

Coupling Statistical and Agent-Based Models in the Optimization of Traffic Signal Control

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Abstract. There have been two directions to target to the problem of Traffic Signal Control (TSC): macroscopic and microscopic. On one hand, macroscopic help to find the optimal solution with an assumption of homogenization (both for vehicles and environment). On the other hand, microscopic one can take into account heterogeneity in vehicles as well as in environment. Therefore, it is very important to couple the two directions in the study of TSC. In this paper, we proposed to couple statistical and agent-based models for TSC problem in one intersection. The experiment results indicated that the proposed model is sufficient good in comparison with some others TSC strategies.

1 Introduction

Recently, Traffic Signal Control (TSC) is an important problem that has attracted the attention of many researchers [1, 12–19]. There are many different approaches to study TSC such as Markov state transition model [19], Mixed-integer programming [16], Kinematic wave model [12], fundamental diagram [13], Agent-based model [17, 18]. These approaches can be divided into two groups: macroscopic model and microscopic one. The strengths of the macroscopic models are able to find the optimal solution, but these approaches only consider the homogeneous vehicles as well as the homogeneous environment. In contrast, the strengths of microscopic model are taking into account many different behaviors of vehicles and heterogeneous environment. However, the weakness of the simulation model is hardly finding the optimal solution. Therefore, the combination of statistical and agent-based models uses advantage of each model.

In this paper, we focus on optimizing multiple performance indices (i.e., multi-objective traffic signal control) at each intersection. In the fact that traffic signal control can be viewed as a multi-objective optimization problem in two levels. First, the system level optimizes the routing on the whole transportation network. Second, the local level optimizes the routing at an intersection.

The multi-objective function can have a global objective for the entire road network or there may be different objectives for the different parts of the road network different times of the day for the same part of the road network [1].

Dynamic Traffic Signal Control modeling is the formulation of rigorous mathematical models and Agent-based model that represent the various dynamics of the traffic system. This includes drivers’s behavior in acceleration, deceleration, lane changing, phenomena such as rubbernecking, and behavior change under different weather conditions. Dynamic Traffic Signal Control simulators are used for experimentation and validation of the underlying traffic models and traffic control mechanisms [1, 5].

On the other hand, there are several researchers who are currently exploring full Bayes modeling [9, 10] for evaluating parameters. There are a number of attractive characteristics of the full Bayes approach. In the fact that, Bayesian statistics offers an ideal framework for analyzing uncertainties conditions to identify parameters. However, the exact Bayesian formulation usually require the posterior distributions of modal parameters [9]. This paper, the advantage of Bayes modelling is use to evaluate parameters of signal traffic lights.

In this paper, we investigate the research problem about optimizing traffic signal control by using new hybrid-based model that integrating statistical model and agent-based model. In the fact that two flows of vehicles meet together at an intersection are considered. Traffic signal lights are assumed to use to control these vehicle flows at the intersection. The research problem is finding the time of green lights at intersection to minimize the waiting time of vehicles. To modelling this problem, the other researchers often using mathematical models such as LWR models, cellular automata models, social force model, statistical analysis model [6, 7]. However, mathematical models can not take into account the complex environment and heterogeneous behaviours of vehicles so Agent-based models being used [8]. The disadvantage of agent-based models are computing cost and very difficult to optimize this problem. Thus, we propose a hybrid-based model integrating statistical model and Agent-based model by using advantage of each model that see Fig. 1.

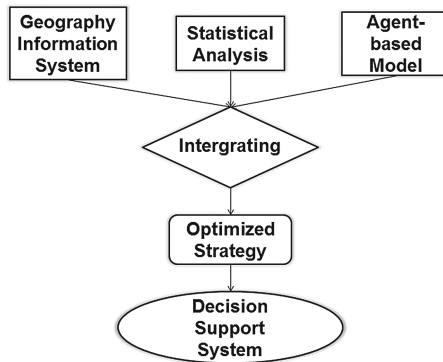


Fig. 1. Hybrid-based model compound by GIS, statistical model and agent-based model.

