Performance Analysis of Nakagami and Rayleigh Fading for 2 × 2 and 4 × 4 MIMO Channel with Spatial Multiplexing

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Abstract. This correspondence presents the spectral efficiency analysis of 2 × 2 and 4 × 4 multiple input multiple output (MIMO) system. Analysis in this article has been done over Nakagami-m and Rayleigh fading channel. In this work analytical model as well as simulation has been observed for spectral efficiency of MIMO system over fading environment with an aid of Singular Value Decomposition (SVD) and waterfilling algorithm. It has also been observed the performance of Nakagami-m channel probabilistic model which one-to-one aligning between Nakagami-m and Rayleigh fading distributions in MIMO system.

Keywords: MIMO · Spectral efficiency · Singular value decomposition Waterfilling algorithm

1 Introduction

The spectral efficiency is elemental parameter in the design of wireless mobile communication systems as it increases the data throughput of the system. Still it is tolerated to fading, which deteriorate the performance of wireless mobile communication system [1, 2]. The normalized capacity of the bandlimited Additive White Gaussian Noise (AWGN) channel, which is due to Shannon limit. However, the cost of reaching the larger spectral efficiency is an increase in the SNR per bit. Therefore, any digital modulation schemes are convenient for communication channels that are bandwidth limited, where is desired a channel capacity to bandwidth ratio greater than one [3]. Power and bandwidth are major challenging resources [4]. The recent perception of multiple-input multiple-output system especially in wireless communication is one of the most impressive interest in research work. So now widely referred to as MIMO technology, this concept can extremely improve data throughput and link performance in wireless networks [5].

The basic facts which makes reliable wireless transmission is time-varying multipath fading [6]. Improving the quality or decreasing the efficient probability of error in a multipath fading channel is very difficult. The large channel capacities associated with MIMO channels are based on the premise that a rich scattering environment, provides independent transmission paths from each transmit to each receive antenna [4]. Now considering linear regression as singular value decomposition provides a MIMO system transmitter and receiver antenna pair to increase the data throughput without increasing

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bandwidth usage or transmit power. MIMO system increases throughput linearly with the help of spatial multiplexing [7].

In classical approach of MIMO system, multiple links provides a spatial diversity which leads to linear scale large channel capacity and improved quality of received signal compared to single antenna system [6]. MIMO system can be modelled with a combination of transmit and receive diversity scheme. Three most basic performance evaluations of wireless communication are data throughput, coverage and seamless connectivity.

Under appropriate flat fading environment, having together multiple antenna configurations, provide an added spatial dimension for communication system and yields a degree of- freedom. The channel capacity of such a MIMO channel with n_t transmit and n_r receive antenna is proportional to n, where $n = min(n_t, n_r)$ [6]. In this paper, simulations have been carried out for 2×2 and 4×4 MIMO system for high rich scattering environment using Nakagami-m probabilistic channel model with the help of MATLAB tool.

The flow of this paper is as follows: The wireless system model is defined in Sect. 2. In Sect. 3 exact expression of channel capacity is derived with the help of singular value decomposition and waterfilling algorithm. Section 4 describes fading distributions. Finally, Sect. 5 incorporate the concluding remarks of this proposed research work.

2 System Model

Consider an $n_r \times n_t$ point-to-point MIMO system $n_r = n_t$, where n_t and n_r denote the number of transmit and receive antennas, respectively. Assume channel state information (CSI) to be known perfectly at both ends. Let $\bar{x} = (x_1, \ldots, x_{n_t})$ be the vector of symbols transmitted by the n_t transmit antennas (Fig. 1).

