of arcs representing path connecting vertices, i.e.  $A = \{(v_i, v_j) \mid v_i, v_j \in V, i \neq j\}$ .

Let  $v_0$  be the depot (also the landfill).  $V' = V \setminus \{v_0\}$  is the subset of  $V$  that includes  $n$  collection centres. Let  $S$  be a subset of  $V'$  containing all routes that satisfy all constraints of our objectives. Let  $C$  be the matrix of non-negative travel costs, where  $c_{ij}$  denotes the cost of traveling from  $v_i$  to  $v_j$ . Let  $m$  be the number of vehicles available. Each vehicle has a capacity of  $q$ . All vehicles stop at the landfill.

We define some relevant variables as follows:

- Let  $A(S) = \{(v_i, v_j) \in V' \mid v_i \in S, v_j \in S\}$  be the set of edges joining all pairs of collection centers in  $S$ ;
- Let  $x_{ij}$  be the number of times that edge  $(v_i, v_j)$  is traveled.  $X$  is a matrix of  $x_{ij}$ ,  $X = [x_{ij}]_{i,j=\overline{0},n}$ ;
- Let a positive  $q_i$  be the weight of solid waste in collection centre  $i$ ,  $i = 1, \dots, n$ .

### 2.2 The Objective Function

We consider two factors for the objective function. The first factor is transportation cost, which is given by

$$F_c = \sum_{i,j=0}^n c_{ij}x_{ij}. \tag{1}$$

The second one is the emission factor as proposed by [12], which is given by

$$F_e = \sum_{k=1}^m W_k D_k \frac{ER_k}{CR_k \times LF_k}, \tag{2}$$

where  $W_k$  is the total weight of waste transported by truck  $k$ ,  $ER_k$  is the carbon dioxide emission rate of fuel (kgCO<sub>2</sub>/l),  $CR_k$  stands for the fuel consumption rate (km/l),  $LF_k$  is the load factor for truck  $k$ , representing the average weight

of waste for each truck.  $D_k$  is the total transport distance of truck  $i$  (km). Let  $A_k \subseteq A$  be the set arcs traveled by truck  $k$ . Since the cost matrix  $C$  is relatively calculated on the distances between collection centres,  $D_k$  can be expressed as

$$D_k = \frac{1}{\mu} \sum_{(v_i, v_j) \in A_k} c_{ij} x_{ij}, \quad (3)$$

where  $\mu$  is the travel cost per kilometer of a truck (dollar/km).

The final objective function for optimal path calculation is formulated to minimize the cost for the municipal solid waste collection system with an effort to minimize the impact of carbon dioxide emission. In general, we cannot form an objective function by adding the two objective factors because they do not have the same dimensions, namely dollar per hour and ton per hour, respectively. To deal with this problem we use an approach of the weighted sum method [20], where each factor is assigned a weight to determine its importance. The combined objective function (4) also includes a penalty coefficient

$$\sigma = \frac{\max(F_c)}{\max(F_e)} (\text{km/dollar})$$

that is necessary to normalize the dimension and value range of each factor. Due to the characteristics of the constraints in our following sections, both  $\max(F_c)$  and  $\max(F_e)$  can be easily calculated using any optimization method to minimize an affine function over a polyhedral convex set. The objective function is defined as

$$F_w = wF_c + (1 - w)\sigma F_e, \quad (4)$$

where  $w$  is a user-specified weight, which can be any number between 0 and 1. In this paper, we consider the case that two factors are treated equally by setting  $w$  equal to 0.5. Equation 4 is rewritten as follows:

$$F = F_c + \sigma F_e. \quad (5)$$

### 2.3 Integer Linear Programming Model

In Eq. 2, considering the case same type of truck is used, we assume  $ER_i = ER$ ,  $CR_i = CR$ ,  $LF_i = LF$ ,  $W_i = q$ ,  $i = 1, \dots, m$ .