This paper is divided into four sections: Introduction that represents the important role of artificial Intelligent and research objects in dynamic traffic signal control and related works are concerned. The second section, methodology of optimization traffic signal control is investigated. Concretely, integrating two approaches to enhance effective optimization dynamic traffic signal control is represented in this section. The third section, implementation and applications for particular cases in Hanoi, Vietnam at an intersection are established; The last subsection is conclusion and discussion.

2 Methodology

The research problem consider two flows of vehicles meet together at an intersection are considered. At the intersection, traffic signal lights are assumed to use to control these vehicle flows. Finding the time of green lights at intersection to minimize the waiting time of vehicles is solved by integrating two methods.

2.1 Initial Assumption

Assumption. We will list assumptions used in this paper as follows:

- An intersection is a cross-point between two one-way roads with the same length. It has only two traffic lights for controlling the flow of transportation.
- Time interval in green state of traffic lights at A and B are integers belonged in the interval $[30, 60]$.
- Time interval in green state of a traffic light on road A (B) is a random variable followed exponential distribution.
- When a traffic light on a road stays in green or yellow state, the other must stay in red state.
- Number of vehicles that arrives to the intersection between road A and B in one second is a random variable followed Poisson distribution.
- The pattern for switching states process of traffic lights is: red \rightarrow green \rightarrow yellow \rightarrow red.
- Time period in a yellow state constantly equals ye seconds.
- A traffic light's period is defined as a time interval that starts at the beginning of a green state and stops at the end of next red state.
- There is no one that has to wait more than a traffic light's period (condition for no traffic jam) (Fig. 2).

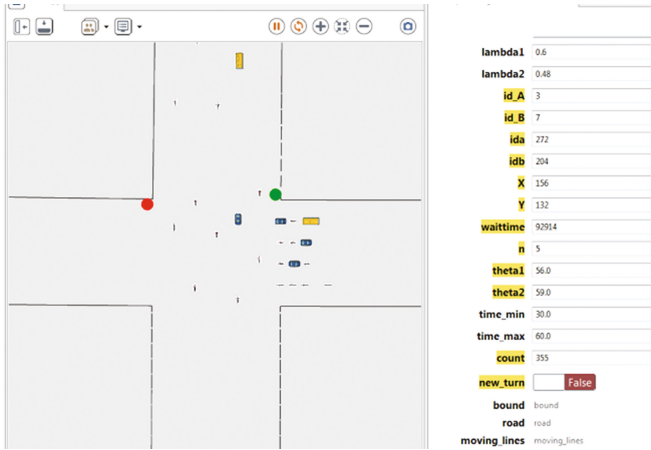


Fig. 2. Intersection is a cross-point between two one-way roads. Two traffic lights for controlling the flow of transportation. (Color figure online)

Variables Are Used in This Paper. Some notations of parameters and variables that will be used in Bayes estimation are described in the table as follows:

Variable	Description
A, B	The roads cross at intersection
Fl^A, Fl^B	Random variables that describe the number of vehicles entering the intersection from road A, B
X, Y	Random variables that describe the number of waiting vehicles on roads A, B during a traffic light's period
λ_1, λ_2	Average number of traffic capacity that are counted on the number of vehicles entering the intersection from roads A, B during 1 s
θ_1, θ_2	Random variables that describe the time interval in green state of traffic lights
$W_{vehicle}$	The number of vehicles waiting for traffic light on intersection in the interval $[0, T]$
W_{time}	The total of waiting time of all vehicles that have to wait at the intersection in the interval $[0, T]$
$\overline{\theta_1}, \overline{\theta_2}$	Mean value of random sample of θ_1, θ_2

Constant Parameter is Used in This Paper. The constant parameters being used in this method are listed in the table as follows:

Constant	Description
ye	Time interval in yellow state of traffic lights
a	Minimum interval time of green light
b	Maximum interval time of green light
T	The maximum time is considered in our model
N	The number of traffic light's period in the interval $[0, T]$

2.2 Statistical Models

Purpose of this subsection is estimate the expected number of vehicles entering the intersection and the time interval in green state of traffic lights which optimize this problem with the number of waiting vehicles.

Estimate the Expected Number of Vehicles Entering the Intersection.

Estimate λ_1, λ_2 Let Fl^A, Fl^B be random variables represent the number of vehicles on the road A, B in one second. Many researchers assume that the flow of vehicles is Poisson process. Thus, the same the other researcher we assume that Fl^A, Fl^B are Poisson random variables with rate λ_1, λ_2 in one second [11]. We need to estimate two parameters λ_1, λ_2 .