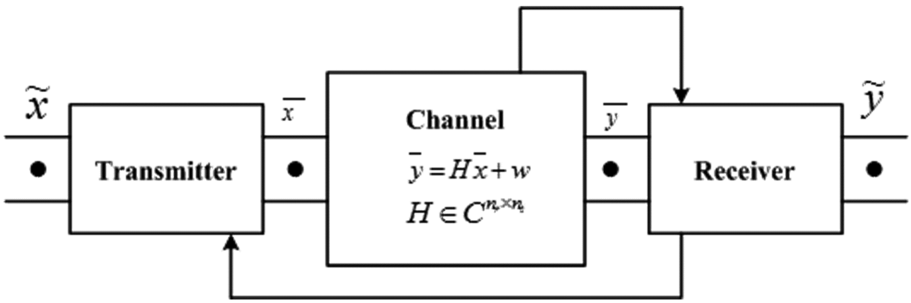

Fig. 1. System model

Let $H = \{h_{ij}\}$, $i = 1, \ldots, n_r$, $j = 1, \ldots, n_t$ be the $n_r \times n_t$ channel matrix are independent and identically distributed (i.i.d) Gaussian variables with zero mean, where h_{ij} is the complex gain between the j^{th} transmit antenna and the i^{th} receive antenna. The $n_r \times 1$ received vector is given by

$$\bar{y} = \mathrm{H}\,\bar{x} + \mathrm{w} \tag{1}$$

where $\bar{x} \in \mathbb{C}^{n_t}, \bar{y} \in \mathbb{C}^{n_r}$ and $w \sim \mathbb{C}\mathcal{N}(0, N_0)$ denote the transmitter vector, received vector and additive white Gaussian noise vector respectively at a symbol time. The channel matrix $H \in \mathbb{C}^{n_r \times n_t}$ is deterministic. Specifically, assumed the instantaneous channel gains, known as the channel state information, are known perfectly at transmitter or at receiver in order to improve the spectral efficiency and to reduce probability of error [7].

The noise vector is statistically i.i.d complex-valued Gaussian random variables with zero mean. Consider total power constraint $\mathbb{E}\left[\|\bar{\mathrm{x}}\|^2\right] = \mathrm{p}$ at the transmit antennas.

3 MIMO System Channel Capacity

The capacity of MIMO system can be derived with the help of linear regression method which linear transforms the MIMO channel into parallel sub channels. From basic theory of linear algebra, all linear transformation can be derived as a composition of three steps: a rotation, a scaling, and once again rotation. Now consider singular value decomposition of a channel matrix H. A matrix H can be represented as:

$$\mathrm{H} = \mathrm{U}\,\mathrm{D}\,\mathrm{V}^{\mathrm{H}} \tag{2}$$

where $(.)^H$ denotes the conjugate transpose; $V \in \mathbb{C}^{n_t \times n_t}$ and $U \in \mathbb{C}^{n_r \times n_r}$ are (rotation) unitary matrices with left and right singular vectors of H as their columns and $\mathrm{D} \in \mathfrak{R}^{n_r X n_t}$ is a matrix whose diagonal elements are positive real numbers and whose other channel coefficients are zero. The diagonal coefficients $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{n_{min}}$ are the ordered singular values of the channel matrix H, where $n_{min} := \min(n_r, n_t)$ and $n_{max} := \max(n_r, n_t)$. These are important conditions of the singular value decomposition. The number of nonzero singular values is equal to the rank of the channel matrix H [7]. Since the squared singular values λ_i^2 are the eigenvalues of

$$\begin{cases} \mathrm{HH^H}, & \text{if } \mathrm{n_r} \leq \mathrm{n_t} \\ \mathrm{H^H H}, & \text{if } \mathrm{n_r} > \mathrm{n_t} \end{cases} \tag{3}$$

Here, singular values transforms the channel into n_{min} parallel sub channels $\lambda_i,\ 1 \leq n \leq n_{min}$. Now, it can be expressed as

$$\mathrm{H} = \sum_{\mathrm{i}=1}^{\mathrm{n_{min}}} \lambda_{\mathrm{i}} \mathrm{u_i} \mathrm{v_i}^* \tag{4}$$

