Outage Probability for Cognitive Heterogeneous Networks with Unreliable Backhaul Connections

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Abstract. To enhance the spectrum scarcity of cooperative heterogeneous networks (HetNets) with unreliable backhaul connections, we examine the impact of cognitive spectrum sharing over multiple small-cell transmitters in Nakagami-m fading channels. In this system, the secondary transmitters are connected to macro-cell via wireless backhaul links and communicate with the secondary receiver by sharing the same spectrum with the primary user. Integrating cognitive radio (CR), we address the combined power constraints: (1) the peak interference power and (2) the maximal transmit power. In addition, to exclude the signaling overhead for exchanging channel-state-information (CSI) at the transmitters, the selection combining (SC) protocol is assumed to employ at the receivers. The closed-form statistics of the end-to-end signal-to-noise (SNR) ratio are derived to attain the exact formulas of outage probability and its asymptotic performance to reveal further insights into the effective unreliable backhaul links.

Keywords: Cognitive Radio · Cooperative system
Wireless backhaul · Outage probability · Nakagami-m fading

1 Introduction

Wireless broadband services have driven high transport capacity requirements among cellular networks. As a result, the deployment of wireless infrastructure will get more dense and heterogeneous in the near future [1]. To achieve such higher data rate systems, backhaul links is becoming an emerging technology in heterogeneous networks (HetNets). Wired backhaul has high reliability but would lead to an ineffective increase in the costs of maintenance. For this reason, wireless backhaul is considered as an alternative solution since it offers cost-efficiency and flexibility. However, wireless backhaul is not completely reliable compared to wired backhaul due to the existence of non-line-of-sight (n-LOS) propagation and fading of transmission signals [2].

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There are several existing studies that investigated unreliable wireless backhaul links. For example, in [3], the impact of unreliable backhaul connections on Coordinated Multi-Point (CoMP) systems has been analyzed in the cooperative downlink system. Taking into account the limited resources such as the number of transmitters, interferers and backhaul reliability, the performance of a cooperative wireless network has been examined in [4,5]. For most of research works, backhaul reliability is shown as one of the key parameters that have significant impact on the system performance.

Cooperative systems in dense networks aim to extend the coverage or enhance the system capacity [6]. Hence, the diversity gain can be improved by taking advantage of the multiple receptions at various transmitters and transmission paths. For relay selection over Rayleigh fading channels, the authors in [7] investigated the secrecy performance of three different diversity combining schemes, namely maximum ratio combining (MRC), selection combining (SC), switch-and-stay combining (SSC). For a cyclic-prefix single carrier (CP-SC) system, the best relay selection scheme has been employed to analyze the performance in cognitive radio (CR) sharing spectrum [8].

As the demand for additional bandwidth continues to grow exponentially [9], many experts have sought solutions to efficiently deploy the available licensed spectrum. In recent years, the investigation on CR technologies has attracted the research community as a key factor to improve the spectrum scarcity in HetNets [10,11]. Under Nakagama-m fading, the authors in [12] analyzed the performance impacts of amplify-and-forward (AF) protocol subject to the transmit power constraints at the source and relay node. In [13], the authors investigated the transmit antenna selection with receive generalized selection combining (TAS/GSC) in CR networks.

Since the spectrum in primary networks has not been well utilized, it is important to integrate the CR technologies in the dense communication networks. To the best of the authors' knowledge, most of previous works only considered CR by neglecting the impact of unreliable backhaul [14,15]. Considering the cognitive HetNets, we take into account the scarcity of the spectrum utilization together with the wireless backhaul links reliability. Excluding the signaling overhead, we employ the SC protocol at the transmitters [16]. Moreover, the Nakagami-m fading is used to model the communication and interference channels since it provides various empirical scenarios for simulation [17]. Based on the derived statistics of the end-to-end SNR of the proposed systems, we derive the closed-form expression of the outage probability along with the asymptotic expressions in high-SNR regime. Thus, the analytical results are validated using Monte Carlo simulation.

Notation: $\mathcal{CN}(\mu, \sigma_n^2)$ denotes the complex Gaussian distribution with mean μ and variance σ_n^2; $F_\lambda(\gamma)$ and $f_\lambda(\gamma)$ denote the cumulative distribution (CDF) and probability density function (PDF) of the random variable (RV) λ, respectively; $\mathbb{E}_\lambda\{f(\gamma)\}$ denotes the expectation of $f(\gamma)$ with regard to the RV λ. In addition, $\binom{\tau_1}{\tau_2} = \dfrac{\tau_1!}{\tau_2!(\tau_1-\tau_2)!}$ denotes the binomial coefficient.

2 System and Channel Models

As illustrated in Fig. 1, we consider a cognitive network in a cooperative spectrum sharing system consisting of a macro base station (macro-BS) which is connected to the backbone network, K small-cells $\{\mathrm{SC}_1, ..., \mathrm{SC}_k, ..., \mathrm{SC}_K\}$ as the secondary network transmitters (SU-T_k) are connected to the macro-BS via unreliable wireless backhaul links, one secondary receiver (SU-D) and one primary user (PU-P). The K transmitters communicate with the secondary receiver SU-D by sharing the same spectrum with the primary user PU-P. All nodes are assumed to be equipped with a single antenna and operate in half-duplex mode.