We assume that we have observation data of road A, B are $(Fl_1^A, Fl_2^A, \dots, Fl_{N_A}^A), (Fl_1^B, Fl_2^B, \dots, Fl_{N_B}^B)$ respectively. To estimate two parameters λ_1, λ_2 , the maximum likelihood estimation method is used. Concretely, $\hat{\lambda}_1$ is estimated as follows:

$$\begin{aligned}
 \hat{\lambda}_1 &= \arg \max_{\lambda_1} (p(Fl^A, \lambda_1)) = \arg \max_{\lambda_1} \left(\prod_{i=1}^N p(Fl_i^A | \lambda_1) \right) \\
 &= \arg \max_{\lambda_1} \left(\prod_{i=1}^N \frac{\lambda_1^{Fl_i^A} e^{-\lambda_1}}{Fl_i^A!} \right) = \arg \max_{\lambda_1} \left(\ln \left(\prod_{i=1}^N \lambda_1^{Fl_i^A} e^{-\lambda_1} \right) \right) \\
 &= \arg \max_{\lambda_1} \left(\ln(\lambda_1) \left(\sum_{i=1}^N Fl_i^A \right) - N \lambda_1 \right) \tag{1}
 \end{aligned}$$

We consider λ_1 as a variable then take derivate the Eq.1 then take it equal 0 then we have:

$$\begin{aligned}
 \frac{\sum_{i=1}^N Fl_i^A}{\lambda_1} - N &= 0 \\
 \Leftrightarrow \lambda_1 &= \frac{\sum_{i=1}^N Fl_i^A}{N} \tag{2}
 \end{aligned}$$

By the maximum likelihood estimation method, (2) is unbiased estimation of λ_1 . Same as λ_1 , we have an unbiased estimation of λ_2 is $\frac{\sum_{i=1}^N Fl_i^B}{N}$ ($(Fl_1^B, Fl_2^B, \dots, Fl_N^B)$ is a random sample of Fl^B).

Bayes Estimation Model. Bayesian estimation is used to estimation parameters of stochastic processes. Bayesian estimation is investigated to find out the unknown parameters duration green light time. In our approach, the current and previous duration green light time become the prior for the next time step. This estimation is considered more stable and more adaptable to the changing environment dynamics. That is if a change occurs in the network dynamics such as rush hours the controller using this probability estimation can handle the traffic efficiently by the way that optimizes the various performance indices (e.g., waiting time, queue lengths, etc.) in the congested periods.

Concretely, let X be a stochastic random variable, S_X be the sample space of X . We assume that X follow the distribution that depends on parameter θ on the sample space Θ . We collect an observer sample space (X_1, X_2, \dots, X_n) of the random experiment generated from the random variable X .

Let θ be random variable has prior probability density function $h(\theta)$, $\theta \in \Theta$. First, we define *likelihood* function:

$$p(X|\theta) = \prod_{i=1}^n p(X = X_i|\theta)$$

Using Bayes' theorem, the posterior density function $p(\theta|X)$ is defined:

$$p(\theta|X) = \frac{p(X|\theta).h(\theta)}{p(X)} = \frac{\text{likelihood.prior}}{p(X)}$$

New Statistical Model. Considering a traffic light's period on road A . We will calculate the number of waiting vehicles \bar{X} based on $\bar{\theta}_1, \bar{\theta}_2$.

We conduct some remarks as follows:

- All vehicles that arrive the intersection during the time interval from 0 to second $(\bar{\theta}_1)$ can go through the intersection.
- All vehicles that arrive the intersection during time interval from second $(\bar{\theta}_1)$ to $(\bar{\theta}_1 + \bar{\theta}_2 + ye)$ must wait in front of stop boarder (wA with vehicles on road A and wB with road B).
- Vehicles that arrive the intersection after second $(\bar{\theta}_1 + \bar{\theta}_2 + ye)$ can pass the intersection in green state of the next traffic light's period.

First, we consider a traffic light's period on road A . Our goal is to calculate the number of waiting vehicles X based on θ_1, θ_2 . We will use notation ye as the yellow state duration ($ye = const$) for further inferences.

