

# Sensitivity Analysis of Phase Matched Turning Point Long Period Fiber Gratings

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**Abstract.** This study presents characterization of phase matched turning point long period gratings. It helps in optimizing the grating parameters of these long period gratings viz. grating period, length of grating, for maximum sensitivity. We have calculated spectral variation of refractive indices, effective refractive indices of fundamental and circularly symmetric cladding modes and grating periods. Phase matching curves for first 14 cladding modes have been obtained. Weakly-guiding analysis is used to compute effective refractive indices for the fundamental guided mode and cladding modes. Ultra high sensitivity at turn around points have been verified analytically with the help of general sensitivity factor. LP<sub>12</sub> cladding mode is observed to be the most sensitive.

**Keywords:** LPFG (Long Period Fiber Grating)  
SRI (Surrounding Refractive Index) · LP (Linearly Polarized)

## 1 Introduction

In recent years Refractometric sensors based on periodic refractive index modulation in optical fibers namely long period fiber gratings have emerged and illustrated in many applications that include physical parameter sensing [1–3], Adulteration detection [4–6], radiation detection [7], detection of bacteria [8, 9]. These Refractometric sensors offer advantages of being ultra high sensitive, capable for remote sensing and able to operate in harsh environments.

Ultra high sensitivity of long period gratings can be achieved by optimizing the pitch, length and index of modulation of these gratings [10, 11]. Index of modulation is mainly governed by fabrication process. The objective of the article is to compute general sensitivity factor for coupling of fundamental and first 14 cladding modes and finding the parameters at which this factor is maximum. Solutions of the Coupled mode theory based on assumption of weakly guiding regime is used for optimization of long period fiber gratings (LPFGs) for maximum sensitivity.

The paper is organized as follows. In Sect. 2 basic theory of LPFGs is discussed. In Sect. 3 results of the simulations for mode effective refractive indices and their spectral variations have been presented. Grating period calculations at resonant wavelengths have been done to plot phase matching curves. Sensitivity factor has been calculated and its values at turn around points have been highlighted. Finally, conclusions have been drawn in Sect. 4.

## 2 Mode Coupling in LPFGs

The Spatial and periodic modulation of refractive index of the order of hundreds to thousand micrometers in optical fiber causes coupling of fundamental core mode with either co-propagating or counter propagating cladding modes. Propagation characteristics of modes in optical fiber with LPFGs are strong function of the refractive index of surrounding medium.

Coupling of co-propagating fundamental guided mode  $LP_{01}$  and cladding modes represented by  $LP_{0m}$  takes place in LPFGs according to the phase matching condition, which results in series of attenuated resonance peaks in transmission spectrum [1]. The phase matching condition is given by-

$$\lambda_{res} = [n_{effco}(\lambda) - n_{effcl,m}(\lambda)]\Lambda \quad (1)$$

where  $\lambda_{res}$  is the resonance wavelength,  $n_{effco}$  is effective refractive index of the fundamental mode,  $n_{effcl,m}$  is effective refractive index of  $m^{th}$  cladding mode and  $\Lambda$  is period of grating.

Phase matching curves drawn between resonance wavelength  $\lambda_{res}$  and grating period  $\Lambda$  indicate presence of turn around point where the slope of the curve changes sign from positive to negative. LPFGs fabricated at these turn around points appear to be ultrahigh sensitive as the slope  $\frac{d\lambda_{res}}{d\Lambda}$  of the dispersion curve is infinite. For higher order modes these turn around points occur at higher wavelengths inside the optical communication window.

Mathematical expression for calculating shift in resonance wavelength with respect to change in surrounding refractive index [12] is given by-

$$\frac{d\lambda_{res}}{dn_{surr}} = \lambda_{res} \cdot \gamma \cdot \Gamma_{res} \quad (2)$$

where  $\lambda_{res}$  is resonance wavelength,  $n_{surr}$  is surrounding refractive index,  $\gamma$  is general sensitivity factor and  $\Gamma_{res}$  represents surrounding refractive index dependence on waveguide dispersion respectively. Turning points offer ultrahigh sensitivity because a change in wavelength corresponding to change in grating period is infinite.