In the practical systems, the transmit powers at each transmitter SU-T_k are constrained due to the interference of the secondary network and must not exceed the peak interference power $\mathcal{I}_p$ at the receiver PU-P. In addition, each transmitter is allowed to transmit up to their maximum power $\mathcal{P}_T$ [12,13]. Under the combined power constraints, the transmit power at the transmitter SU-T_k can be mathematically written as

$$\tilde{P}_k = \min\left(\mathcal{P}_T, \frac{\mathcal{I}_p}{|h_k^p|^2}\right). \tag{1}$$

where $h_k^p, k \in \{1, 2, ..., K\}$ denotes the channel coefficients of the interference links SU-$\mathrm{T}_k \rightarrow$ PU-P. Recall that $\mathcal{I}_p$ denotes the peak interference power at the receiver PU-P [18]. Let define S-SNR as the end-to-end SNR at the receiver SU-D. Without considering the backhaul reliability, the S-SNR over the channel from the transmitter SU-T_k to the receiver SU-D is given as

$$\gamma_k^s = \min\left(\bar{\gamma}_{\mathcal{P}}|h_k^s|^2, \frac{\bar{\gamma}_{\mathcal{I}}}{|h_k^p|^2}|h_k^s|^2\right), \tag{2}$$

where $h_k^s, k \in \{1, 2, ..., K\}$ denotes the channel coefficients of the communication links SU-$\mathrm{T}_k \rightarrow$ SU-D. The average SNR of the primary and secondary network is given as $\bar{\gamma}_{\mathcal{I}} = \mathcal{I}_p/\sigma_n^2$ and $\bar{\gamma}_{\mathcal{P}} = \mathcal{P}_T/\sigma_n^2$, respectively, with σ_n^2 representing the noise variance.

Due to the unreliable nature backhaul links, the signal received at the receiver SU-D via the transmitter SU-T_k is given by

$$r^{k,s} = \sqrt{\tilde{P}_k}(h_k^s)(\mathbb{I}_k)x + n^{k,s}, \tag{3}$$

where $\tilde{P}_k$ recalls the combined constraints transmit power at the transmitter SU-T_k and $n^{k,s} \sim \mathcal{CN}(0, \sigma_n^2)$. Since the message is transmitted from the core network to the receiver, it must go through the backhaul links and perform the success/failure transmission due to the characteristic of wireless links. Thus, the backhaul reliability $\mathbb{I}_k$ of the transmitter SU-T_k is modeled as Bernoulli process [19] with successful probability $\{\Lambda_k, \forall k\}$, i.e., the SU-T_k will successfully receive the message from macro-BS and forward to the receiver SU-D. Otherwise, the transmitter SU-T_k does not send anything with failure probability being $(1-\Lambda_k)$.

We denote x as the desired symbol transmitted by the small-cell transmitters and assume that $\mathbb{E}\{x\} = 0$ and $\mathbb{E}\{|x|^2\} = 1$.

Herein, we assume the SC protocol at the receiver SU-D[1] by selecting the small-cell station which has the best SNR over the received signals from K transmitters. Upon applying the SC protocol, it can be defined as

$$k^* = \max \arg_{[k \in K]}(\gamma_k^s \mathbb{I}_k), \tag{4}$$

is the selected transmitter SU-T$_k$ index. Consequently, the instantaneous S-SNR at the receiver SU-D can be obtained as

$$\gamma_S = \min\left(\bar{\gamma}_{\mathcal{P}}|h_{k^*}^s|^2, \frac{\bar{\gamma}_{\mathcal{I}}}{|h_{k^*}^p|^2}|h_{k^*}^s|^2\right)\mathbb{I}_{k^*}. \tag{5}$$

As can be seen from (5), the end-to-end SNR is decided by the unreliable backhaul of the considered HetNets, i.e., the Bernoulli RV $\mathbb{I}_k$. In addition, we assume all channels undergo Nakagami-m fading, i.e., a set of channel coefficients $\{h_k^s, \forall k\}$ of the links SU-T$_k$ → SU-D and a set of channels $\{h_k^p, \forall k\}$ of the links SU-T$_k$ → PU-P are distributed according to the gamma distribution, which is denoted by $|h_k^s|^2 \sim \text{Ga}(\mu_k^s, \eta_k^s)$ and $|h_k^p|^2 \sim \text{Ga}(\mu_k^p, \eta_k^p)$, respectively. Hence, The PDF and CDF of the RV $\chi \sim \text{Ga}(\mu_\chi, \eta_\chi)$, where $\chi \in \{h_k^s, h_k^p\}$ are given, respectively, as