From initial conditions, we infer the following remarks:

- Vehicles that arrive the intersection during the time interval from 0 to second θ_1 can go through the intersection.
- All vehicles that arrive the intersection during time interval from second θ_1 to $(\theta_1 + \theta_2 + ye)$ must wait in red state of traffic light.
- Vehicles that arrive the intersection after second $\theta_1 + \theta_2 + ye$ can pass the intersection in green state of the next traffic light's period.

We imply from remarks above that the number of waiting vehicles in a traffic light's period equals the number of arriving vehicles during time interval from second (θ_1) to $(\theta_1 + \theta_2 + ye)$ in the same period. Since the assumptions that number of vehicles arriving the intersection from road A follows Poisson distribution with average number per second λ_1 , we can imply the following formula:

$$\begin{aligned} p(X|\theta_1, \theta_2) &= poisson(X, (\theta_2 + \theta_1 + ye - ye - \theta_1 + ye)\lambda_1) \\ &= poisson(X, (\theta_2 + ye)\lambda_1) \end{aligned}$$

So we imply that:

$$\begin{aligned} p(X|\theta_1, \theta_2) &= poisson(X, (\theta_2 + ye)\lambda_1) \\ &= \frac{((\theta_2 + ye)\lambda_1)^X e^{-(\theta_2 + ye)\lambda_1}}{X!} \end{aligned} \tag{3}$$

In the same manner we can see that:

$$p(Y|\theta_1, \theta_2) = \frac{((\theta_1 + ye)\lambda_2)^Y e^{-(\theta_1 + ye)\lambda_2}}{Y!} \tag{4}$$

According to (3) and (4), we have

$$E(X|\theta_1, \theta_2) = \lambda_1(\theta_2 + ye) \tag{5}$$

$$E(Y|\theta_1, \theta_2) = \lambda_2(\theta_1 + ye) \tag{6}$$

We next consider time interval T that quite greater than the traffic light's period. Define $W_{vehicle}$ as the number of waiting vehicles in this duration. From these assumptions, we can estimate the average number of waiting vehicles during T based on (5), (6):

$$W_{vehicle} = \sum_{i=1}^N (\lambda_1(\theta_2(i) + ye) + \lambda_2(\theta_1(i) + ye)) + G \tag{7}$$

Define N as the total number of light's periods during T , G is the number of waiting vehicles in the last light's period (when time interval T equals N times of light's period, $G = 0$). We can assume $\lambda_1 \geq \lambda_2$ that without losing the generality of our inferences:

We assign:

$$\begin{aligned} \bar{\theta}_1 &= \frac{\sum_{j=1}^N \theta_1(j)}{N} \\ \bar{\theta}_2 &= \frac{\sum_{k=1}^N \theta_2(k)}{N} \end{aligned}$$

Because G can be estimated by $\lambda_1\theta'_2 + \lambda_2\theta'_1$ with θ'_1, θ'_2 are red state durations on road A, B (θ'_1 and θ'_2 can be equal to 0), respectively, we can transform formula (7) into:

$$\begin{aligned} W_{vehicle} &= \sum_{i=1}^N (\lambda_1 - \lambda_2)\theta_2(i) + N\lambda_1ye + \sum_{i=1}^N \lambda_2(\theta_1(i) + \theta_2(i) + ye) + \lambda_1\theta'_2 + \lambda_2\theta'_1 \\ &= N((\lambda_1 - \lambda_2)\bar{\theta}_2 + \lambda_1ye) + (\lambda_1 - \lambda_2)\theta'_2 + \lambda_2T \end{aligned} \tag{8}$$

From the condition that A is quite greater than a traffic light's period, we can use approximate estimation without any significant changes in the result:

$$N((\lambda_1 - \lambda_2)\bar{\theta}_2 + \lambda_1 ye) + (\lambda_1 - \lambda_2)\theta_2' \approx \frac{T((\lambda_1 - \lambda_2)\bar{\theta}_2 + \lambda_1 ye)}{\bar{\theta}_1 + \bar{\theta}_2 + ye}$$