We consider the following integer linear programming model, of which objective function is derived from (5):

$$\min F = \left( \sum_{(v_i, v_j) \in A} c_{ij} x_{ij} \right) \left( 1 + \sigma \mu q \frac{ER}{CR \times LF} \right) \quad (6)$$

subject to:

$$\sum_{j=0, j \neq i}^n x_{ji} = \sum_{j=0, j \neq i}^n x_{ij}, \quad i = 1, \dots, n; \tag{7}$$

$$\sum_{i=0, i \neq j}^n x_{ij} \geq 1, \quad j = 1, \dots, n; \tag{8}$$

$$\sum_{j=0, j \neq i}^n x_{ij} \geq 1, \quad i = 1, \dots, n; \tag{9}$$

$$\sum_{i=0}^n x_{i0} = m; \tag{10}$$

$$\sum_{j=0}^n x_{0j} = m; \tag{11}$$

$$\sum_{v_i \notin S} \sum_{v_j \in S} x_{ij} \geq \sum_{v_k \in S} \frac{q_k}{q}, \quad \forall S \subseteq V', S \neq \emptyset; \tag{12}$$

$$x_{ij} \in \{0, 1\}, \forall i, j \in \{0, \dots, n\}. \tag{13}$$

The classical assessment restrictions (7)–(9) ensure that each collection centre is visited at least one time. Constraints (10) and (11) indicate the number of trucks in our model and make sure that all the trucks finish their routes at the landfill. Constraints (12) are the capacity cut constraints, which impose that the routes must be connected and that the demand on each route must not exceed the vehicle capacity. These are known to include an exponential number of constraints. Some suggestions are proposed such as to consider a smaller subset of these constraints, or limit to the ones which have a polynomial cardinality (MTZ constraints, see [4, 14]). Constraints (13) represent an usual assignment decision logic.

### 2.4 Mixed Integer Linear Programming Model

Apart from variables and definitions in Sect. 2.1, we add some variables as follows:

- Let  $y_{ij}$  present the total remaining load of all vehicles when crossing arc  $(v_i, v_j) \in A$ .  $Y$  is a matrix of  $y_{ij}$ ,  $Y = [y_{ij}]_{i, j = \overline{1, m}}$ ;
- Let  $z_i$  be the number of times that collection centre  $i$  is visited by vehicles.

The problem is formulated with the same objective function of (6) and constraints replaced with (14)–(23).

$$\sum_{j|(v_i, v_j) \in A} x_{ij} = \sum_{j|(v_i, v_j) \in A} x_{ji} = z_i, \quad \forall i = 1, \dots, n; \tag{14}$$

$$\sum_{j|(v_i, v_j) \in A} y_{ij} - \sum_{j|(v_i, v_j) \in A} y_{ji} = \begin{cases} -q_i & \text{if } i \neq 0 \\ \sum_{i \in V'} q_i & \text{if } i = 0 \end{cases}; \quad (15)$$

$$y_{ij} \leq qx_{ij} \quad \forall (v_i, v_j) \in A; \quad (16)$$

$$\sum_{v_i \in V'} z_i \leq n + m - 1; \quad (17)$$

$$0 \leq x_{ij} \leq m, x_{ij} \in \mathbb{N}, \quad \forall (v_i, v_j) \in A(V) \setminus A(V'); \quad (18)$$

$$x_{ij} \in \{0, 1\} \quad \forall (v_i, v_j) \in A(V'); \quad (19)$$

$$\sum_{v_i \in V'} \frac{q_i}{q} \leq z_0 \leq m, z_0 \in \mathbb{N}; \quad (20)$$

$$1 \leq z_i \leq m, \quad \forall i = 1, \dots, n; \quad (21)$$

$$y_{ij} \geq 0, \quad \forall (v_i, v_j) \in A; \quad (22)$$

$$y_{0i} = q, \quad \forall i \neq 0. \quad (23)$$

Constraints (14), (18)–(22) are obvious, followed by the way we define variables. Constraints (15) ensure the load balance. Constraints (16) prevent the vehicles from overloading. Constraints (17) limit the number of times a collection center is visited [2]. The final constraints (23) ensure the vehicles to leave the depot with empty load.