It can be seen that the rank of H is precisely the number of diagonal values. It can be defined as

$$\widetilde{y} = U^H \bar{y},\ \widetilde{x} := V^H \bar{x}, \widetilde{w} := U^H w \tag{5}$$

Substituting (2) and (5) into (1) the received signal vector can be rewritten as

$$\widetilde{y} = D\widetilde{x} + \widetilde{w} \tag{6}$$

where D is diagonal coefficients vector. Thus, the powers of $\bar{x}$ and $\widetilde{x}$ are the same, as well as $\bar{y}$ and $\widetilde{y}$, w and $\widetilde{w}$. The equivalent model of the system can be depicted in Fig. 2, which shows that the MIMO channel is converted into n_{min} parallel sub-channels through SVD.

$$\widetilde{y}_i = \lambda_i \widetilde{x}_i + \widetilde{w}_i \qquad i = 1, 2, \ldots, n_{min} \tag{7}$$

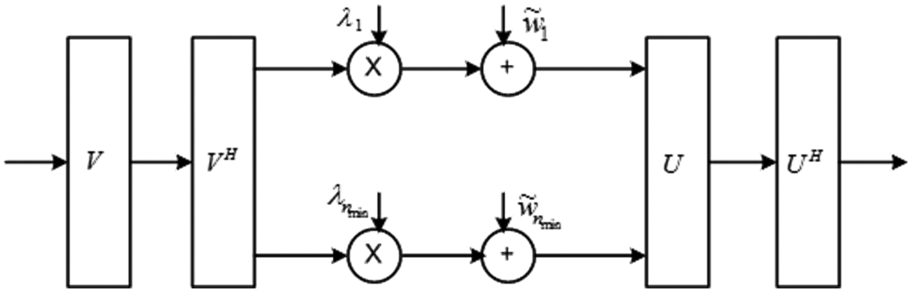

Fig. 2. MIMO system and equivalent model [5]

SVD is obtainable when H is known to both the ends. The optimal power denotes by P_i the power associated with the i^{th} symbol. That is

$$P_i \triangleq \mathbb{E}\left\{ |\widetilde{x}|^2 \right\} \tag{8}$$

Main goal is to find the optimum distribution for the set $\{P_i\}$ under the constraint that their sum is a fixed value.

$$\sum_{i=1}^{n_{min}} P_i = P \tag{9}$$

Finding the optimum set of $\{P_i\}$ is a standard optimization problem, which can be solved using the Lagrange multiplier technique. The resulting algorithm that implements this solution is called the waterfilling algorithm.

The optimal power allocation strategy has been shown in Fig. 3. For a faithful communication, power allocated to the i^{th} sub channel should be such that the total power constraint is met.

$$\max_{P_{1,\ldots,P_{n_{min}}}} \sum_{i=1}^{n_{min}} log\left(1 + \frac{P_i \lambda_i^2}{N_0}\right) \tag{10}$$

subject to Eq. (9). The power P is the average power constraint.

$$\mathrm{P}_{\mathrm{i}}^{*} = \left(\frac{1}{\beta} - \frac{\mathrm{N}_0}{\lambda_i^2} \right)^{+} \tag{11}$$

where β, a constant is called the Lagrange multiplier and satisfies condition

$$\sum_{i=1}^{n_{min}} \left(\frac{1}{\beta} - \frac{\mathrm{N}_0}{\lambda_i^2} \right)^{+} = P, \tag{12}$$

$$\mathrm{P}_{\mathrm{i}}^{*} = \left(\mu - \frac{\mathrm{N}_0}{\lambda_i^2} \right)^{+} \tag{13}$$

where, $\mu \triangleq \frac{1}{\beta}$. These Values of $\mathrm{P}_{\mathrm{i}}^{*}$ and μ are used to iteratively compute the set of values for $\{P_i\}$ that maximize the channel capacity. Optimal capacity is achieved by summing the capacities of individual sub channels [7]. Thus,

$$C = \sum_{i=1}^{n_{min}} log_2 \left(1 + \frac{P_i^* \lambda_i^2}{N_0} \right) \text{ bps /Hz} \tag{14}$$