Therefore, the problem of finding $\{\theta_1(i)\}_{i \leq N}$ and $\{\theta_2(i)\}_{i \leq N}$ to get the smallest W_T can be recognized as the following problem:

$$\min_{\{\bar{\theta}_1, \bar{\theta}_2\}} \left(\frac{T((\lambda_1 - \lambda_2)\bar{\theta}_2 + \lambda_1 ye)}{\bar{\theta}_1 + \bar{\theta}_2 + ye} \right) \tag{9}$$

Where there conditions must be satisfied:

- (i) $\theta_1, \theta_2 \in [a, b]$
- (ii) $T \gg b$
- (iii) $a \gg ye$

Solving problem (9) relates to the problem of optimizing a two-variable rational function. It can be solved easily in specific cases. After solving (9), we can select two satisfied constant sets $\{\theta_1(i)\}_{i \leq N}$ and $\{\theta_2(i)\}_{i \leq N}$ with corresponding values $\bar{\theta}_1, \bar{\theta}_2$.

2.3 Agent-Based Model

Modellers using Agent-based models can take into account different behaviours of vehicles depended on city and complex environment such as geometric information system (GIS).

- Each passenger has a motorcycle, a car, a truck, a bus, etc. is an agent has different behaviours such as its own size that occupies space of road, velocity, accelerate, target, etc.
- $\hat{\theta}_1, \hat{\theta}_2$ in this model could be consider with a range replace the average value of statistical model.

Geometric Agents. We built geometric agents based on the real data of an intersection from GIS database. These agents contain the data of roads, start - waiting - end points, and traffic lanes, and thus have all geometric properties (i.e.: position, length, etc.) and do not possess any behaviors.

Vehicle Agents. Vehicle agents represent active vehicles on an intersection. They are the main factors of our model as always be in any traffic models. Vehicle agents have following properties: lanes, current speed. In addition, each mean of transportation has different limitations in speed, size, and minimal distance to others, etc. Vehicle agents' behaviors are described below (Fig. 3):

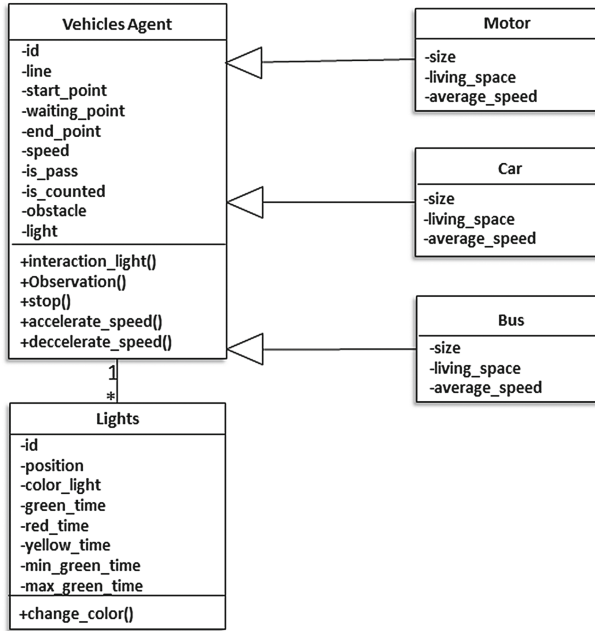


Fig. 3. Interaction between vehicles with control traffic lights

- Moving: Current position changes of the agent follow a given direction.
- Waiting: Vehicles stop in front of a wait-boarder of the road when the traffic light is red state or there is no free space to move (this condition is equivalent to ahead vehicle has an obstacle).

Behavior performing steps are illustrated in the following diagram (in Fig. 4).

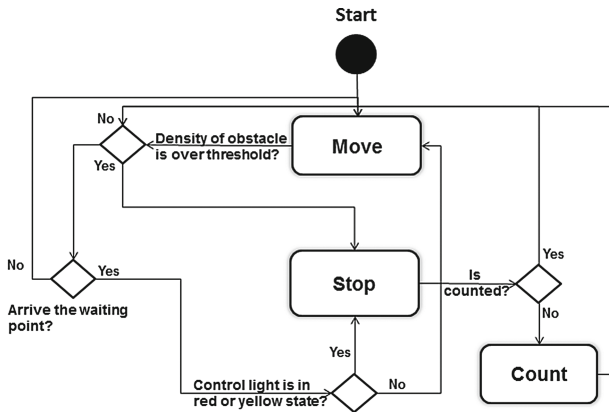


Fig. 4. Behavior of vehicles

Light Agents. Light agents control the traffic flows on an intersection. Their properties includes: color of current state, time intervals of red and yellow state, maximum and minimum time intervals of green state. Light agents' behaviors are shown below:

- As a time counter: Counting time interval of the light agent's current state.
- Color changing: When time counter meets a predetermined threshold, the light agent changes its state as following: green → yellow, yellow → red, red → green.