$$\begin{aligned} f_\chi(x) &= \frac{1}{(\mu_\chi - 1)!(\eta_\chi)^{\mu_\chi}} x^{\mu_\chi - 1} e^{(-x/\eta_\chi)}, \\ F_\chi(x) &= \left(1 - e^{(-x/\eta_\chi)} \sum_{i=0}^{\mu_\chi - 1} \frac{1}{i!} \left(x/\eta_\chi\right)^i\right), \end{aligned} \tag{6}$$

where $\mu_\chi \in \{\mu_k^s, \mu_k^p\}$ represents the positive fading severity parameter [5,22,23], with channel powers $\{\Omega_k^s, \Omega_k^p\}$, and $\eta_\chi \in \{\eta_k^s = \Omega_k^s/\mu_k^s, \eta_k^p = \Omega_k^p/\mu_k^p\}$ indicates the scale factor on the corresponding channel.

3 Closed-Form Statistics of S-SNR in Cognitive Heterogeneous Systems

In this section, our challenges are how to derive the statistical properties of the S-SNR with respect to the backhaul reliability and the combined power constraints at SU-T$_k$. Without loss of generality, we assume that all channels follow the independent and identically distributed (i.i.d.) Nakagami-m fading, i.e., $\mu_s = \mu_k^s, \eta_s = \eta_k^s, \Lambda = \Lambda_k, \mathbb{I} = \mathbb{I}_k, \forall k \in K$ for transmission signals respect to the receiver SU-D and $\mu_p = \mu_k^p, \eta_p = \eta_k^p, \forall k \in K$ for the interference signals at the receiver PU-P, respectively. We first obtain the CDF of S-SNR for the signal between the particular SU-T$_k$ and SU-D, which is given in the following lemma.

[1] In the literature in unreliable backhaul [4,5], the perfect knowledge of CSI is not required at the transmitters, which is different from maximum ratio transmission (MRT) protocol [20,21].

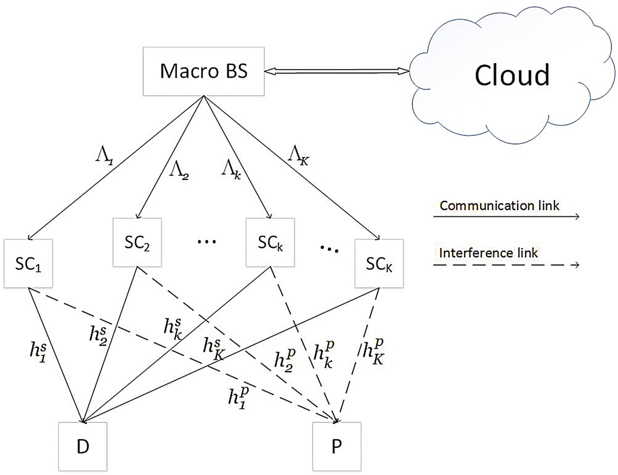

Fig. 1. A cognitive HetNet with unreliable backhaul links.

Lemma 1. *For a cognitive HetNet with unreliable backhaul links, where transmitter SU-T_k utilizes the sharing spectrum with the primary user PU-P, the CDF of the S-SNR for particular transmitter, γ_k^s, is given as*

$$F_{\gamma_k^s \mathbb{I}_k}(x) = 1 - \Lambda(\Theta_1(x) + \Theta_2(x)), \tag{7}$$

where $\Phi = \dfrac{\Upsilon\left(\mu_p, \bar{\gamma}_{\mathcal{I}}/\bar{\gamma}_{\mathcal{P}}\eta_p\right)}{\Gamma(\mu_p)}$, $\epsilon = \dfrac{\bar{\gamma}_{\mathcal{I}}\eta_s}{\eta_p}$ *and*

$$\begin{aligned}
\Theta_1(x) &= \Phi e^{-(x/\bar{\gamma}_{\mathcal{P}}\eta_s)} \sum_{i=0}^{\mu_s-1} \frac{1}{i!}\left(x/\bar{\gamma}_{\mathcal{P}}\eta_s\right)^i, \\
\Theta_2(x) &= \sum_{j=0}^{\mu_s-1} \sum_{g=0}^{\mu_p+j-1} \binom{\mu_p+j-1}{\mu_p-1} \frac{1}{g!(\bar{\gamma}_{\mathcal{P}}\eta_s)^g} \\
&\quad \epsilon^{\mu_p} e^{-(\bar{\gamma}_{\mathcal{I}}/\bar{\gamma}_{\mathcal{P}}\eta_p)} \frac{x^j e^{-(x/\bar{\gamma}_{\mathcal{P}}\eta_s)}(x+\epsilon)^g}{(x+\epsilon)^{\mu_p+j}}.
\end{aligned} \tag{8}$$

Proof. The proof is given in Appendix A.

In (7), $\Gamma(.)$ and $\Upsilon(.,.)$ are the Gamma function [24, Eq. (8.310.1)] and the lower incomplete Gamma function [24, Eq. (8.350.1)], respectively. Next, the corresponding CDF and PDF for the received S-SNR at the receiver SU-D will be derived in the following theorem.