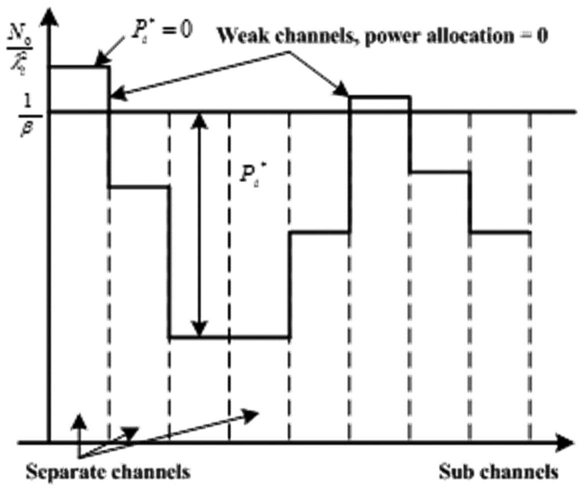

Fig. 3. Waterfilling power allocation [5]

In this way, the MIMO system can explore the spatial multiplexing of multiple streams.

4 Fading Wireless Channels

In this paper consider two fading distributions, Rayleigh and Nakagami-m multipath fading has been considered for analysis.

Rayleigh fading channel: The Rayleigh random variable r has the probability distribution:

$$f_R(r) = \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)$$

Where, $\mathbb{E}\{r^2\} = 2\sigma^2$ *and* $r \geq 0$ is the power is exponentially distributed.

Nakagami fading channel: The instantaneous power distribution is expressed as

$$f_R(r) = \frac{2m^m r^{2m-1}}{\Gamma(m)\Omega^m}\exp\left(-\frac{m}{\Omega}r^2\right), r \geq 0$$

where $\{r^2\} = \Omega, m = \frac{\mathbb{E}^2\{r^2\}}{Var\{r^2\}}$, $\Gamma(.)$ is the Gamma function and Ω denotes average fading power and controls the spread of the distribution.

A close observation of Table 1 reveals that Nakagami-m fading model results in the special case $m = 1$ which towards Rayleigh distribution. For $m > 1$, the fluctuations of the signal reduce compare to Rayleigh fading, and Nakagami-m tends to be more line of sight components.

Table 1. Nakagami and Rayleigh fading distributions for variation of m & σ^2

σ^2	m	Pdf of Nakagami channel	Pdf of Rayleigh channel
0.5	0.5	$8x^3\exp(-2x^2)$	$2x\exp(-x^2)$
	1	$2x\exp(-x^2)$	$2x\exp(-x^2)$
	2	$x\exp(-x^2/2)$	$2x\exp(-x^2)$

5 Simulation Results

Simulation parameters has been shown in Table 2.

Table 2. Simulation parameters

Sr. no.	Parameters	Description
1	Number of iterations	10,000
2	Execution time	9 min
3	Fading channels	Nakagami & Rayleigh
4	SNR range	−10 to 20 dB
5	m values	0.5, 1 & 2

Spectral efficiency for 2 × 2 and 4 × 4 MIMO systems over Nakagami and Rayleigh fading have been shown in Figs. 4 and 5 for m = 0.5, 1 and 2.