## 2.5 Algorithm for Finding Routes

We use an algorithm to obtain a feasible solution of the MSW problem, if exists, from a solution after solving the given ILP or MILP. We have this done by matching each pair of an incoming arc and an outgoing arc so that they satisfy the flow constraints.

Considering the solution obtained from the ILP/MILP, it can be seen that  $y$  variables only exist in MILP model. However, whether or not, the idea is quite the same. We now provide a pseudo code of procedure FINDROUTES, followed by a detailed explanation. The statements in square brackets are only applied for solutions obtained from MILP model and excluded when being used for ILP model.

At the beginning of the algorithm, we sort arcs in  $\text{Out}(i)$  in order of non-increasing value of  $y$  variables, while  $\text{In}(i)$  in order of non-decreasing value of  $y$  variables instead. We sequentially consider each arc in  $\text{Out}(i)$  and assign it to the first unassigned arc in  $\text{In}(i)$ . When the assigned incoming arc is arc  $(0, i)$ , if  $x_{0i} > 1$ , the current load of  $(0, i)$  is set equal to the load of outgoing arc,  $y_{0i}$  is decreased by the same amount, and  $x_{0i}$  is decreased by one. Otherwise, we assign the current load of arc  $(0, i)$  to be equal to  $y_{0i}$ .



**Algorithm 1.** Route finding algorithm

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1: procedure FINDROUTES
2:   for  $i = 1, \dots, n$  do
3:     [Determine  $\text{Out}(i)$  the set of non-increasing positive  $y_{ij}, j = 1, \dots, n$ ]
4:     [Determine  $\text{In}(i)$  the set of non-decreasing positive  $y_{ti}, t = 1, \dots, n$ ]
5:     for each arc  $(i, j) \in \text{Out}(i)$  do
6:       Match arc  $(i, j)$  to the first arc  $(t, i)$  in  $\text{In}(i)$  [such that  $y_{ti} \geq y_{ij}$ ]
7:       if there is no such arc then
8:         the Procedure terminates, no route feasible
9:       end if
10:      Set  $L_{ij} = (t, i)$ 
11:      if  $t \neq 0$  then
12:        [Set  $y_{L_{ij}} = y_{ti}$ ]
13:        Remove  $(t, i)$  from  $\text{In}(i)$ 
14:      else
15:        if  $x_{0i} > 1$  then
16:          Set  $[y_{L_{ij}} = y_{ij}, y_{0i} = y_{0i} - y_{ij}]x_{0i} = x_{0i} - 1$ 
17:          Insert arc  $(0, i)$  in the sorted set  $\text{In}(i)$ 
18:          [according to the new value of  $y_{0i}$ ]
19:        else
20:          [Set  $y_{L_{ij}} = y_{0i}$ ]
21:          Remove  $(0, i)$  from  $\text{In}(i)$ 
22:        end if
23:      end if
24:    end for
25:  end for
26: end procedure