Theorem 1. *For the i.i.d. Nakagami-m fading channels between K cooperative transmitters and the secondary receiver SU-D in the cognitive spectrum sharing with the primary user PU-P, the CDF of the RV* $\gamma_S \triangleq \max(\gamma_1^s \mathbb{I}_1, ..., \gamma_K^s \mathbb{I}_K)$ *with* *respect to SC protocol and unreliable backhaul links is given by (9) in the top of next page.*

$$F_{\gamma_S}(x) = 1 + \sum_{k=1}^{K} \binom{K}{k} (-1)^k \widehat{\sum_{k,\mu_s,\mu_p,\Lambda,\Phi}} \frac{x^{\widetilde{\varphi_1}} e^{-\beta x}}{(x+\epsilon)^{\widetilde{\varphi_2}}}, \tag{9}$$

where $\widetilde{L_{a_n}}$ *is defined as* $\widetilde{L_{a_n}} \triangleq \sum_{b_n=0}^{\mu_p+n-2} b_n a_{b_n+1}$, $\beta \triangleq k/\bar{\gamma}_{\mathcal{P}}\eta_s, \widetilde{\varphi_1} \triangleq \sum_{\vartheta=0}^{\mu_s-1} \vartheta u_{\vartheta+1} + \sum_{t=0}^{\mu_s-1} t w_{t+1} + c_1 + c_2 + ... + c_{\mu_s}$, $\widetilde{\varphi_2} \triangleq \sum_{t=0}^{\mu_s-1} (\mu_p + t) w_{t+1}$ *and* $\widehat{\sum_{k,\mu_s,\mu_p,\Lambda,\Phi}}$ *is a shorthand notation of*

$$\widehat{\sum_{k,\mu_s,\mu_p,\Lambda,\Phi}} \triangleq \sum_{l=0}^{k} \binom{k}{l} \sum_{u_1 \ldots u_{\mu_s}}^{k-l} \sum_{w_1 \ldots w_{\mu_s}}^{l} \sum_{a_{1,1} \ldots a_{1,\mu_p}}^{w_1} \sum_{a_{2,1} \ldots a_{2,\mu_p+1}}^{w_2} \cdots \sum_{a_{\mu_s,1} \ldots a_{\mu_s,\mu_p+\mu_s-1}}^{w_{\mu_s}}$$
$$\frac{(k-l)!}{u_1! \ldots u_{\mu_s}!} \frac{l!}{w_1! \ldots w_{\mu_s}!} \frac{w_1!}{a_{1,1}! \ldots a_{1,\mu_p}!} \frac{w_2!}{a_{2,1}! \ldots a_{2,\mu_p+1}!} \cdots \frac{w_{\mu_s}!}{a_{\mu_s,1}! \ldots a_{\mu_s,\mu_p+\mu_s-1}!}$$
$$\prod_{t=0}^{\mu_s-1} \binom{\mu_p+t-1}{\mu_p-1}^{w_{t+1}} \frac{1}{\prod_{\vartheta=0}^{\mu_s-1} \left(\vartheta! (\bar{\gamma}_{\mathcal{P}}\eta_s)^{\vartheta}\right)^{u_{\vartheta+1}}} \frac{1}{\prod_{b_1=0}^{\mu_p-1} \left(b_1! (\bar{\gamma}_{\mathcal{P}}\eta_s)^{b_1}\right)^{a_{1,b_1+1}}}$$
$$\frac{1}{\prod_{b_2=0}^{\mu_p} \left(b_2! (\bar{\gamma}_{\mathcal{P}}\eta_s)^{b_2}\right)^{a_{2,b_2+1}}} \cdots \frac{1}{\prod_{b_{\mu_s}=0}^{\mu_p+\mu_s-2} \left(b_{\mu_s}! (\bar{\gamma}_{\mathcal{P}}\eta_s)^{b_{\mu_s}}\right)^{a_{\mu_s,b_{\mu_s}+1}}} \sum_{c_1=0}^{\widetilde{L_{a_1}}} \sum_{c_2=0}^{\widetilde{L_{a_2}}} \cdots \sum_{c_{\mu_s}=0}^{\widetilde{L_{a_{\mu_s}}}}$$
$$\binom{\widetilde{L_{a_1}}}{c_1} \binom{\widetilde{L_{a_2}}}{c_2} \cdots \binom{\widetilde{L_{a_{\mu_s}}}}{c_{\mu_s}} \Lambda^k \Phi^{k-l} e^{-(\bar{\gamma}_{\mathcal{I}} l/\bar{\gamma}_{\mathcal{P}}\eta_p)} \epsilon^{\left(\widetilde{L_{a_1}} + \widetilde{L_{a_2}} + \ldots + \widetilde{L_{a_{\mu_s}}} + \mu_p l - (c_1 + c_2 + \ldots + c_{\mu_s})\right)}. \tag{10}$$

Proof. The proof is given in Appendix B.