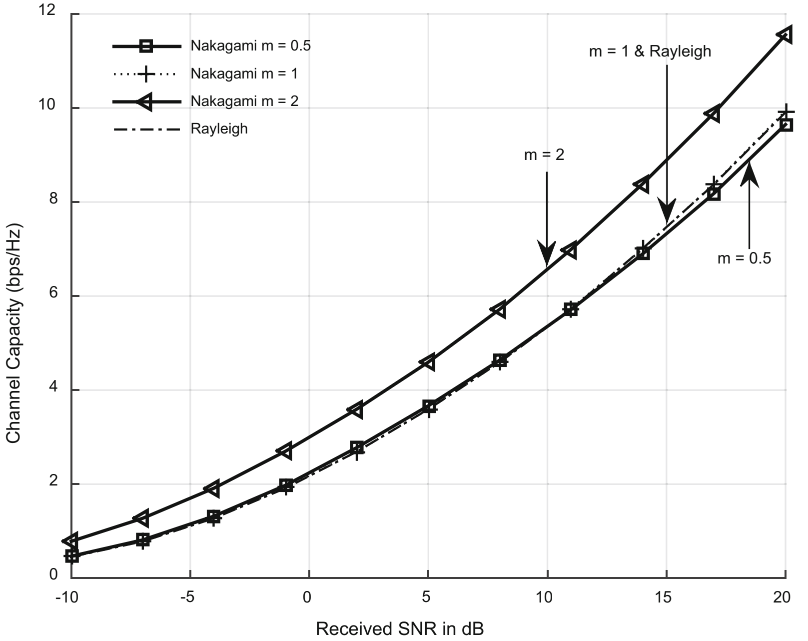

Fig. 4. Spectral efficiency for 2 × 2 MIMO system over Nakagami and Rayleigh fading channel

For 2 × 2 MIMO system it has been observed that, as the received average signal to noise ratio varies from 5 to 20 dB, channel capacity rises from 5 to 11 bps/Hz for Nakagami fading channel and 3 to 9 bps/Hz for Rayleigh fading channel. Similar analysis of 4 × 4 MIMO system reveals that here also channel capacity rises from 7 to 21 bps/Hz in Nakagami fading channel and 6 to 18 bps/Hz in Rayleigh fading channel.

The empirical distributions (histogram) of singular values for 2 × 2 and 4 × 4 MIMO system with 10,000 iteration for random realization of H have been shown in Figs. 6 and 7 respectively.

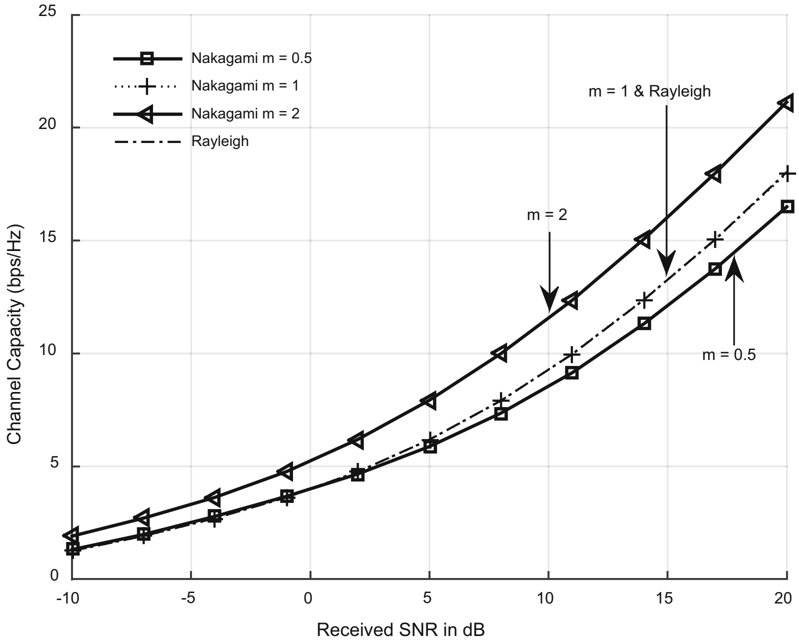

Fig. 5. Spectral efficiency for 4 × 4 MIMO system over Nakagami and Rayleigh fading channel