The relation between light agents' behaviors are illustrated in the diagram below (in Fig. 5):

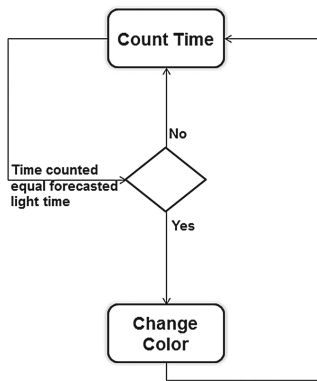


Fig. 5. Behavior of light agents

2.4 Algorithms of the Hybrid Method Integrating Statistical Model and Agent-Based Model

See Fig. 6.

Algorithm 1: Estimated light time by statistical estimation.

Input: Random samples $(Fl_1^A, Fl_2^A, \dots, Fl_{M_A}^A)$, $(Fl_1^B, Fl_2^B, \dots, Fl_{M_B}^B)$
 (X_1, X_2, \dots, X_N) and (Y_1, Y_2, \dots, Y_N)

T, ye

Output: $\hat{\theta}_1, \hat{\theta}_2$ and λ_1, λ_2

Estimate λ_1 and λ_2 using (2)

Estimate θ_1 and θ_2 using (9)

Algorithm 2: Simulation transportation by using Agent-Based Model

Input: GIS data
 Ratio of car/bus/motor
 $\hat{\theta}_1, \hat{\theta}_2, \lambda_1, \lambda_2, T, ye$

Output: $(X_1^{new}, X_2^{new}, \dots, X_N^{new})$ and $(Y_1^{new}, Y_2^{new}, \dots, Y_N^{new})$

Generate the vehicles cars, bus, motors entering the road A, B having different behaviors
 Counting the number of waiting vehicles
 Change θ_1, θ_2 follow different scenarios
 return $(X_1^{new}, X_2^{new}, \dots, X_N^{new})$ and $(Y_1^{new}, Y_2^{new}, \dots, Y_N^{new})$

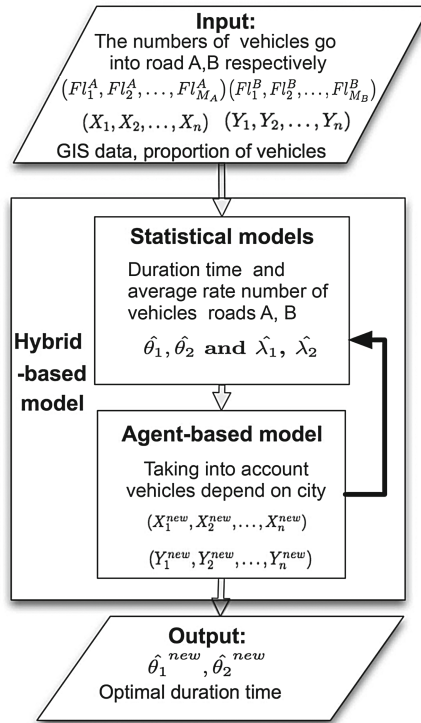


Fig. 6. Hybrid-based model compound by statistical model and agent-based model.

Algorithm 3: Hybrid Based Algorithm

Input: Random samples (X_1, X_2, \dots, X_N) and (Y_1, Y_2, \dots, Y_N) $\lambda_1, \lambda_2, \rho_1, \rho_2$

Output: $\{\theta_1(i)\}$ and $\{\theta_2(i)\}$ that make $W_{vehicle}$ minimum

Repeat

Estimate $\hat{\theta}_1, \hat{\theta}_2$ using Algorithm 1.

Using Algorithm 2.