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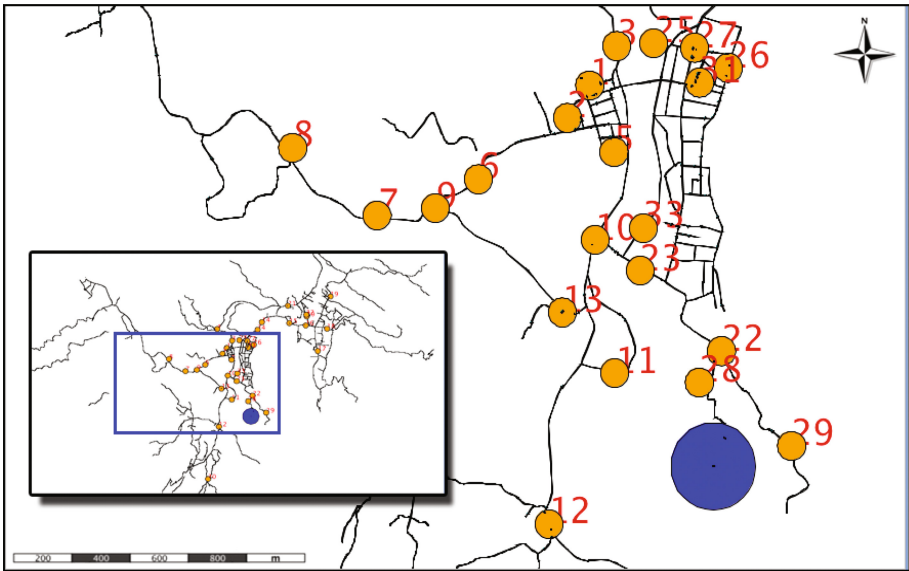
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### 3 Implementation and Application for Case Study in Ha Giang, Vietnam

Hagiang has 33 collection centres as shown in Fig. 3 (the yellow circles denote the collection centres, while the bigger blue circle indicates the location of depot). The waste generation rate is 181.4 tones or 365.4 m<sup>3</sup>. Hagiang has two types of trucks (35 m<sup>3</sup> and 50 m<sup>3</sup>) that are responsible for the collection and transportation at two regions. Each vehicle collects twice a day (in the morning and in the afternoon).

We present some computational experiments on eight scenarios described in Table 1. In the first four scenarios, we apply two proposed models with the bigger type of trucks. While the rest is for the smaller one. The optimized plans found are described in Table 2.

The comparison of the results obtained for the real plan and the optimized plans of each model are presented in Table 3 and Fig. 4. In Fig. 4, the blue columns indicate the results on current plan, while the white columns and the red columns indicate the results after being optimized when using ILP Model and MILP Model, respectively.