Hence, the PDF of the received S-SNR can be derived as follows

$$f_{\gamma_S}(x) = \sum_{k=1}^{K} \binom{K}{k} (-1)^k \widehat{\sum_{k,\mu_s,\mu_p,\Lambda,\Phi}} \frac{e^{-\beta x}}{(x+\epsilon)^{\widetilde{\varphi_2}+1}}$$
$$\left((\widetilde{\varphi_1} - \widetilde{\varphi_2} - \epsilon\beta) x^{\widetilde{\varphi_1}} + \widetilde{\varphi_1} \epsilon x^{\widetilde{\varphi_1}-1} - \beta x^{\widetilde{\varphi_1}+1} \right). \tag{11}$$

4 Outage Probability Analysis

To investigate the performance of the proposed cognitive HetNets with unreliable backhaul connections over i.i.d. Nakagami-m fading channels, we focus on the outage probability, where the exact formula of the outage probability are presented based on the statistics derived in Sect. 3.

Given a certain SNR threshold γ_{th}, the outage probability of the S-SNR is defined as the probability that the S-SNR is below the threshold γ_{th}, which can be written as

$$\mathcal{P}_{out}(\gamma_{\text{th}}) \triangleq \Pr(\gamma_S \le \gamma_{\text{th}}) = F_{\gamma_S}(\gamma_{\text{th}}). \tag{12}$$

In other words, the outage probability can be expressed as the CDF of the S-SNR at the given $\gamma_{\rm th}$. By substituting (9) into (12), the outage probability is derived in the following theorem.

Theorem 2. *The outage probability closed-form expression for the proposed cognitive HetNets with respect to the unreliable backhaul links is derived as*

$$\mathcal{P}_{out}(\gamma_{th}) = 1 + \sum_{k=1}^{K} \binom{K}{k} (-1)^k \widehat{\sum_{k,\mu_s,\mu_p,\Lambda,\Phi}} \frac{{\gamma_{th}}^{\widetilde{\varphi_1}} e^{-\beta\gamma_{th}}}{(\gamma_{th}+\epsilon)^{\widetilde{\varphi_2}}}. \tag{13}$$

To provide insight into how the fading parameters and backhaul reliability impact the network performance, we next derive the asymptotic outage probability in the high-SNR regime of the considered system. In this case, we assume the peak interference threshold $\bar{\gamma}_{\mathcal{I}}$ is proportional to the maximum transmit power $\bar{\gamma}_{\mathcal{P}}$. The asymptotic outage probability is given in the following theorem as

Theorem 3. *At the high-SNR regime with respect to $\bar{\gamma}_{\mathcal{P}}$ as $\bar{\gamma}_{\mathcal{P}} \to \infty$ in the cognitive sharing system with K cooperative transmitters and unreliable backhaul links, the asymptotic outage probability is given by*

$$\mathcal{P}_{out}^{Asy}(\gamma_{th}) \overset{\bar{\gamma}_{\mathcal{P}}\to\infty}{=} (1-\Lambda)^K = \Xi. \tag{14}$$

Proof. The proof is given in Appendix C.

5 Numerical Results and Discussions

In this section, we present the numerical results of the outage probability to verify the analysis under the impact of unreliable backhaul links. The "Sim" curves indicate the link-level Monte Carlo simulation results, while the "Ana" and "Asy" curves represent the analytical results and asymptotic performance at high-SNR regime, respectively. We fix the S-SNR threshold $\gamma_{\rm th} = 3$ dB. Without loss of generality, we assume that the secondary user SU-D and the primary user PU-P are located at point $[0, 0]$ and $[0.5, 0.5]$, respectively. Those small-cell transmitters SU-T$_k$ are located at $[0, 0.5]$. Hence, the channel mean powers are calculated by $\Omega_k^s = \Omega_k^p = \left(\sqrt{(x_k - x_u)^2 + (y_k - y_u)^2}\right)^{\zeta}$, where $u \in \{D, P\}$ and $\zeta = 4$ as the path-loss exponent. In this setting, we obtain the mean power of all links is equal to 16. We also assume the ratio of the interference power $\bar{\gamma}_{\mathcal{I}}$ and the maximum transmit power $\bar{\gamma}_{\mathcal{P}}$ is constant for all numerical results. We define the average SNR as $\bar{\gamma} = \bar{\gamma}_{\mathcal{P}}$.

5.1 Outage Probability Analysis

Figures 2, 3 and 4 show the outage probability for various scenarios. In Fig. 2, we verify the accurate of the derived analytical outage probability versus the average SNR with the simulation. Assuming ($\Lambda_1 = 0.98, \Lambda_2 = 0.98, \Lambda_3 = 0.98$) for $K = 1$, $K = 2$, $K = 3$, respectively. The fading severity parameters are initialized as $\mu_\chi = \{\mu_k^s = 1, \mu_k^p = 1, \forall k\}$. From this figure, it can be observed that all curves converge to the asymptotic limitation as $\bar{\gamma}$ increases. Furthermore, the outage probability values get lower when more transmitters cooperate due to the correlation of multiple signals at the receiver SU-D.