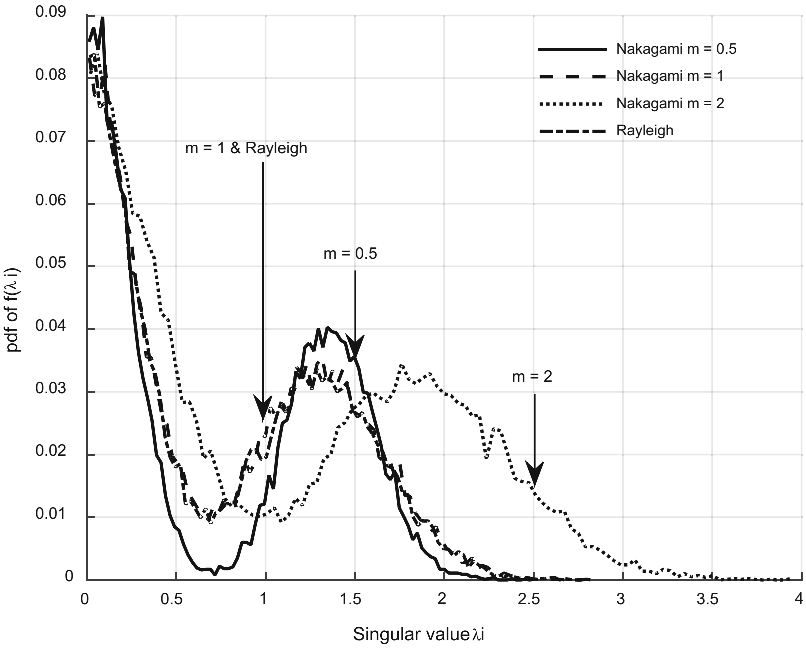

Fig. 6. Probability density function for 2 × 2 MIMO system over Nakagami and Rayleigh fading

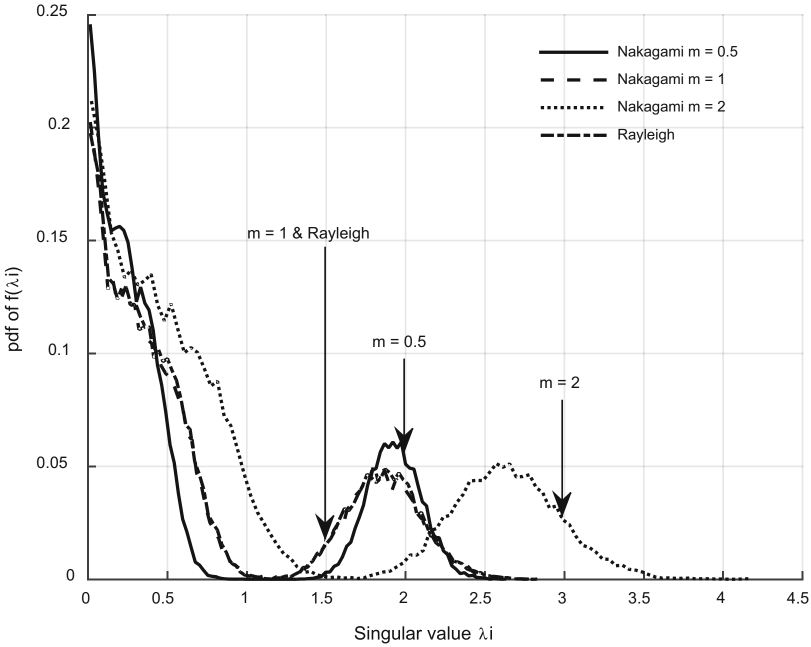

Fig. 7. Probability density function for 4 × 4 MIMO system over Nakagami and Rayleigh fading

6 Conclusion

In this research work spectral efficiency analysis of 2 × 2 and 4 × 4 MIMO systems have been done in both ways analytically as well as through simulation. Here, improvements in MIMO channel capacity and distribution function has been also analyzed among that the variation of parameter m from 0.5 to 2. The Nakagami distribution approaches towards Rayleigh distribution by varying m from 0.5 to 1. From above observations it can be concluded that m parameter is a well suited quantity for characterization of scattering effects.

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