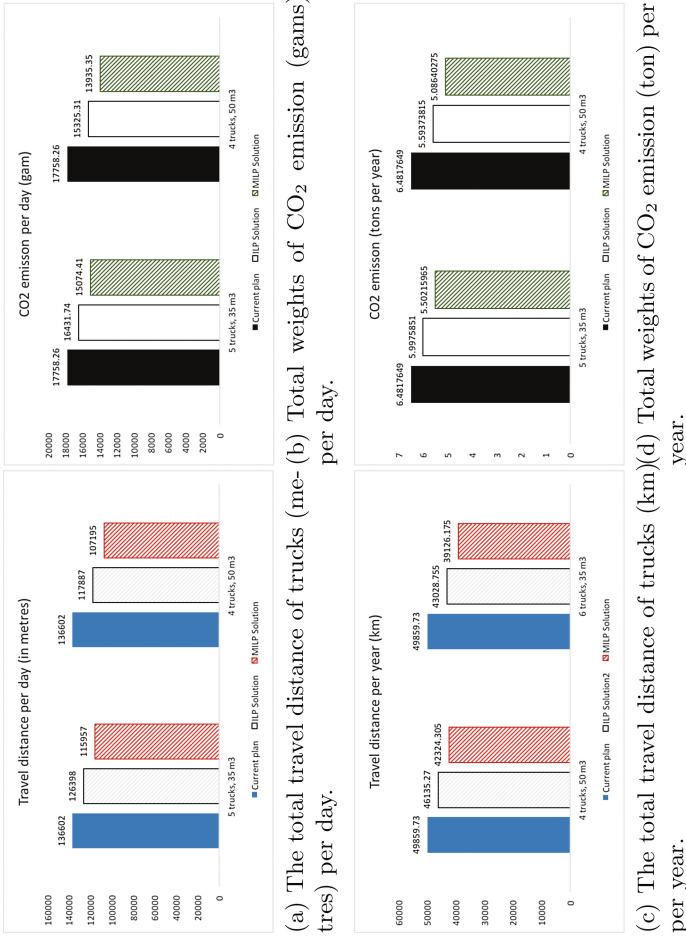


**Fig. 3.** GIS map of solid waste collection centres and depot in Ha Giang, Vietnam. The smaller map inside describes the locations of overall investigated region; the bigger map is a zoomed view of a smaller part; the yellow circles denote the collection centres, while the bigger blue circle indicates the location of depot. (Color figure online)

**Table 1.** Scenarios

|            | Number of trucks | Type of trucks    | Collection time | Methodology |
|------------|------------------|-------------------|-----------------|-------------|
| Scenario 1 | 4                | 50 m <sup>3</sup> | Morning         | ILP         |
| Scenario 2 | 4                | 50 m <sup>3</sup> | Morning         | MILP        |
| Scenario 3 | 4                | 50 m <sup>3</sup> | Afternoon       | ILP         |
| Scenario 4 | 4                | 50 m <sup>3</sup> | Afternoon       | MILP        |
| Scenario 5 | 5                | 35 m <sup>3</sup> | Morning         | ILP         |
| Scenario 6 | 5                | 35 m <sup>3</sup> | Morning         | MILP        |
| Scenario 7 | 5                | 35 m <sup>3</sup> | Afternoon       | ILP         |
| Scenario 8 | 5                | 35 m <sup>3</sup> | Afternoon       | MILP        |

Substantial differences can be observed in the values obtained both from the optimization of routes for distance and CO<sub>2</sub> based on MILP model, when compared to the ones estimated for the actual plan, and also ILP. In this case, the optimized solution finding by MILP is better than the solution finding by ILP.



**Fig. 4.** The results are compared among scenarios by different methodologies. The blue columns indicate the results on current plan, while the white columns and the red columns indicate the results after being optimized when using ILP model and MILP model, respectively. (Color figure online)

**Table 2.** Computational results

| Scenario              | Schedule  | Travel distance (metre) | CO <sub>2</sub> emission (gam) | Objective value |
|-----------------------|---|-------------------------|--------------------------------|-----------------|
| Real plan (morning)   | Route 1: 1 → 32 → 27 → 28 → 24 → 1  | 76674                   | 9967.62                        | 153348          |
|                       | Route 2: 1 → 22 → 21 → 33 → 18 → 19 → 20 → 17 → 16 → 25 → 28 → 34 → 24 → 1                |                         |                                |                 |
|                       | Route 3: 1 → 29 → 30 → 23 → 6 → 2 → 1   |                         |                                |                 |
|                       | Route 4: 1 → 4 → 5 → 3 → 7 → 8 → 9 → 10 → 14 → 1  |                         |                                |                 |
|                       | Route 5: 1 → 24 → 31 → 13 → 12 → 11 → 6 → 2 → 24 → 1                                      |                         |                                |                 |
| Real plan (afternoon) | Route 1: 1 → 27 → 28 → 34 → 24 → 1  | 59928                   | 7790.64                        | 119856          |
|                       | Route 2: 1 → 25 → 15 → 16 → 17 → 18 → 19 → 20 → 21 → 22 → 26 → 1                          |                         |                                |                 |
|                       | Route 3: 1 → 29 → 23 → 6 → 5 → 4 → 2 → 1  |                         |                                |                 |
|                       | Route 4: 1 → 3 → 7 → 8 → 9 → 10 → 14 → 13 → 12 → 11 → 1                                   |                         |                                |                 |
|                       | Route 5: 1 → 24 → 1   |                         |                                |                 |
| Scenario 1            | Route 1: 1 → 19 → 20 → 18 → 17 → 16 → 22 → 33 → 21 → 5 → 2 → 4 → 9 → 8 → 10 → 1           | 62820                   | 8166.60                        | 125640          |
|                       | Route 2: 1 → 24 → 1   |                         |                                |                 |
|                       | Route 3: 1 → 32 → 27 → 25 → 28 → 23 → 30 → 29 → 1   |                         |                                |                 |
|                       | Route 4: 1 → 34 → 12 → 13 → 31 → 11 → 14 → 7 → 3 → 6 → 1                                  |                         |                                |                 |
| Scenario 2            | Route 1: 1 → 24 → 12 → 13 → 31 → 14 → 9 → 8 → 10 → 7 → 3 → 24 → 23 → 1                    | 57941                   | 7532.33                        | 115882          |
|                       | Route 2: 1 → 27 → 25 → 11 → 34 → 28 → 32 → 23 → 30 → 1                                    |                         |                                |                 |
|                       | Route 3: 1 → 28 → 25 → 16 → 21 → 17 → 19 → 18 → 20 → 21 → 33 → 22 → 5 → 4 → 2 → 3 → 6 → 1 |                         |                                |                 |
|                       | Route 4: 1 → 29 → 1   |                         |                                |                 |
| Scenario 3            | Route 1: 1 → 5 → 4 → 6 → 11 → 16 → 19 → 18 → 20 → 21 → 22 → 17 → 15 → 1                   | 55067                   | 7158.71                        | 110134          |
|                       | Route 2: 1 → 25 → 26 → 28 → 27 → 1  |                         |                                |                 |
|                       | Route 3: 1 → 29 → 23 → 24 → 1   |                         |                                |                 |
|                       | Route 4: 1 → 34 → 14 → 13 → 12 → 8 → 9 → 10 → 7 → 3 → 2 → 1                               |                         |                                |                 |