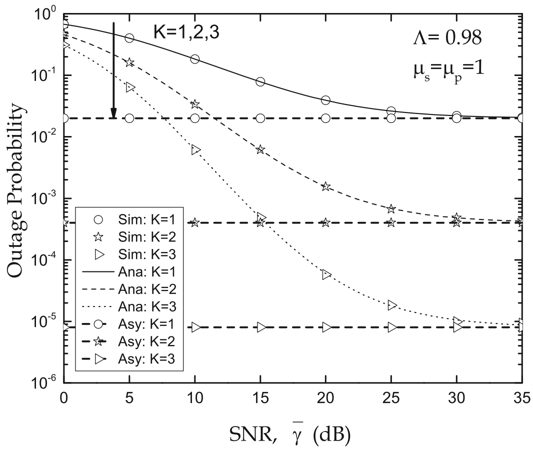

Fig. 2. Outage probability for various level of the degree of transmitter cooperation with fixed unreliable backhaul links.

To investigate the outage probability behavior at the same asymptotic threshold when the degree of transmitter cooperation is changed, we show it in Fig. 3. Assuming $\Xi = 6.4\text{E}{-5}$, we set ($\Lambda_1 = 0.999936$), ($\Lambda_1 = 0.992, \Lambda_2 = 0.992$), and ($\Lambda_1 = 0.96, \Lambda_2 = 0.96, \Lambda_3 = 0.96$) for case $K = 1$, $K = 2$, $K = 3$, respectively. The fading severity parameters are similar as in Fig. 2. We can observe that at the same outage probability asymptotic limitation, the higher degrees of transmitter cooperation converge faster than the others. Moreover, at the same degrees of transmitter cooperation ($K = 1$ or $K = 3$), the outage probability performance gets worst if the backhaul links is more unreliable, otherwise, the receiver SU-D performs the good performance.

Figure 4 plots the outage probability with various Nakagami-m fading severity scenarios at the fixed value ($K = 2, \Lambda = 0.988$). From these curves, it can be seen that the outage probability is strongly affected by the fading severity of the secondary network μ_s rather than the primary network fading severity μ_p. Specifically, the performance at the receiver SU-D tends to be better with the increase of μ_s while the outage probability values seem unchanged with the alternation of μ_p.

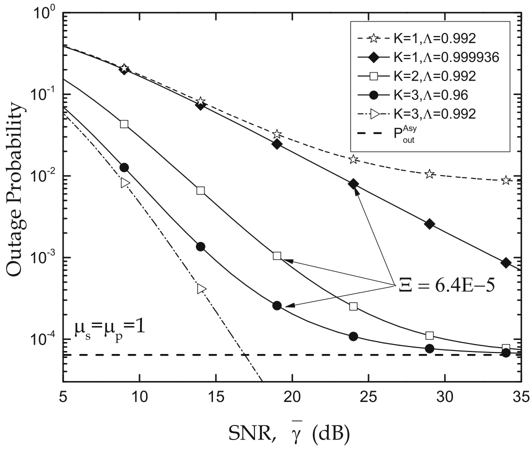

Fig. 3. Outage probability for various level of backhaul unreliability with fixed asymptotic limitation.

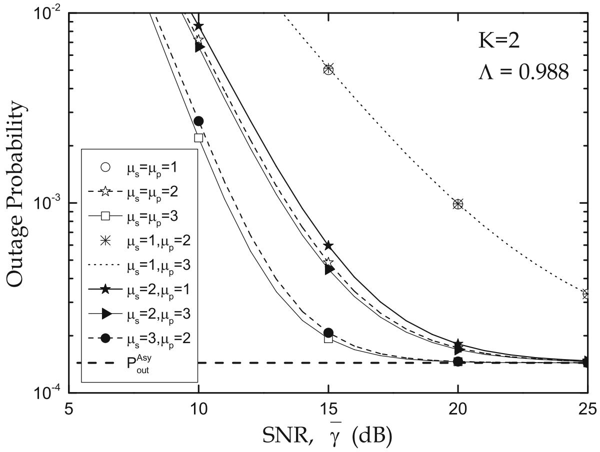

Fig. 4. Outage probability for various Nakagami-m fading severity with $\Xi = 1.44\text{E}{-4}$.

6 Conclusions

In this paper, we have taken into account the cognitive HetNets with unreliable backhaul links over i.i.d. Nakagami-m fading. The constrained transmit power of transmitters have been practically considered, i.e., the peak interference power $\mathcal{I}_p$ and the maximal transmit power $\mathcal{P}_T$. We have derived the closed-form expressions of the outage probability as well as asymptotic performance to obtain study insights. It has been shown that the asymptotic performance is only determined by the unreliable backhaul links in the high-SNR regime. The performance of the proposed system is highly improved proportionally to the degree of cooperation and the fading severity of secondary network. Our analyzed results provide suitable framework for network designers to clearly understand the effects of unreliable backhaul links and decide whether enabling the CR networks for those cooperative transmitters in order to efficiently utilize the spectrum.