## 3 Sensitivity Analysis Results and Discussions

The fiber considered in this paper is of a step-index profile and a three layers structure SMF-28 fiber. The parameters of the fiber are given as: the core radius  $r_1 = 4.61 \mu\text{m}$ , the cladding radius  $r_2 = 62.5 \mu\text{m}$ , core region is made up of 3.1% GeO<sub>2</sub> doped SiO<sub>2</sub> and cladding region is fused silica. Index of modulation is assumed as  $5 \times 10^{-4}$ .

MATLAB R2008a version has been used as software tool for simulations. Wavelength dependent core and cladding indices have been calculated using Sellmeier equation and the surrounding refractive index is taken as 1.0.

$$n^2(\lambda) = 1 + \sum_{i=1}^M \frac{A_i \lambda^2}{\lambda_i^2 - \lambda^2} \tag{3}$$

**Table 1.** Sellmeier Coefficients for Core and cladding composition of SMF-28 fiber

	SiO <sub>2</sub> (%)	GeO <sub>2</sub> (%)	A <sub>1</sub>	λ <sub>1</sub>	A <sub>2</sub>	λ <sub>2</sub>	A <sub>3</sub>	λ <sub>3</sub>
Cladding	100	0	0.6961663	0.0684043	0.4079426	0.1162414	0.8974994	9.896161
Core	96.9	3.1	0.7028554	0.0727723	0.4146307	0.1143085	0.8974540	9.896161

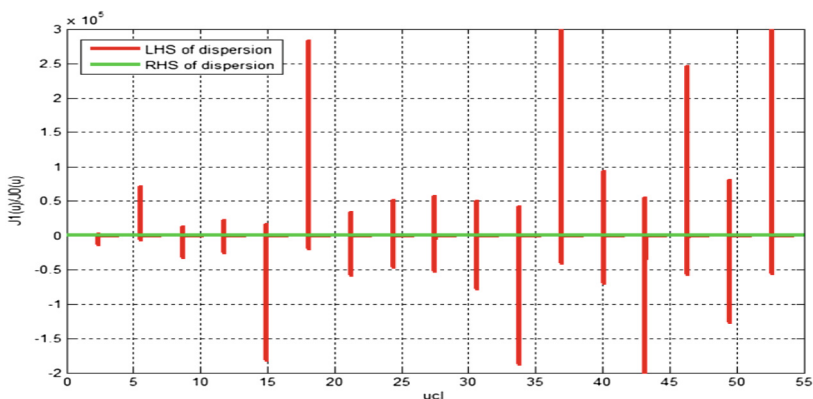
### 3.1 Core and Cladding Modes Effective Indices

The effective refractive indices of core and cladding must first be calculated to predict the resonance wavelengths in transmission spectrum of LPFGs. It helps in choosing a particular grating period of LPFG sensor for any application. Interactions between the core and cladding modes at core-cladding boundary and between cladding and surrounding at cladding-surrounding boundary can be solved using Eigen mode/dispersion equation [13].

The propagation is simplified by assuming linearly polarized (LP) modes. Radial power distribution in the core assuming unity power transmitted by the fundamental LP mode is given by-

$$E = \left( \frac{k_0(w)}{J_0(w)} \right) J_0(u) \tag{4}$$

where  $J_0$  and  $k_0$  are Bessel's and modified Bessel functions of first kind, zero order respectively.



**Fig. 1.** Cladding mode effective refractive indices at wavelength of 1.3 μm

Eigen value in core  $u$  is related to propagation constant  $\beta$  as-

$$u^2 = n_1^2 k^2 - \beta^2 \tag{5}$$

Normalized frequency  $V$  is related to waveguide parameters as-  $u^2 + w^2 = V^2$ . Matching the tangential components of the electric field ( $E_\phi, E_z$ ) and magnetic field ( $H_\phi, H_z$ ) at core cladding interface gives characteristic equation given below which can be solved for Eigen values  $u$  &  $w$ .