(continued)

**Table 2.** (continued)

| Scenario   | Schedule   | Travel distance (metre) | CO <sub>2</sub> emission (gam) | Objective value |
|------------|--|-------------------------|--------------------------------|-----------------|
| Scenario 4 | Route 1: 1 → 24 → 6 → 7 → 8 → 9 → 10 → 14 → 12 → 13 → 11 → 23 → 1                    | 49254                   | 6403.02                        | 198508          |
|            | Route 2: 1 → 27 → 28 → 15 → 16 → 20 → 19 → 17 → 18 → 21 → 22 → 5 → 4 → 2 → 3 → 6 → 1 |                         |                                |                 |
|            | Route 3: 1 → 28 → 25 → 26 → 34 → 29 → 1  |                         |                                |                 |
|            | Route 4: 1 → 29 → 24 → 1   |                         |                                |                 |
| Scenario 5 | Route 1: 1 → 8 → 7 → 10 → 9 → 13 → 31 → 12 → 14 → 1                                  | 67765                   | 8809.45                        | 135530          |
|            | Route 2: 1 → 11 → 6 → 4 → 5 → 2 → 3 → 24 → 1   |                         |                                |                 |
|            | Route 3: 1 → 24 → 1  |                         |                                |                 |
|            | Route 4: 1 → 28 → 25 → 27 → 32 → 1   |                         |                                |                 |
|            | Route 5: 1 → 30 → 16 → 19 → 20 → 18 → 21 → 33 → 22 → 17 → 34 → 29 → 23 → 1           |                         |                                |                 |
| Scenario 6 | Route 1: 1 → 6 → 4 → 5 → 22 → 33 → 21 → 20 → 19 → 18 → 17 → 16 → 25 → 28 → 27 → 1    | 59978                   | 7797.14                        | 119956          |
|            | Route 2: 1 → 23 → 30 → 11 → 13 → 31 → 12 → 24 → 1                                    |                         |                                |                 |
|            | Route 3: 1 → 24 → 29 → 1   |                         |                                |                 |
|            | Route 4: 1 → 29 → 14 → 9 → 8 → 10 → 7 → 3 → 2 → 6 → 1                                |                         |                                |                 |
|            | Route 5: 1 → 34 → 28 → 32 → 1  |                         |                                |                 |
| Scenario 7 | Route 1: 1 → 15 → 28 → 27 → 29 → 1   | 58633                   | 7622.29                        | 117266          |
|            | Route 2: 1 → 21 → 22 → 20 → 19 → 18 → 17 → 16 → 25 → 26 → 1                          |                         |                                |                 |
|            | Route 3: 1 → 23 → 7 → 9 → 8 → 10 → 14 → 12 → 13 → 11 → 34 → 1                        |                         |                                |                 |
|            | Route 4: 1 → 24 → 1  |                         |                                |                 |
|            | Route 5: 1 → 24 → 6 → 3 → 2 → 4 → 5 → 1  |                         |                                |                 |
| Scenario 8 | Route 1: 1 → 9 → 8 → 10 → 7 → 3 → 2 → 4 → 6 → 34 → 1                                 | 55979                   | 7277.27                        | 111958          |
|            | Route 2: 1 → 24 → 14 → 12 → 13 → 11 → 23 → 1   |                         |                                |                 |
|            | Route 3: 1 → 27 → 28 → 15 → 16 → 19 → 18 → 17 → 20 → 21 → 22 → 5 → 4 → 1             |                         |                                |                 |
|            | Route 4: 1 → 28 → 25 → 26 → 1  |                         |                                |                 |
|            | Route 5: 1 → 29 → 24 → 1   |                         |                                |                 |