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A Appendix A: Proof of Lemma 1

According to the definition of RV γ_k^s at particular SU-T$_k$, which was given as $\gamma_k^s = \min\left(\bar{\gamma}_{\mathcal{P}}|h_k^s|^2, \frac{\bar{\gamma}_{\mathcal{I}}}{|h_k^p|^2}|h_k^s|^2\right)$, results the CDF as

$$\begin{aligned} F_{\gamma_k^s}(x) &= \Pr\left\{\min\left(\bar{\gamma}_{\mathcal{P}}|h_k^s|^2, \frac{\bar{\gamma}_{\mathcal{I}}}{|h_k^p|^2}|h_k^s|^2\right) \le x\right\} \\ &= \underbrace{\Pr\left\{|h_k^s|^2 \le \frac{x}{\bar{\gamma}_{\mathcal{P}}}; \frac{\bar{\gamma}_{\mathcal{I}}}{|h_k^p|^2} \ge \bar{\gamma}_{\mathcal{P}}\right\}}_{\mathcal{J}_1} \\ &\qquad + \underbrace{\Pr\left\{\frac{|h_k^s|^2}{|h_k^p|^2} \le \frac{x}{\bar{\gamma}_{\mathcal{I}}}; \frac{\bar{\gamma}_{\mathcal{I}}}{|h_k^p|^2} \le \bar{\gamma}_{\mathcal{P}}\right\}}_{\mathcal{J}_2}. \end{aligned} \tag{A.1}$$

where

$$\mathcal{J}_1 = F_{|h_k^s|^2}\left(\frac{x}{\bar{\gamma}_{\mathcal{P}}}\right) F_{|h_k^p|^2}\left(\frac{\bar{\gamma}_{\mathcal{I}}}{\bar{\gamma}_{\mathcal{P}}}\right), \text{ and} \tag{A.2}$$

$$\mathcal{J}_2 = \int_{\frac{\bar{\gamma}_{\mathcal{I}}}{\bar{\gamma}_{\mathcal{P}}}}^{\infty} f_{|h_k^p|^2}(y) F_{|h_k^s|^2}\left(\frac{xy}{\bar{\gamma}_{\mathcal{I}}}\right) dy. \tag{A.3}$$

After some manipulations, we obtain the CDF of γ_k^s as follows.

$$\begin{aligned} F_{\gamma_k^s}(x) &= 1 - \varPhi e^{-\left(\frac{x}{\bar{\gamma}_{\mathcal{P}}\eta_s}\right)} \sum_{i=0}^{\mu_s-1} \frac{1}{i!}\left(\frac{x}{\bar{\gamma}_{\mathcal{P}}\eta_s}\right)^i \\ &\quad - \sum_{j=0}^{\mu_s-1} \binom{\mu_p+j-1}{\mu_p-1} \epsilon^{\mu_p} e^{-\left(\frac{\bar{\gamma}_{\mathcal{I}}}{\bar{\gamma}_{\mathcal{P}}\eta_p}\right)} \\ &\quad \frac{x^j e^{-\left(\frac{x}{\bar{\gamma}_{\mathcal{P}}\eta_s}\right)} \sum_{g=0}^{\mu_p+j-1} \frac{1}{g!(\bar{\gamma}_{\mathcal{P}}\eta_s)^g} (x+\epsilon)^g}{(x+\epsilon)^{\mu_p+j}}, \end{aligned} \tag{A.4}$$

with the help of [24, Eq. (8.352.4)]. The PDF of a particular RV $\gamma_k^s \mathbb{I}_k$ is modeled by the mixed distribution as

$$f_{\gamma_k^s \mathbb{I}_k}(x) = (1 - \Lambda)\delta(x) + \Lambda \frac{\partial F_{\gamma_k^s}(x)}{\partial x}, \tag{A.5}$$

where $\delta(x)$ indicates the Dirac delta function. Hence, the CDF of the RV $\gamma_k^s \mathbb{I}_k$ can be written as follows

$$F_{\gamma_k^s \mathbb{I}_k}(x) = \int_0^\infty f_{\gamma_k^s \mathbb{I}_k}(x) dx = 1 - \Lambda(\Theta_1(x) + \Theta_2(x)). \tag{A.6}$$

B Appendix B: Proof of Theorem 1

From the definition of S-SNR γ_S in (5), which is given by

$$\gamma_S = \max_{k \in K} \left(\gamma_1^s \mathbb{I}_1, \gamma_2^s \mathbb{I}_2, ..., \gamma_k^s \mathbb{I}_k, ..., \gamma_K^s \mathbb{I}_K \right). \tag{B.1}$$