For weakly guided approximation, solution of dispersion equation yields very accurate values of  $u, w$  and  $\beta$  [13].

$$u \left( \frac{J_1(u)}{J_0(u)} \right) = w \left( \frac{k_1(w)}{k_0(w)} \right) \tag{6}$$

Evaluation of core and cladding modes effective refractive indices have been accomplished by finding value of waveguide parameter  $u$  at which left and right side of dispersion Eq. 5 intersect.

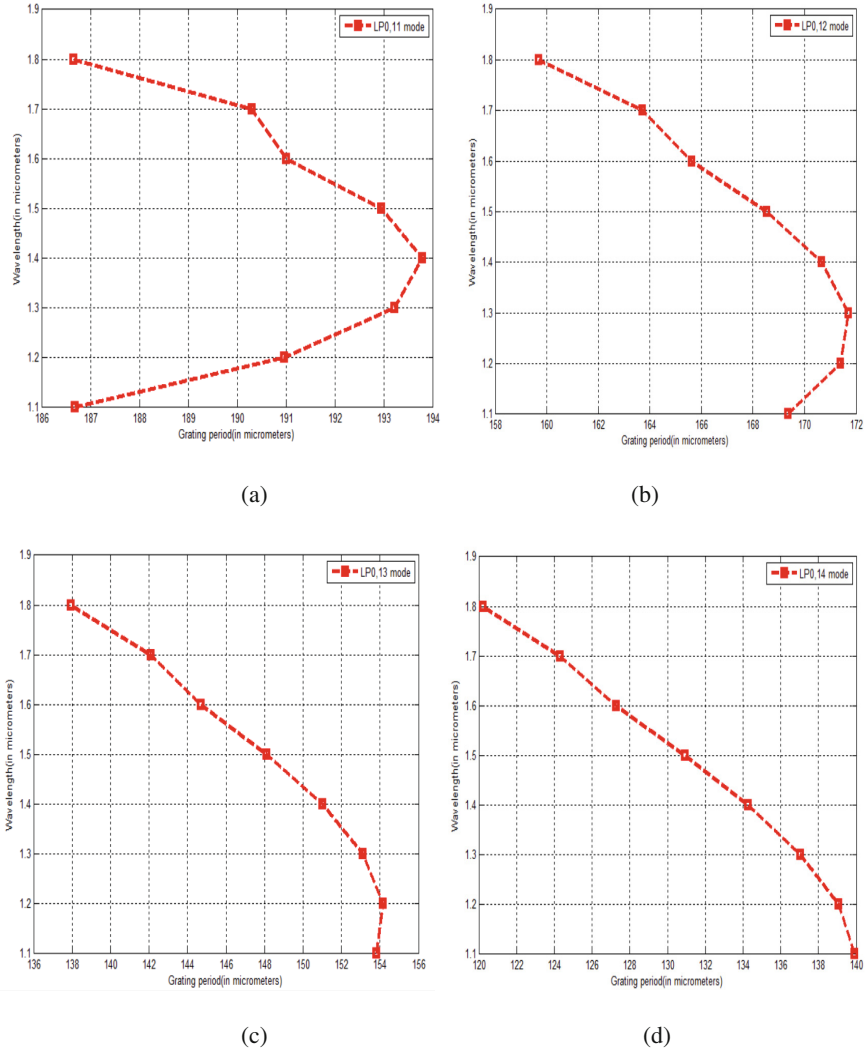
Higher dimensions of cladding results in satisfaction of characteristic equation for a number of  $u$  values as indicated in Fig. 1. Using Eq. 5, effective refractive indices of first 14 cladding modes have been calculated. Refractive indices of higher order cladding modes  $LP_{11}$  to  $LP_{14}$  are given in Table 1.