**Table 3.** Result comparison table. The table below compares the computational results among current plan, ILP and MILP optimized plan.

| Methodology  | Type of truck                         | Travel cost | Emission cost | Total cost | Percentage saved |
|--------------|---------------------------------------|-------------|---------------|------------|------------------|
| Current plan | 35 m <sup>3</sup> & 50 m <sup>3</sup> | 49859.73    | 6.4817549     | 99719.46   | 0%               |
| ILP          | 35 m <sup>3</sup>                     | 46135.27    | 5.9975851     | 92270.54   | 7.5%             |
|              | 50 m <sup>3</sup>                     | 43028.755   | 5.59373815    | 86057.51   | 13.7%            |
| MILP         | 35 m <sup>3</sup>                     | 42324.305   | 5.50215965    | 84648.61   | 15.1%            |
|              | 50 m <sup>3</sup>                     | 39126.175   | 5.08640275    | 78252.35   | 21.5%            |

## 4 Conclusion and Discussion

This paper proposes two models to optimize the collection and transportation of municipal solid waste, in term of costs and carbon dioxide emissions. These models compose of two steps: (i) firstly, the real and GIS data of municipal solid waste collection and transportation are collected and analyzed; (ii) secondly, the GIS data are integrated with vehicle routing problem in order to find optimized solutions. In the first model, we use an integer linear programming (ILP) approach, while it is the mixed integer linear programming (MILP) approach for the second one. The result of each model is then applied with an algorithm to determine the collection plan.

We presented some experiments in a case study of Hagiang, Vietnam. The results show that the optimized solution finding by MILP is better than the solution finding by ILP. Concretely, the total cost the MSW collection using the ILP proposed model is reduced by from 7% to 13.7%, and MILP proposed model is reduced by from 15.1% to 21.5%.

The achieved results of this paper are as follows:

- Proposing new models for optimizing municipal solid waste collection;
- Integrating two models (GIS analysis, ILP or MILP) are feasible;
- Giving the algorithm for finding route;
- Analysing and comparing path cost, carbon dioxide emission and total cost of eight scenarios to find the best strategy for the decision maker.

Last but not least, in the future works, we will consider multiple types of vehicles in the model and other pollutants (such as NO, PM and so on). Moreover, the objective function of this paper is linear, it is difficult to cover models with complicated factors such as the velocity of vehicles. Therefore, the objective function will be in non-linear form that is a challenge for researchers.

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