Since all RVs $\gamma_k^s \mathbb{I}_k$ are independent and identically distributed with each other, the CDF of SNR γ_S can be written as

$$\begin{aligned} F_{\gamma_S}(x) &= F_{\gamma_k^s \mathbb{I}_k}^K(x) \\ &= 1 + \sum_{k=1}^{K} \binom{K}{k} (-1)^k \Lambda^k (\Theta_1(x) + \Theta_2(x))^k \\ &= 1 + \sum_{k=1}^{K} \binom{K}{k} (-1)^k \Lambda^k \sum_{l=0}^{k} \binom{k}{l} \Theta_1(x)^{k-l} \Theta_2(x)^l. \end{aligned} \tag{B.2}$$

Applying multinomial theorem provides the following expression

$$\begin{aligned} \Theta_1(x)^{k-l} &= \left(\Phi e^{-\left(\frac{x}{\bar{\gamma}_{\mathcal{P}} \eta_s}\right)} \sum_{i=0}^{\mu_s - 1} \frac{1}{i!} \left(\frac{x}{\bar{\gamma}_{\mathcal{P}} \eta_s} \right)^i \right)^{k-l} \\ &= \sum_{u_1 \dots u_{\mu_s}}^{k-l} \frac{(k-l)!}{u_1! \dots u_{\mu_s}!} \frac{\Phi^{k-l} e^{-((k-l)/\bar{\gamma}_{\mathcal{P}} \eta_s) x} x^{\sum_{\vartheta=0}^{\mu_s - 1} \vartheta u_{\vartheta+1}}}{\prod_{\vartheta=0}^{\mu_s - 1} \left(\vartheta! (\bar{\gamma}_{\mathcal{P}} \eta_s)^{\vartheta} \right)^{u_{\vartheta+1}}}. \end{aligned} \tag{B.3}$$

Again multinomial and binomial theorem give the following expression for $\Theta_2(x)^l$ as

$$\Theta_2(x)^l = \sum_{w_1 \ldots w_{\mu_s}}^{l} \frac{l!}{w_1! \ldots w_{\mu_s}!} \prod_{t=0}^{\mu_s-1} \binom{\mu_p+t-1}{\mu_p-1}^{w_{t+1}}$$
$$e^{-(\bar{\gamma}_{\mathcal{I}} l/\bar{\gamma}_{\mathcal{P}}\eta_p)} \epsilon^{\mu_p l} e^{-(l/\bar{\gamma}_{\mathcal{P}}\eta_s)x} x^{\sum_{t=0}^{\mu_s-1} t w_{t+1}}$$
$$\underbrace{\prod_{t=0}^{\mu_s-1} \left(\sum_{g=0}^{\mu_p+t-1} \frac{1}{g!(\bar{\gamma}_{\mathcal{P}}\eta_s)^g} (x+\epsilon)^g \right)^{w_{t+1}}}_{\mathcal{J}_3} \left(\underbrace{\prod_{t=0}^{\mu_s-1} \left((x+\epsilon)^{\mu_p+t} \right)^{w_{t+1}}}_{\mathcal{J}_4} \right)^{-1}. \tag{B.4}$$

Let denotes $\widetilde{L_{a_n}} = \sum_{b_n=0}^{\mu_p+n-2} b_n a_{b_n+1}$. By expanding $\mathcal{J}_3$ and $\mathcal{J}_4$, together with (B.2), (B.3), (B.4), yields (9).

Appendix C: Proof of Theorem 3

From (7), we can rewrite it as the Gamma form as

$$F_{\gamma_k^s \mathbb{I}_k}(x) = 1 - \Lambda\Phi \frac{\Upsilon\left(\mu_s, \frac{x}{\bar{\gamma}_{\mathcal{P}}\eta_s}\right)}{\Gamma(\mu_s)} - \Lambda \sum_{j=0}^{\mu_s-1} \frac{\epsilon^{\mu_p} x^j \Gamma\left(\mu_p+j, \frac{x+\epsilon}{\bar{\gamma}_{\mathcal{P}}\eta_s}\right)}{j!\Gamma(\mu_p)(x+\epsilon)^{\mu_p+j}}. \tag{C.1}$$

It can be easily seen that as y goes to infinity,

$$\lim_{y\to\infty} \frac{\Upsilon(\mu_\chi, x/y)}{\Gamma(\mu_\chi)} \approx 0 \text{ and}$$
$$\lim_{y\to\infty} \frac{\Gamma(\mu_\chi, x/y)}{\Gamma(\mu_\chi)} \approx 1. \tag{C.2}$$

Substituting (C.2) into (C.1) with the given outage threshold γ_{th}, we can obtain

$$\mathcal{P}_{out}^{Asy}(\gamma_{\text{th}}) \overset{\bar{\gamma}_{\mathcal{P}}\to\infty}{=} \prod_{k=1}^{K} \left(1 - \Lambda \frac{1}{\left(1+\frac{x}{\epsilon}\right)^{\mu_p}} \right)$$
$$\overset{\bar{\gamma}_{\mathcal{P}}\to\infty}{=} \prod_{k=1}^{K} (1-\Lambda), \tag{C.3}$$

where $\sum_{j=0}^{\mu_s-1}(.)$ is dominated by $j=0$ as $\bar{\gamma}_{\mathcal{P}} \to \infty$.

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