**Table 2.** Effective refractive indices of  $LP_{11}$  to  $LP_{14}$  cladding modes

Wavelength ( $\mu\text{m}$ )	$n_{\text{effcl\_11}}$	$n_{\text{effcl\_12}}$	$n_{\text{effcl\_13}}$	$n_{\text{effcl\_14}}$
1.1	1.4461076	1.4455052	1.4448490	1.4441390
1.2	1.4444162	1.4436985	1.4429166	1.4420704
1.3	1.4425721	1.4417287	1.4408097	1.4398151
1.4	1.4407757	1.4397962	1.4387289	1.4375731
1.5	1.4388260	1.4377000	1.4364729	1.4351446
1.6	1.4368232	1.4355402	1.4341419	1.4326280
1.7	1.4347670	1.4333164	1.4317353	1.4300233
1.8	1.4325566	1.4309278	1.4291521	1.4272291

### 3.2 Phase Matching Curves

Sensitivity of LPPGs greatly affected by the choice of grating period. Grating periods at various wavelengths have been calculated. Figure 2(a)–(d) depict phase matching curves for  $LP_{11}$ ,  $LP_{12}$ ,  $LP_{13}$  and  $LP_{14}$  cladding modes and turn around point for these modes appears to occur approximately at 193, 171, 154 and 140  $\mu\text{m}$  respectively.



**Fig. 2.** Phase matching curves of (a) LP<sub>11</sub> mode (b) LP<sub>12</sub> mode (c) LP<sub>13</sub> mode (d) LP<sub>14</sub> mode

### 3.3 Sensitivity at TAP in LPFGs

Region of ultrahigh sensitivity for LPFGs can be theoretically demonstrated by magnitude of the sensitivity factor. At turn around points, sensitivity factor exhibits highest values.

In order to compute the sensitivity factor given by Eq. (7) an LPFG, relationship between  $\lambda$  and  $\Lambda$  for  $m = 1$  to 14 cladding modes is calculated.

$$\gamma = \frac{\frac{d\lambda_{res}}{d\Lambda}}{n_{eff}^{co} - n_{eff}^{cl,m}} \quad (7)$$

Highlighted value of sensitivity factor magnitude corresponds to turn around points on phase matching curves for LP<sub>11</sub> to LP<sub>14</sub> modes. As highlighted in Table 2, sensitivity factor comes out to be highest at/near turn around points (Table 3).

**Table 3.** Sensitivity factor for coupling of LP<sub>11</sub> to LP<sub>14</sub> cladding modes

Wavelength (in $\mu\text{m}$ )	Sensitivity factor (LP <sub>11</sub> )	Sensitivity factor (LP <sub>12</sub> )	Sensitivity factor (LP <sub>13</sub> )	Sensitivity factor (LP <sub>14</sub> )
1.1–1.2	3.9584	7.6038	<b>40.1620</b>	<b>-14.535</b>
1.2–1.3	7.0474	<b>46.2442</b>	-12.1475	-5.8058
1.3–1.4	<b>26.3850</b>	-12.6013	-5.5850	-3.7754
1.4–1.5	-16.4900	-5.7680	-3.7335	-2.8847
1.5–1.6	-6.6045	-3.8680	-2.8805	-2.3747
1.6–1.7	-17.1036	-5.4079	-3.4715	-2.6801
1.7–1.8	-3.0679	-2.3864	-2.0229	-1.7985

## 4 Conclusion

Characterization of LPFGs require calculations of wavelength dependent refractive indices, mode effective refractive indices and grating periods. An analytical solution has been presented in this paper for calculation of parameters involved in the characterization of gratings in standard SMF-28 fiber. Sensitivity factor is found to be highest for LP<sub>12</sub> cladding mode. Ultra high sensitivity of long period grating sensors can be fully utilized if parameters are optimized to operate these gratings at turn around points.

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