Replace (X_1, X_2, \dots, X_n) and (Y_1, Y_2, \dots, Y_n) with
 $(X_1^{new}, X_2^{new}, \dots, X_n^{new})$ and $(Y_1^{new}, Y_2^{new}, \dots, Y_n^{new})$

Estimate $\hat{\theta}_1^{new}, \hat{\theta}_2^{new}$ using Algorithm 1.

until $\|\hat{\theta}_1^{new} - \hat{\theta}_1\| + \|\hat{\theta}_2^{new} - \hat{\theta}_2\| < \epsilon$

return $hat\theta_1^{new}, hat\theta_2^{new}$

3 Experiments and Results

We set up and designed four scenarios which are given in the first table. The result about the total number of waiting vehicles are shown in the second table and are also represented in Fig. 7.

	Description
Scenario 1	θ_1, θ_2 are constants
Scenario 2	θ_1, θ_2 are random variables following truncated Gaussian distribution with mean is mean of t_{min}, t_{max}
Scenario 3	θ_1, θ_2 following truncated Gaussian distribution and estimated by Bayes estimation
Scenario 4	θ_1, θ_2 are constants and obtained by optimal calculation

	θ_1	θ_2	ye	T	λ_1	λ_2	W_{time}	$W_{vehicle}$
Scenario 1	50	40	5	4000	0.6	0.48	72579.4	2598.1
Scenario 2	45	45	5	4000	0.6	0.48	72158.9	2653.5
Scenario 3	59	58	5	4000	0.6	0.48	84772.4	2590.6
Scenario 4	60	30	5	4000	0.6	0.48	76402.4	2529.2

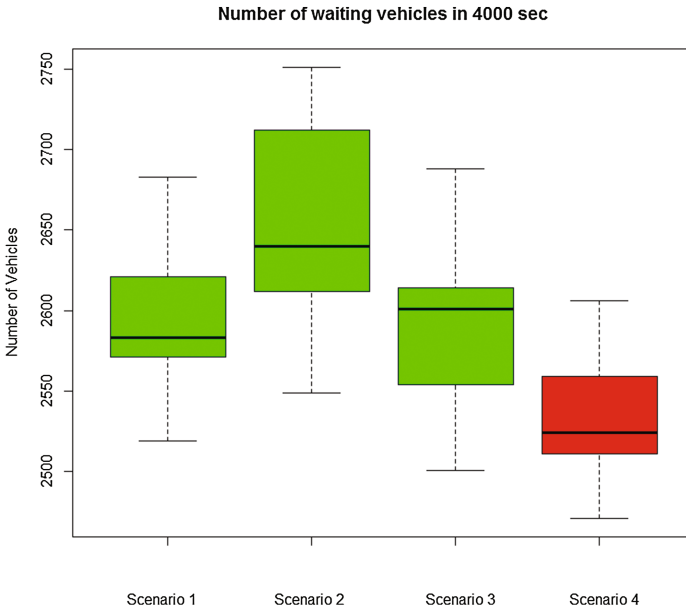


Fig. 7. Results of 4 scenarios that the proposed models is better than the other models.

4 Conclusion and Discussion

This paper opens a protocol for coupling statistical and agent-based models for optimizing TSC. There are three principal messages in the paper. The first is that, based on statistical model from actual and empirical data. The second message is a caution against the homogeneous of behaviour of vehicles and environment so Agent-based model is used. Combining statistical model and ABM to create a new hybrid method. To this end, a number of issues that are critical to the proper conduct of ABM evaluations were raised and illustrated based on recent experience.

Mathematical method using statistical model, agent-based model, hybrid-based model support different and complementary views of the traffic system. Then, we illustrate the agreement between these models. Thus, we show that our proposal model integrating statistical model and Agent-based model by using advantage of each model. Conclusively, the results of this paper are obtained:

- Integrating three models (GIS analysis, statistical estimation model, and Agent-based model) are feasible.
- Integrating model is more effective than GIS analysis approach by considering the dynamic context.
- Integrating model is less timing simulation cost than that of ABM.
- Integrating simulation allows to visualize the vehicles and lights at intersection traffic.

Finally, in future research activity, we need to extend to the case of large-scale environment. In this paper, we just focus on the number of waiting vehicle. In the future, we need to focus on the waiting time and total waiting time.

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